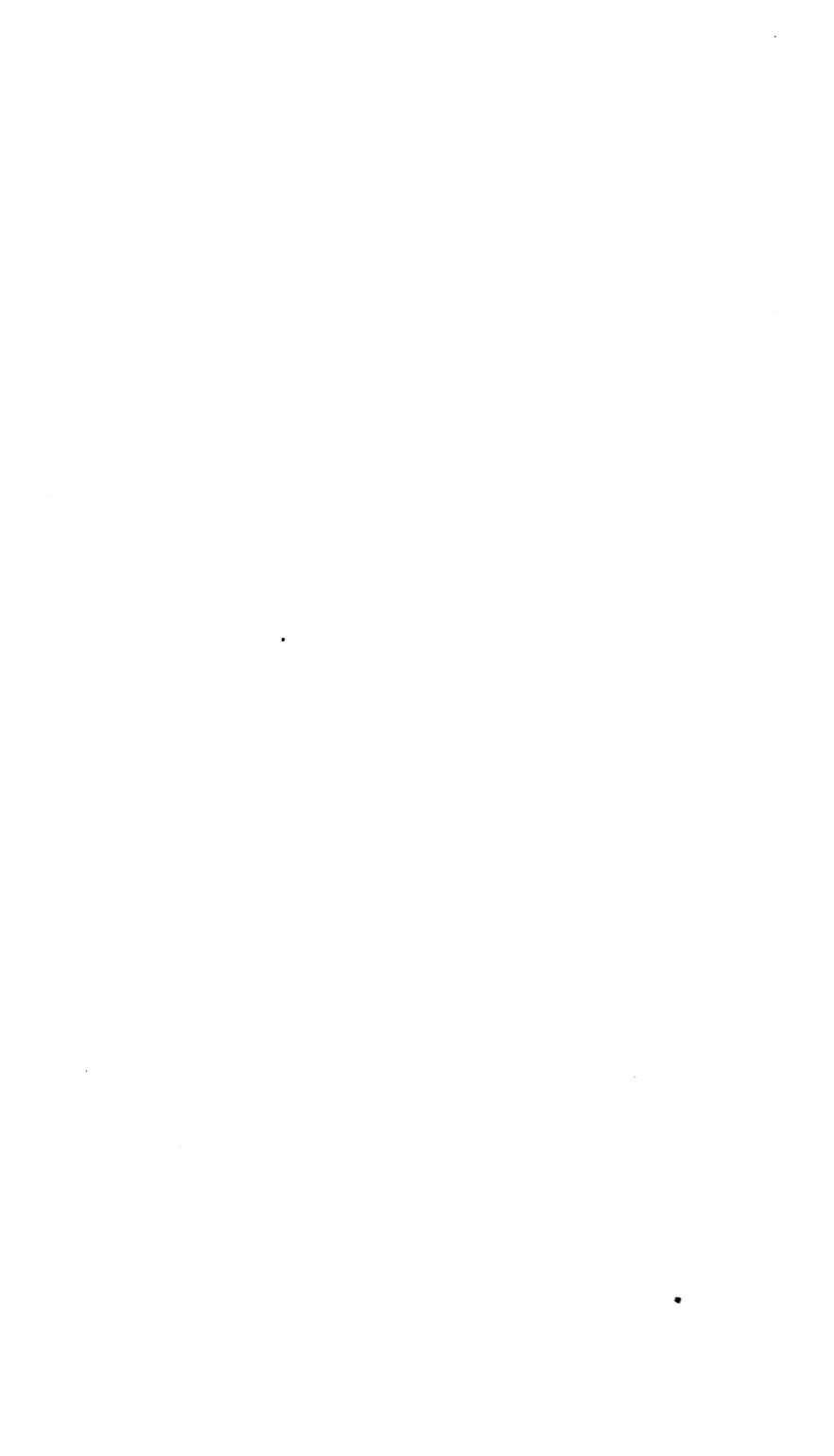




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Trinity College. Tor.



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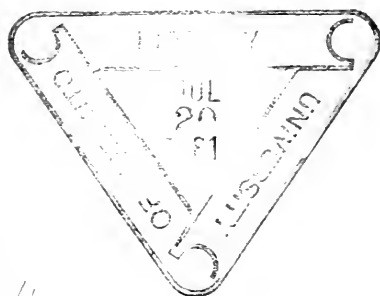
INSTITUTE OF ACTUARIES.

"I hold every man a debtor to his profession, from the which as men of course do seek to receive countenance and profit, so ought they of duty to endeavour themselves by way of amends to be a help and ornament thereunto."—BACON.

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INSTITUTE OF ACTUARIES.

On the Necessity for a Universal Notation. Report presented to the International Congress of Actuaries held at Brussels, September 1895, by A. BÉGAULT, Actuary of the Compagnie Belge d'Assurances générales sur la Vie, Brussels. Translated from the French by G. KING., V.P.I.A.

[Read before the Institute, 30 March 1896.]

THE favour with which the convening of the present Congress has been received by actuaries of all nations, and the very questions submitted to it by the members, fully show that everywhere the same desire has sprung up, namely that actuaries should know each other personally, so that later on there may be more effective inter-communication of ideas by the interchange of the writings which are periodically published. The question of language is a great obstacle, which it is in our interest to reduce to a minimum. A foreign tongue becomes easier to us, when the technical symbols employed, assuming a form which is familiar to us, indicate the drift of the paragraph. On the other hand, when the same symbol represents, according to the author using it, diametrically opposite ideas, the passage is made more difficult to understand, and the formulas, instead of being an index to its meaning, become a source of error.

In all sciences of recent development—electricity for instance—the necessity for a uniform notation has made itself felt. The

Electrical Congress of 1881 fixed electrical units, and succeeding Congresses found that their labours had thereby been greatly lightened.

The science of the actuary is more than any other international in its scope. Everywhere the same questions crop up, whether we are conducting scientifically financial operations in their infinite variety, or whether we are engaged in the serious problem, which is met with in every country, in connection with provision for old age and assurance against sickness. This department of the social question can be dealt with, with safety and equity, only by the aid of actuarial science. It must be said, and repeated with emphasis, that every legislative effort which goes beyond the rigorous laws laid down by mathematics, bears within itself the germ of decay which surely results in destruction. Equity will be absent, and the apparent advantages secured by the earlier beneficiaries will be certainly and dearly paid for by the more deserving, those, namely, who are called upon to make sacrifices, without a guarantee, other than honeyed words, for the carrying out of the promises made to them.

The questions of invalidity and sickness will give rise in different countries to numberless researches. Statistics will be collected, and tables of every kind will be calculated. How then will it be possible to compare the work done in one country with that done in another, if we are not first in accord upon the definition of the subjects with which we are dealing? What is "invalidity"? What is meant by "a day's sickness"?

In the report presented by M. Léon Marie in the name of the Statistical and Financial Commission of Friendly Societies, we see expressed the same idea in connection with a comparison of the sickness tables of different countries. He therein says:

"Sickness of Soldiers: Here we have the only statistics of 'sickness where the meaning of the word 'sickness' is clearly defined. That which in the army, in all countries of the world, is meant by 'a day's sickness' is a day of incapacity for duty caused by a pathological state," and further on:

"The definition of 'sickness' of the soldier (incapacity for duty) being distinctly the same in every country, it results that the statistics of different armies are closely comparable. (Report of Dr. Bertillon)."

It is no more our intention, than it is in our power, to force a notation on the actuaries of different countries; but if the

actuaries come to understand that the notation which we prefer is simple, rational, and complete, it is to be hoped that they will employ it in preference to any other. Once the movement started, those authors who wish to be read, will certainly follow it; and once more free discussion will have achieved that which it is impossible to obtain by pressure. We therefore propose to examine

- (1) The notations in use in different countries.
- (2) Whether we can find one among them which satisfies all the conditions of a notation universally acceptable.

THE PRINCIPAL NOTATIONS IN USE IN DIFFERENT COUNTRIES.

To assist in the examination of this question, the reader is begged to refer to the synoptical table in the appendix. It shows:

- (1) The notation employed by English and American actuaries. (The Institute of Actuaries and the Actuarial Society of America.)*
- (2) The notations used by the French actuaries (Dormoy, and the Committee of four Life Assurance Companies).
- (3) The notations employed in Germany (Karup, senior).
- (4) The notations employed in Holland (C. Landré).

The first two groups have many points in common, as also have the second two groups. That evidently follows from the respective characters of the languages.

A suggestive symbol has always been sought, and the initial of the word has naturally occurred to the mind. Hence the differences arise from the differences in the words employed in languages of German origin, and those of Latin origin. The terminology of English actuarial science is almost entirely of Latin origin. Nevertheless, this rule of initials can never

* The commutation system employed in America is that of Farr. That is to say that the American N_x corresponds to the English N_{x-1} . Moreover, the Actuarial Society of America has adopted the following symbols:

$$k_x = \frac{d_x}{l_{x+1}} = \frac{C_x}{D_{x+1}}$$

$$u_x = \frac{l_x}{l_{x+1}} (1+i) = \frac{D_x}{D_{x+1}}$$

S = sum assured.

K = cost of the assurance = $q(S - V)$.

Π = the single premium (as also does A).

be absolute; because, in view of the great number of terms employed, duplication would be unavoidable. Hence, for the less frequent benefits, letters have been adopted without reference to the benefits which they represent. This also explains the impossibility of selecting one part of a system of notation from one scheme and another part from another. The letters employed in the first would most likely be found in the second, and with other meanings. It is therefore absolutely necessary, unless we should decide on creating an entirely new system, to adopt in its entirety that system which is the most perfect; it being understood that there will be imported into it later on, such additions as may be rendered necessary by the development of actuarial science.

The systems in force are based upon the following ideas :

- (1) The use of a suggestive symbol to indicate the function in question.

England and the United States	l	d	a	A	V
France	l, v	$-, d$	X, a	P, π	V, G
Germany	λ	τ	1R	P	\mathfrak{R}
Holland	l	d	r	C	V

- (2) The use of suffixes placed below, and to the right of the principal symbol, to indicate the ages of the lives in question.

England and the United States	l_x	a_{xy}	A_{xyz}
France	l_x, v_x	X_{xy}, a_{xy}	P_{xyz}, Π_{xyz}
Germany	λ_a	$R_{a, b}$	$P_{a, b, c}$
Holland	l_a	$r_{a, b}$	$C_{a, b, c}$

- (3) The use of supplementary indices to the right or to the left, to indicate whether the benefit is temporary or deferred.

England and the United States	${}_nA$	${}_nA$	${}_nA$
France	${}_nX, a^{(n)}$	${}_nP, -$	X^n, a^n
Germany	${}_nR$	${}_nP$	${}_nR$
Holland	${}_nR$	$-$	$-$

- (4) The indication of fractional parts of a year in the benefit. When the number of these becomes infinite, the benefit is said to be continuous.

England and the United States	$a^{(m)}$	\bar{a}
France	$\frac{1}{m}X, -$	$\bar{X}, -$
Germany	$\frac{m}{m}R$	$-$
Holland	$R^{\frac{n}{n}}$	\bar{R}

- (5) Lastly in the English system alone, we meet with special signs to indicate the one ($^{\circ}$) that the benefit is complete (complete expectation of life, annuities payable to the moment of death), the other (\wedge) that in the case of a survivorship annuity, the epoch of the first payment becoming due depends on the date of death of the life tenant, and not on the date of the initiation of the transaction. These benefits are represented thus.

$$\bar{e}, a, \bar{a}.$$

Thus, each of the different systems solves in a manner, more or less complete, the problem of notation.

The systems in use by German and Dutch actuaries are found, however, in certain respects to be inferior to the others. The German actuaries have varied the principal symbol according to the benefit to be represented. Thus ${}_nP_x$, ${}^n\mathbb{P}_x$, ${}^n\mathfrak{P}_x$ represent the single premium for a Temporary Assurance, an Endowment, and an Endowment Assurance for n years, respectively. Nothing in the symbols P, \mathbb{P} or \mathfrak{P} indicates that difference to the reader. We can thus say that the German notation is not suggestive. Compare it with the English Notation for the same benefits.

$$A_{x:n}^1, A_{x:n}^{\frac{1}{x}}, A_{x:n}^-.$$

We see here that A represents the single premium for the assurance, and the index indicates to us the nature of the benefit. ${}_{x:n}^-$ means payable at the termination of the status ${}_{x:n}^-$, that is to say by the death of x or at the expiration of the fixed term of n years. That is the Endowment Assurance. ${}_{x:n}^{\frac{1}{x}}$ means payable at the termination of the status ${}_{x:n}^-$ provided that the life drops first, that is to say, dies before the expiration of n years. That is a Temporary Assurance. ${}_{x:n}^1$ means payable at the termination of the status, if the n years run out before the death of x , that is to

say x must still be alive after the n years. This is the Endowment. In fact very few of the symbols adopted by the German actuaries agree with those of the other systems.

The Dutch system set forth in the work of M. M. G. SNOER, (*Lijfrente en Levensverzekering*) and more recently by M. CORNEILLE L. LANDRÉ (*Wiskundige Hoofdstukken voor levensverzekering*), while it is derived principally from the German notation,* includes a certain number of English symbols such as i, v, l, d, V ; but it does not give symbols for several important groups of benefits, such as deferred benefits, survivorship benefits, and endowment assurances. It does not even mention benefits depending on several lives where the order of survivorship is fixed in advance. None of the systems so far examined include this.

The system of Dormoy bears a certain analogy to the English system, but it is open to the objection, especially from the French point of view, that it does not respect the law of initials. In fact, Q, X, Y, do not suggest to the mind in any way the words Endowment, Annuity, and Variable Annuity. Lastly, the probabilities of life, of death, and of contingent survivorship, whether simple or compound, and Endowment Assurances, are not specially represented.

The system of the Committee of the four French Life Assurance Companies resembles most, without question, the system of the Institute of Actuaries. The actuaries who compiled it took for their model the system adopted in England, but introduced certain modifications to which we shall refer again later on.

We shall examine successively these two systems in so far as they relate to the principal symbol, the various indications of age, of duration, and of the conditions of the benefit. We shall make note of the similarities, and of the differences, and we shall see whether the modifications adopted by the French actuaries have in reality simplified or improved the English system.

(1) The principal symbols (pages 2 and 3 of the appendix) i, d, p, q, a , have the same meaning, while l, μ, A, P, V of the English system are replaced in the French system by v, χ, Π, π, G .

(2) Special symbols (pages 4 *et seq* of the appendix). The indication of the group of lives on which the benefit depends is given in the two systems in the same way.

$px, a_{xyz}, \&c.$

* One portion of the Dutch notation has been borrowed from the work of A. Zillmer, *Mathematische Rechnungen*, 1st Edition.

It is not so when this group becomes a status* according to the phrase in use in England, that is to say when the benefit has reference not merely to the ages of one or several lives, but to other conditions, such as the continued existence or the disappearance of the whole group, or a determined order of death in the lives involved, &c. Thus for example

(a) $p_{\overline{xy}}$ represents in both notations the probability that the survivor, that is, at least one of the lives shall be in existence at the end of . . . years.

$q_{\overline{xy}}$ is used by English actuaries to signify the death of the survivor, and by the French, the probability of the dissolution of the group by the death of one or of both. English actuaries have given to the horizontal bar above the suffix the meaning of last survivor, as have also the French in the symbol $a_{\overline{xy}}$ representing an annuity payable until the last death. Further, English actuaries write

$$1 - p_{xy} = q_{xy}$$

$$1 - p_{\overline{xy}} = q_{\overline{xy}}$$

analogous to $1 - p_x = q_x$

equations which do not exist in the French system.

(b) Survivorship benefits (annuities for instance) are indicated
in the English system $a_{y|x}$;
in the French system $\Pi_{\overline{x}}^y$

the one as simple and as clear as the other.

(3) Deferred and temporary benefits.—The limitation of the benefit for n years is indicated

in the English system by ${}_na$, or $a_{x:\overline{n}}$,

in the French system by a^n .

and the postponement of the benefit

in the English system by ${}_na$,

and in the French system by a^n ,

notations equally satisfactory from the point of view of clearness. French actuaries claim for their notation the advantage of avoiding the index on the left of the principal

* The word "status" has a wider meaning in England than that given to it here. It includes a single life on which a benefit depends. Thus, in a_x , x is a status.—G. K.

symbol. We do not see anything objectionable in this index, nor even that it presents any difficulty to the reader. On the contrary the vertical bar is in conformity with the English principle of the separation of the statuses.

${}_na$ means that n years will elapse before entering on the annuity.

$|_na$ signifies that 0 years will elapse (the benefit is, therefore, immediate) and that it will run for n years.

Moreover, the index on the left is only avoided for the moment; and whenever it is desired to indicate in the French system that the annuities are to be payable half-yearly, quarterly, or n times a year, it will be necessary to place the new index on the left of, or above, the principal symbol.

Lastly, a^n is an algebraical symbol, which by universal consent means the n^{th} power of a .

(4) *Assurances*.—In this department the French system is entirely incomplete, the different benefits having no individual symbols. It has been thought sufficient to take Π and π for the single and the annual premiums respectively for all assurances payable only on the death of the Life Assured, and $\Pi^{(n)}$ and $\pi^{(n)}$ for all benefits, assurances or annuities, which last for only a limited number of years.

Evidently this branch of the question was left on one side. In preparing “*l’Aide-mémoire*” the Committee were content to supply only a guide for the clerks charged with the duty of calculating rates by given formulas. From this narrow standpoint it was not found inconvenient that $\Pi_x^{(n)}$ should represent the single premium for a temporary assurance, an endowment assurance, or an insurance for a term-certain, a temporary life annuity or a deferred life annuity, or an endowment, a compound assurance, &c. The heading of the page made all mistake impossible; and from this purely utilitarian point of view, the principle is not open to criticism. Let us say, however, that in all the symbols, where the two systems, English and French, are comparable, there are only trivial differences, and that it is impossible to say whether one is or is not more complex than the other. It is, therefore, to be presumed that, when the Institute of French Actuaries resumes the work begun by the Committee of the Four Companies, with a view to extend to assurances the general principles adopted for annuities, it will be difficult, nay even impossible, to simplify in the slightest degree the English system. Let us see, therefore, whether this

system can be applied to the conditions which French-speaking peoples have a right to require.

2. EXAMINATION FROM THE FRENCH STANDPOINT OF THE SYSTEM OF NOTATION OF THE ENGLISH INSTITUTE OF ACTUARIES.

As we showed at the commencement of this enquiry, the greatness of the number of symbols required in the science of life contingencies does not permit of borrowing one part of a scheme of notation from one system, and another part from another. On the other hand, it would not be reasonable to advise actuaries to adopt a system entirely out of harmony with the requirements which would be laid down, were an entirely new system to be devised.

Let us therefore examine the English symbols which the French actuaries have rejected, and let us see if there is any real impossibility in adopting them.

1. The instantaneous death rate $= \mu$; μ is the limit of m , and m at once suggests the word "mortality" (French, mortalité).

2. Single Premium (present value) for an assurance $= A$; in French likewise A is the initial of "assurance" (French, assurance).

3. Annual premium for a benefit $= P$. The letter P at once makes us think of "premium" (French, prime).

4. Value of a Policy (reserve) $= V$. Once again the initial V is that of "value" (French, valeur).

We must call special attention to this expression which explains perfectly the reserve.

In theory, a constant equation must exist between the assets and the liabilities; but the equality which exists at the commencement, between the premiums to be received and the corresponding assurance, disappears. The policy, the value of which is zero before the payment of the first premium, and 1 (if unity is the sum assured) at the moment of death, passes through a series of intermediate values. The expression "value of the policy" is therefore a perfectly happy one. The word "reserve" has already a more commercial meaning, because while it is true that we must have at each valuation

$$\Sigma \text{ Reserves} = \Sigma \text{ value of policies,}$$

we may say that, taken individually, it is not so, as recent policies for example may have negative values.

Number living $= l$. This from the French standpoint is the only letter for which nothing can be said. The letter v suggests the word "vivant"; and from this point of view it is not easy to justify its rejection. We should not even see much objection to admitting it, if v had not already with English actuaries its own meaning $= \frac{1}{1+i}$. Now the adoption of v for $\frac{1}{1+i}$, which gets rid of negative exponents in formulas, and much simplifies the operation of writing, seems to us very desirable, and the more so because v has a meaning easy to remember, namely the present value (French, valeur) of one franc payable in a year.

If, therefore, we find that in French v and consequently $v^2, v^3, \dots v^n$ are useful symbols to represent the present value of one franc payable in 1, 2, 3, $\dots n$ years, we should abandon the meaning of v assigned to it by French actuaries, and take away from it, its signification as the number living.

Moreover the rule has been but little observed hitherto by French actuaries, for we find $l_x, y, f(x)$ used by different authors.

Surely it is not possible that this single letter should be an insurmountable obstacle to the unification of the systems of notation. We have too much confidence in the practical good sense of our colleagues, to believe that this single objection should outweigh with them the gain that would follow order evolved from the chaos which reigns supreme to-day.

CHANGES INTRODUCED BY THE COMMITTEE OF FRENCH COMPANIES.

1. Adjustments for selection. The mortality tables H^M and $H^{M(5)}$ are the mathematical embodiment of the influence of medical selection during the first five years of whole-life assurance.

The annuity tables just published by the Institute of Actuaries are the embodiment of the same influence in so far as it concerns annuity nominees (Joint Life Annuity Tables for Lives of both sexes, and also Single Life Annuity Tables, deduced from the mortality experience of Government Life annuitants). In truth the law of mortality varies with the age at entry, but mortality tables constructed on this principle are based upon observations too restricted, and are therefore untrustworthy. Moreover, to employ them practically is quite impossible. It has, however, been attempted to take account of

the influence of selection specially manifested in annuity nominees above 65 years of age. The ages $x, y, z \dots$ to meet the change in the condition of the lives observed have been represented by the Greek letters $\xi, \eta, \zeta \dots$.*

2. Valuation premiums, and commercial premiums. The loading on the net premium for the different life benefits includes two very distinct portions. The first is destined to meet the outgo which will be incurred by the Company as long as the contract lasts, and which recurs each year (cost of conducting the business and provision for bonus). This may be called "cost of working" (*frais de gestion*). The second is to provide for the special expenses incurred in acquiring new business (remuneration to agents) and the cost of collecting the premium throughout the duration of the contract. This has been called "procuration expenses." These two reasonable divisions have produced two distinct premiums, one taking account of "cost of working", and called Valuation premium, the other bringing in also the "procuration expenses", and called Commercial premium. This last is the premium given in the prospectus. The necessity for the Valuation premium has already been felt, because the English Companies in their valuations introduce an adjustment (whole-life loading) for those contracts under which the premiums cease before the sum assured becomes payable.

Following out these ideas we could designate by A' and P' the Single and Annual Valuation Premiums, and by A'' and P'' the Single and Annual Commercial or Office Premiums corresponding to the net premiums A and P .

COMMUTATION SYMBOLS.

$$D_x = v^x l_x = (1+i)^{-x} l_x$$

has everywhere the same signification.

N_x has among English actuaries† that value which divided by D_x gives the life annuity a_x .

In America and France there has been adopted for N_x the value which divided by D_x gives for its quotient $1 + a_x$, that is the annuity-due. This difference is of small importance, and to take

* On the English method of marking Selection, see Translator's Note on p. 16.—G. K.

† In England the notation has also been adopted $\mathfrak{N}_x = D_x + D_{x-1} + \&c.$ —G. K.

note of it is sufficient. The French and American notations take account principally of the annuity as an auxiliary quantity, to be used as divisor in the calculation of the annual premiums for life benefits. From a didactic point of view, and looking upon annuities as a class of benefits distinct in themselves, like annuities-certain, no valid reason can be urged in favour of the adoption for N_x of the value $D_x + D_{x+1} + \dots + D_w$; annuities, unless there be a special stipulation to the contrary, being always understood as making the first payment fall due at the end of one year. But this is of little consequence. It is sufficient to pay careful attention to their construction, in using commutation columns constructed on one or on the other system.

$$S_x = \sum_x N_x.$$

This being a function of N , will have, following what we have just said, different values according to the method of constructing N .

$$C_x = v^{x+1} d_x$$

but the French actuaries have assigned to it the value

$$C_x = (1+i)^{-(x+\frac{1}{2})} d_x.$$

This modification is the mathematical embodiment of the hypothesis that all deaths take place in the middle of the year. We need not remark that this hypothesis is but a modification of the English hypothesis* that all deaths take place at the end of the year. It must be said, however, in defence of the English hypothesis, that it excellently answers the theoretical purposes of actuarial science, and that supplementary calculations enable us by it to approximate as closely as may be desired to the real facts, without even the necessity of assuming that deaths are uniformly distributed between two consecutive ages.

The only objection that can be made to the French value of C_x , from the strictly mathematical point of view, is in its application to a table of mortality other than that from which it first had its rise, a table where the values of $l_x, l_{x+1}, l_{x+2} \dots$ &c. are replaced by $l_{x+\frac{1}{2}}, l_{x+\frac{3}{2}} \dots$ &c. In fine, if $C_x = (1+i)^{-(x+\frac{1}{2})} d_x$ is a practical notation, it cannot be accepted from the stand-point of rigorous mathematical reasoning.

$$M_x = \sum_x C_x$$

* The English hypothesis does not refer to the time of death, but is based upon the original practice of paying claims six months after death, that is, approximately at the end of the year of death.—G. K.

This being a function of C_x will therefore have for value in the English notation

$$(1+i)^{-(x+1)}d_x + (1+i)^{-(x+2)}d_{x+1} + \&c.$$

and in the French notation

$$(1+i)^{\frac{1}{2}}[(1+i)^{-(x+1)}d_x + (1+i)^{-(x+2)}d_{x+1} + \dots \&c.]$$

$$R_x = \Sigma M_x$$

This will be derived naturally from the value of the function represented by M .

CONCLUSIONS.

The notation adopted by the Institute of Actuaries, the Faculty of Actuaries in Scotland, and the Actuarial Society of America, is rational, and for this reason does not require any effort of memory. It is complete, because it includes all cases met with in practice; and the principles upon which it is based, allow of its extension to the most difficult combinations. All other notations have been inspired by it, and none of them have simplified it. Moreover, it has been employed for many years by numberless English authors, whose works form a veritable monument in science.

It is not this last reason which induces us to give preference to the notation of the Institute of Actuaries, but with the advantages which it possesses over all other systems actually in use, and seeing that the desire has been expressed by members of the profession not to introduce an entirely new notation unless such were found to be absolutely necessary, this circumstance is certainly one argument the more in favour of adopting the English system.

We therefore submit to the Congress the following two resolutions:—

- (1) That the notation of the Institute of Actuaries, be adopted by Actuaries of all countries.
- (2) That such modifications in it as may be found necessary in future, be considered by future International Congresses.

APPENDIX.

COMMUTATION SYMBOLS.

FUNCTION ENGLISH NOTATION (Age x)	Institute of Actuaries (England)	Actuarial Society (America)	Committee of Four Companies (France)	Dormoy (France)	Karup, Sear, (Germany)	Landré (Holland)
$\left\{ \begin{array}{l} v^x l_x \\ (1+i)^{\omega-x} l_x \end{array} \right.$	D_x	D_x	D_x	D_a	D_a	λ_a
$\left\{ \begin{array}{l} v^x l_x + v^{x+2} l_{x+2} + \dots + v^{\omega} l_{\omega} \\ v^x l_x + v^{x+1} l_{x+1} + \dots + v^{\omega} l_{\omega} \end{array} \right.$	N_x^*	N_x	N_x	N_a	G_a	$\Sigma \lambda_a$
$\left\{ \begin{array}{l} (1+i)^{\omega-x} l_x + (1+i)^{\omega-(x+1)} l_{x+1} + \dots + l_{\omega} \\ v^{x+1} l_{x+1} + 2v^{x+2} l_{x+2} + \dots + (\omega-x)v^{\omega} l_{\omega} \\ v^x l_x + 2v^{x+1} l_{x+1} + \dots + (\omega-x+1)v^{\omega} l_{\omega} \end{array} \right.$	S_x	S_x	S_x	S_a	$\Sigma (D_a)$	$\Sigma \Sigma \lambda_a$
$\left\{ \begin{array}{l} v^{x+1} d_x \\ v^{x+\frac{1}{2}} d_x \end{array} \right.$	C_x	C_x	C_x	C_a	τ_{a+1}	τ_{a+1}
$\left\{ \begin{array}{l} v^{x+1} d_x + v^{x+2} d_{x+1} + \dots + v^{\omega+1} d_{\omega} \\ v^{x+\frac{1}{2}} d_x + v^{x+\frac{3}{2}} d_{x+1} + \dots + v^{\omega+\frac{1}{2}} d_{\omega} \end{array} \right.$	M_x	M_x	M_x	M_a	$\Sigma \tau_{a+1}$	$\Sigma \Sigma \tau_{a+1}$
$\left\{ \begin{array}{l} v^{x+1} d_x + 2v^{x+2} d_{x+1} + \dots + (\omega-x+1)v^{\omega+1} d_{\omega} \\ v^{x+\frac{1}{2}} d_x + 2v^{x+\frac{3}{2}} d_{x+1} + \dots + (\omega-x+1)v^{\omega+\frac{1}{2}} d_{\omega} \end{array} \right.$	R_x	R_x	R_x	R_a		

* In England the symbol \mathbf{M}_x has been adopted for $D_x + D_{x-1} + \dots$, &c.—G. K.

SYSTEMS OF NOTATION.

	Dormoy	Committee of Four French Companies	Institute of Actuaries	Karup, Senr.	C. Landré
GENERAL SYMBOLS.					
I. Interest.					
Annual Interest per Monetary Unit	t	i	i	z	i
Discounted Value $\frac{1}{1+i}$ of 1 payable a year hence . . .	$(1+t)^{-1}$	$(1+i)^{-1}$	v	ρ	v
Nominal Rate of Interest convertible m times a year, when the effective rate is i , $m \{ (1+i)^{\frac{1}{m}} - 1 \}$			$j_{(m)}$		
$\text{Log}_e(1+i)$; Force of Discount	ρ	ι	δ		
Value of i discounted, $\frac{i}{1+i}$ $= 1-v=vi$		$\frac{i}{1+i}$	d		ρ
Present Value of an Annuity of n payments			a_n		
II. Mortality.					
Number Living	$l, f(x), y$	v	l	λ	l, l'
Number Dying		d	d	τ	d, t

	Dormoy	Committee of Four French Companies	Institute of Actuaries	Karup, Senr.	C. Landré
Population			L		
Probability of Life . . .		p	p		W
Probability of Death . .		q	q	w	w
Limiting Age of the Table	ω		ω		
Correction for Selection .		γ	*		
Age corrected for the In- fluence of Selection . . .		ξ, η, ζ	*		
Force of Mortality . . .	τ	z	μ		μ
Central Death Rate . . .			m		
Expectation of Life . . .			e		
Makeham's Formula . . .	$\frac{k}{a^x} g^{qx}$	$k s^x g^{cx}$	$k s^x g^{cx}$		$ck^x g^{qx}$
III. Benefits.					
Endowment	Q^n	P^n	${}_nE$	${}_n\mathfrak{P}$	
Annuity, Life Interest . .	X	a	a	1R	r
Annuity-due			$a, 1+a$	R	R
Single Premium for an Assurance	P	Π	A	P	C
Annual Premium	p	π	P	p	p
Policy Value, Reserve . .	V	G	V	\mathfrak{R}	V

* In England Dr. Sprague's way of marking selection has been officially adopted, viz.:—

$l_{[x]}$ = living aged x , just selected.

$l_{[x-1]+1}$ = living aged x , selected a year ago,

&c.

&c.

G. K.

	Dormoy	Committee of Four French Companies	Institute of Actuaries	Karup, Senr.	C. Landré
SPECIAL SYMBOLS.					
I. Designation of the Lives to which the Principal Symbol refers.					
Number Living	$l_x, f(x)$	v_x	l_x	λ_a	$l_a, \phi, f(x)$
Number of pairs existing. Two Joint Lives	$l_x \times l_y ;$ $f_x \times f_y$	v_{xy}	l_{xy}	$\lambda_a \times \lambda_b$	
Expectation of Life. One Life			e_x		
Annuity on One Life . .	X_x	a_x	a_x	1R_a	r_a
Joint Life Annuity ceasing on first death	$X_{xyz \dots}$	$a_{xyz \dots}$	$a_{xyz \dots}$	${}^1R_{a, b, c}$	$r_{a, b, c}$
Joint Life Assurance pay- able on first death	P_{xyz}	Π_{xyz}	A_{xyz}	$P_{a, b, c}$	$C_{a, b, c}$
Probability that at least one of three Joint Lives will fail within a year		q_{xyz}	q_{xyz}		
II. Indication of Increase in Age of all the Lives mentioned in the Suffix.					
Number living, aged $x+n$	f_{x+n}	v_{x+n}	$l_{x+n} : {}^n l_x$	λ_{a+n}	$l_{x+n} ;$ $f(x+n)$
Number of pairs of Lives existing, aged $x+n$ and $y+n$	$l_{x+n} \times l_{y+n}$ $f'(x+n)$ $\times f'(y+n)$	$v_{x+n} v_{y+n}$	${}^n l_{xy}$ also $l_{x+n : y+n}$	$\lambda_{a+n} \lambda_{b+n}$	
Probability that three Joint Lives, aged $x+n, y+n$, and $z+n$, will all live a year . .		$p_{x+n, y+n, z+n}$	${}^n p_{xyz}$		
Annuity on two Joint Lives, aged $x+n$ and $y+n$.	$X_{x+n : y+n}$	$a_{x+n : y+n}$	$a_{x+n : y+n}$ ${}^n a_{xy}$	${}^1R_{a+n, b+n}$	

	Dormoy	Committee of Four French Companies	Institute of Actuaries	Karup, Senr.	C. Landré
<p>III. Indication of the number of years over which the probability or benefit extends, as shown in the principal symbol.</p> <p>Probability that a Life aged x will live n years</p> <p>Probability that both the Lives x and y will live n years</p> <p>Annual Premium limited to n years</p> <p>Value after n years of a Policy on two Joint Lives .</p> <p>Endowment payable if two Joint Lives, x and y, both survive n years</p>		p_x^n p_{xy}^n $\pi^{(n)}$ G_n P_{xy}^n	${}_np_x$ ${}_np_{xy}$ ${}_n\mathbf{P}$ ${}_nV_{xy}$ ${}_nE_{xy}$	 ${}_n\mathbf{P}$ ${}_n\mathbf{P}[P_{a,b}]$ ${}_n\mathbf{P}[p_{a,b}]$ ${}_n\mathbf{P}_{a,b}$	
<p>IV. Indication of the Survivors of Lives.</p> <p>Probability that the Survivor of x and y will live n years</p> <p>Probability that the last Survivor of x, y, and z, will die within a year</p> <p>Annuity payable in full during Joint Lives and to Survivor of x and y . . .</p>	 ${}_nV_{xy}$ Q_{xy}^n $X_{\overline{ab}}$	 P_{xy}^n q_{xyz} a_{xy}^-	 ${}_n\overline{p}_{xy}$ $q_{\overline{xyz}}$ a_{xy}^-	 $R_a \ b \ (*)$ 1	

* General Remark.—In German Notation R represents the annuity-due. There may, therefore, not be complete analogy between the function in the fourth column and the others on the same line.

	Dermoy	Committee of Four French Companies	Institute of Actuaries	Karup, Senr. C. Landré
Annual Premium payable until the second death for an Assurance on the Survivor of two Lives	\overline{p}_{xy}	$\pi_{x y}$	\overline{P}_{xy}	
Annual Premium payable until the first death for an Assurance on the Survivor of two Lives		$\pi_{x y}$		
Probability that at least r Lives out of m Lives will sur- vive n years			$n\overline{p}_{x_1, 2, \dots, (m)}^r$	
Annuity on the last r Sur- vivors of m Lives			$a_{x_1, 2, \dots, (m)}^r$	
Annuity payable while exactly r Lives out of m Lives are in being, and ceasing on the death of one of the r Lives			$a_{x_1, 2, \dots, (m)}^r$	
Assurance on the last r Sur- vivors of m Lives			$A_{x_1, 2, \dots, (m)}^r$	
Annuity payable during the Life of the last Survivor of the three Lives abc , jointly with all the three Lives xyz .			$a_{xyz : \overline{abc}}$	
Annuity payable during the Life of the last Survivor of abc , jointly with the last Sur- vivor of xyz			$a_{xyz} \overline{abc}$	
7. Indication of Survivorship.				
Contingent Annuity to x after y	P	$\Pi_{\overline{y}}^x$	$a_{y x}$	$R_{a,b}$ e

	Dormoy	Committee of Four French Companies	Institute of Actuaries	Karup, Senr.	C. Landr
Annual Premium for a Con- tingent Annuity		π_y x	$Pa_{y x}$	$p_{a,b}$ e	
Contingent Annuity on the Status s after the failure of the Status s'			$a_{s s}$		
Contingent Assurance on the Survivor of x and y against z			$\left\{ \begin{array}{l} A_{x xy} \\ A_{z:xy} \end{array} \right.$		
Contingent Annuity on the last Survivor of abc , after the death of the last Survivor of xyz			$a_{xyz abc}$		
VI. Indication of the Order of Death.					
Probability that of xyz , x will die first within n years . .			$ nq_{xye}^1$		
Probability that x will die before y in the n th year . . .			$n-1 q_{xy}^1$		
Total probability that x will die before y			Q_{xy}^1		
Assurance payable on the death of w if he die fourth, the other deaths following the order shown in the suffix . . .			A_{wxyz}^4 321		
Compound Survivorship An- nuity to x , after the Survivor of y and z , z having died first			$\left\{ \begin{array}{l} a_{y2 x} \\ a_{yz x}^2 \end{array} \right.$		

	Dormoy	Committee of Four French Companies	Institute of Actuaries	Karup, Senr. C. Landré	
VII. Indication of the Order of Succession.					
Assurance payable on the death of the n th Life to be nominated in succession after the Life in possession			$\frac{A_{\overline{x(y)(z)}}}{A_{w(1)(2)(3) \dots (n)}}$		
VIII. Indication of a Limit- ation of time.					
Probability that x and y will both die within n years .		$q_{xy}^{(n)}$	${}_n q_{xy}^-$		
Temporary Annuity . . .	${}_n X_x$	$a_x^{(n)}$	${}_n a_x; a_{xn}^-$	${}_n R_a$	${}_n r_a$
Temporary Assurance . . .	${}_n P_x$		${}_n A_x; A_{xn}^1$	${}_n P_a$	${}_n C_a$
Single and Annual Premium for an Endowment	$Q_x^n; p$	$P_x^n; \pi_x^n$	$A_{xn}^1; P_{xn}^1$	${}_n \mathfrak{P}_a^1; {}_n \mathfrak{P}_a$	
Single and Annual Premium for an Endowment Assurance			$A_{xn}^-; P_{xn}$	${}_n \mathfrak{P}_a^-; {}_n \mathfrak{P}_a$	
Endowment Assurance on the Survivor of two Lives . .		$\Pi_{xy}^{(n)}$	$A_{xy}^- \bar{n}$		
Annual Premium for an Endowment Assurance for n years, the Premium being payable for n' years. . . .		$\pi_x^{(n \ n')}$	${}_n P_{xn}^-$		
Temporary Contingent An- nuity		$\Pi_y^{(n)} \bar{x}$	${}_n a_{yx}$	${}_n R_{a \ b}$	

	Dormoy	Committee of Four French Companies	Institute of Actuaries	Karup, Senr.	C. Landré
IX. Indication of a deferred term.					
Annuity deferred n years .	X_x^n	a_x^n	${}_n a_x$	${}_nR_a$	
Assurance deferred n years			${}_nA_x$	${}_nP_a$	
Annual Premium for a Deferred Annuity on the Survivor of two lives . . .			$P_n a_{xy}$		
Intercepted Annuity . .	${}_mX_x^n$	$a_x^{n(m)}$	${}_n{}_m a_x$	${}_m{}_nR_a$	
Deferred Contingent Assur- ance			${}_nA_{xy}^1$		
X. Indication of Sub-division of the year.					
Annuity payable m times a year	$\frac{1}{m}X_x$		$a_x^{(m)}$	${}_mR_a$	$\frac{n}{r^n a}$
Deferred Annuity payable m times a year on the Survivor of two lives	$\frac{1}{m}X_{xy}^n$		${}_n a_{xy}^{(m)}$		
Annual Premium for ditto			$P_n a_{xy}^{(m)}$		
Assurance payable $\frac{1}{2m}$ of a year after death			$A_x^{(m)}$		

Dermoy	Committee of Four French Companies	Institute of Actuaries	Karup, Senr. C. Landré
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XI. Indication of Continuity.

Continuous Annuity . . .	X_x	\bar{a}_x	\bar{R}_a
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Assurance payable at the
moment of death: Continuous
Assurance

 \bar{A}_x

Endowment Assurance pay-
able at the moment of death .

 $\bar{A}_{x:n}$ **XII. Indication that the
Benefit is to be complete.**

Complete Expectation of Life	\bar{e}_x	${}_m L_a$	e_a
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Deferred Complete Expect-
ation of Life

 ${}_n \bar{e}_x$

Complete, or Apportionable
Annuity

 \bar{a}_x **XIII. Indication that Pre-
miums are to be payable by
Instalments.**

Annual Premium payable by m instalments	$P^{(m)}$	$\bar{P}^{(m)}$
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Policy Value when the
Premium is payable by instal-
ments

 $V^{(m)}$

	Dormoy	Committee of Four French Companies	Institute of Actuaries	Karup, Seur.	C. Landré
XIV. Symbols for Annual Premiums for Fractional or Continuous Benefits.					
Annual Premium for an Assurance payable $\frac{1}{2m}$ of a year after death			$^{(m)}P$		
Policy Value of ditto . .			$^{(m)}V$		
Annual Premium for a Continuous Assurance			$^{(\infty)}P$		
Policy Value of ditto . .			$^{(\infty)}V$		
XV. Symbols for Variable Benefits.					
Annuity commencing at 1 and increasing by 1 each year	Y		(Ia)		
Varying Annuity	Y		(∇a)		
Assurance commencing at 1 and increasing by 1 each year.			(IA)		
Varying Assurance			(∇A)		
Temporary Increasing Annuity	Y		$(Ia)_{\overline{xn}}$		
Temporary Varying Annuity	Y		$(\nabla a)_{\overline{xn}}$		

	Dormoy	Committee of Four French Companies	Institute of Actuaries	Karup, Senr. C. Landré
Temporary Increasing As- surance.			$(IA)_{\bar{x} n}$	
Temporary Varying As- surance.			$(\nabla A)_{\bar{x} n}$	
Whole-Life Annuity In- creasing for n years . . .	Y		$(I_{\bar{n}} a)_x$	
Whole-Life Annuity Vary- ing during n years . . .	Y		$(\nabla_{\bar{n}} a)_x$	
Whole-Life Assurances In- creasing for n years. . . .			$(I_{\bar{n}} A)_x$	
Whole-Life Assurance Vary- ing for n years			$(\nabla_{\bar{n}} A)_x$	

The following is an abstract of the discussion which took place at the Congress on M. Bégault's Report:—

M. BÉGAULT, in submitting the Report said that it was not without misgiving that he had first taken up the subject. The further he had advanced in his studies, and the more he had become acquainted with the conscientious work done in different countries, the more hesitation he had felt in asking actuaries to surrender the notations familiar to them, in order to adopt that of the Institute of Actuaries. What had encouraged him to follow up the task which he had undertaken was, that in several countries actuaries were not in accord in employing a national notation. M. Landré had written that his notation had been derived partly from that of Zillmer, and partly from the English system. M. Karup, who had kindly revised the tables in the appendix to the Report, had said that German actuaries used several systems, and that, so far, they had not come to an agreement. This information, which bore

witness to the service to be rendered to actuarial science, decided him to complete his task.

The Report submitted could be thus summarized:—

1. The German notation of M. Karup, Senr., is neither complete nor suggestive. It requires constant efforts of memory, the same letter, P, representing entirely different functions according to the kind of type used, whether Roman or Gothic. It is therefore not representative, and moreover it is not complete. It presents the same defects as that of Francis Baily.

2. The Dutch notation, which has borrowed many of its symbols from the English system, is incomplete.

3. The notation of Dormoy, incomplete, as also is that of Maas which preceded it, includes symbols which suggest the French names of the functions even less than does the English system itself.

4. Lastly, the notation adopted by the Committee of Four French Life Assurance Companies is based on the principles of the English system.

It is to be remarked that the notation of the Institute of Actuaries could be adopted in its entirety by French-speaking peoples from the point of view of the convenience which results from representing technical terms by their initial letters. It is therefore unnecessary to create an entirely new system, a fact which meets the wish expressed elsewhere by a member of the Institute of French Actuaries.

Moreover, it is undeniable that the slight modifications introduced by the Committee of French Companies into the notation relating to annuities, have not simplified it; and it is to be presumed that analogous alterations in the notation relating to assurances, would not lead to any simplification of the English system.

There is, nevertheless, one improvement introduced by the Committee of Four French Companies, namely, the methods proposed for marking "selection", and for distinguishing valuation premiums and office premiums. It is this which had induced him to submit the second portion of the resolution to be discussed, namely:—That such modifications as may be found necessary hereafter, be considered by future International Congresses.

M. QUIQUET said that, in his capacity of delegate of the Institute of French Actuaries, he had, in accordance with resolutions passed by that Institute, to record his vote for a universal notation, but personally he was not in favour of such a course. Under these circumstances he felt his position to be somewhat delicate. Nevertheless, he wished to explain the reasons which had led him to his own convictions, and, without committing the Institute of French Actuaries, he would oppose the proposals of M. Bégault.

Certainly the idea of a universal notation is attractive, but is it possible? Would it not introduce serious inconveniences?

In the first place, M. Bégault has relied, in order to induce us to adopt the English system, on the example of the Electrical Congress. There is, however, an essential difference between the

resolutions of that Congress and the proposals on which we are asked to vote. We are asked to represent certain functions by appropriate symbols: the electricians had to determine the values of units of measurement. They confined themselves, as regards notation, to a symbol for each of the three fundamental units which have given their name to the system, C.G.S.

The most serious inconvenience which it appeared to him as likely to result from the adoption of a single official system, is the retarding influence it would exercise on initiative. Who has said that no one has imagined a system more simple than that now offered to us? The official adoption of a system would give it a sanction which would make more difficult, and would retard, the adoption of any other.

Who says, on the other hand, that the commutation system is the best? and who will venture to affirm that it will be used for any length of time in the future? Its official adoption will evidently delay, precisely because of the official character attached to it, the substitution of a simplification or an innovation.

Thus, the resolution submitted would have no other effect than to interfere with initiative. It would neither inspire nor stimulate anything. It would tend to enslave science, whose freedom it would be much better to proclaim.

He would go further. A single system would not have the effect of suppressing other systems. We must read the works already published. It will be necessary, therefore, as in the past, to know the systems of notation employed hitherto. That is why the Institute of French Actuaries had proposed the preparation of a dictionary of notations, which should facilitate the study of existing works, and which would be, in his opinion, more useful than a single system. On this account he congratulated M. Bégault on having, in the most perfect manner, arranged the tables in the appendix to his Report. They will be of the greatest assistance to us in our studies.

He did not certainly under-estimate the importance of the labours of his English brethren, and he wished to acknowledge them in passing. They constitute a veritable monument, and that is one of the best arguments that could be urged in favour of adopting their notation. But the publications of the Institute of French Actuaries already form, with the nine volumes which previously appeared during the time of the former Cercle des Actuaires Français, a remarkable series, not written in English notation. The French notation includes certain peculiarities which do not suggest the English system. The difference of symbols, and the question of initials, would not be, as M. Bégault seems to think, an obstacle to its adoption: but the Committee of Four French Companies has invented a special notation for pure premiums, valuation premiums, and office premiums, distinctions which are not to be found in the English system. It follows that this system should at any rate be modified before being agreed to. The system adopted would therefore no longer be the English system, but a new system to be added to the already long list of those with which we must be acquainted.

He would add that the English system appeared to him to be redundant and complicated, and to present, from the typographical point of view, very serious difficulties.

He would therefore ask the Congress to accept the following resolutions:—

1. That there is no need to elaborate a new system of notation.

2. That, without laying down any system, the Congress resolves that authors should, at the head of their works, specify the system of notation used.

3. That the tables prepared by M. Bégault be completed and published.

Mr. MCCLINTOCK said that he shared the opinion of M. Quiquet, that a universal notation is certainly not absolutely necessary. Nevertheless, it appeared to him, as well as to the other American actuaries present at the Congress, that the report of M. Bégault treats the question as well as it could be treated under existing circumstances. Also they were disposed to adopt his conclusions, although they hoped that the modifications in details which the Americans have made in the English system, would be adopted. There are, in fact, two points to be considered. First, is a universal notation desirable? M. Bégault had sufficiently established the necessity of an understanding on this point. Next, should this universal notation be one of the systems already existing? and, if so, which of the systems is most to be preferred? The great seniority of the Institute of Actuaries of London, and the preponderating influence which it has acquired, as well as the important work it has done, seemed to the American actuaries sufficient reasons for adopting its system as the foundation of a universal notation, as proposed in the report of M. Bégault, which justly had been so favourably received. He hoped that the French members of the Congress would accept this proposition.

Mr. PIERSON said the Actuarial Society of America had commissioned him to hand in a memorandum on the question under discussion, in the tenor of the remarks of Mr. McClintock. He had the honour of placing this memorandum on the table.*

M. SAMWER said that M. Karup had expressed the hope that the English system of notation would be adopted by the Congress.

M. BÉGAULT said that M. Quiquet had pronounced in favour of liberty in notation, fearing that initiative would be impeded by the adoption of a uniform system. That fear seemed to him to be baseless. The notation of the Institute of Actuaries of London had been known for a very long time, and he was not aware that so far private initiative had succeeded in introducing any simplifications.

Moreover, since we must learn all systems, in order to read books published in the past, we shall all know the English notation, and it is not asking too much to require all actuaries to use it in the future. M. Quiquet seemed also to think that on account of their official character, obsolete symbols would take too long to disappear; but if, for instance, commutation columns should fall into desuetude, then, *ipso facto*, should they no longer be employed, the symbols attached to them would also disappear. M. Quiquet thought the English

* A copy of this memorandum follows the discussion.

system redundant and complicated. Nevertheless, it was based on a few very simple principles of very easy application. That was his main reason for supporting the recommendations of his report, viz. :—

1. The adoption of the English Notation, and
2. The consideration by future Congresses of modifications to be made in it.

M. QUIQUET said that if modifications were made in the English system, then it would become a new system, and, if the question be remitted to a Commission, to another Congress, when should we realize the benefits of the reform? It would, at the best, be a new system to be studied. The official sanction accorded to a system, he repeated, because the arguments of M. Bégault had not convinced him, would delay progress, because innovations would be with difficulty introduced if they should have for effect to revolutionize part of a universal system.

If he said that the English system is redundant and complicated, he did not mean that as a criticism of the system itself, but he merely wished to point out the difficulties it presents in giving instruction to students. It is plainly impossible, when expounding a theory, to pause every moment, in order to explain the meaning of a complicated symbol employed; but that explanation would be unavoidable in teaching one's students or clerks. The English system was not well fitted for teaching purposes, and that was what he meant when he said it was redundant and complicated.

M. LÉON MARIE said that the question before the Congress was in two parts: 1st, is it desirable to adopt a universal notation? and 2nd, which system should be adopted? He regretted to differ from his colleague M. Quiquet, because he thought it was very desirable that actuaries should speak the same mathematical language. Nevertheless, could we decide on an existing system? He thought not, because, seeing that even the English system, proposed by M. Bégault for adoption, requires modifications, these modifications would require to be examined; and it was not at a meeting of a Congress that they could be discussed. Thus, it would be necessary to come to an understanding as to the meaning of N_x . The letter r , representing the value of 1 due a year hence, did not seem to him to be such an important simplification that it should of necessity be adopted. No mathematician would be troubled, if, instead of bringing into his calculations the symbol r , he were to write $(1 + i)^{-1}$, the negative exponent having nothing to alarm. Then again, the letters A and P each represents a premium; the one the single premium, the other the annual. To employ two similar letters, Π and π for instance, would be more logical. Other modifications for consideration could also be mentioned. Could that be done here? Would it not be better to adjourn the discussion? That is why he would propose:—

1. That the Congress resolve in favour of a universal notation; and
2. That the permanent Committee elaborate a scheme for a universal notation, the question to be decided on at the next Congress.

Mr. KING said that the English actuaries present at the Congress had read with so much interest the Report of M. Bégault that they proposed to ask leave for the Institute of Actuaries to publish a translation of it in their Journal.

The English actuaries had no desire to force their notation on their professional brethren of other nationalities. Nevertheless, that notation was based on very simple principles, and was not nearly so complicated as might be thought at first sight. It had stood, before being finally adopted, the proof of time. It had been employed for many years, and had undergone many modifications before the Institute officially adopted it.

It had been urged against the English notation that it includes special symbols for sundry benefits of various kinds unknown elsewhere. That arose from the peculiarities of the British legal system, whence these peculiar benefits sprang, for which it was necessary to provide suitable symbols; but it would always be easy for other actuaries to omit from the notation those symbols not required by them; and, as it might seem to them fit, but always adhering to the principles on which the system of notation is based, to introduce modifications suggested to them by special conditions unknown in England.

As to the difficulty of introducing into an official system changes as required, that argument did not appear to him to be of much weight. It is not the science which must be limited by the system of notation, but the system itself which must be pliable as the necessity arises. English actuaries have always been guided by this principle. In England from 150 to 200 students annually undergo the course of training of the Institute, and the difficulty in teaching spoken of by M. Quiquet has never been experienced.

Lastly, the English actuaries trusted that the Congress would bear in mind the fact that English books and periodicals for many years past have all been written in the same notation, while the already long catalogue of works by French actuaries are written in various notations. Scarcely any inconvenience could therefore be produced by adopting the English system.

Seeming, however, that it was not the desire of English actuaries to force their system on others, he would suggest slightly to amend the resolutions of M. Bégault as follows, namely:

1. It is desirable that the notation of the Institute of Actuaries should be employed by actuaries of all countries.
2. That such modifications as may hereafter be found necessary be discussed at future Congresses.

The PRESIDENT (M. Mahillon) said that four propositions were before the Congress, namely those of Messrs. Bégault, Quiquet, Marie, and King. He thought the Congress should first of all decide on the question whether a universal system of notation should be adopted; and he asked M. Quiquet whether he saw any objection to this question being in the first place determined.

M. QUIQUET replied in the negative. Although he was personally averse to a universal notation, yet, as delegate of the Institute of French Actuaries, he would vote for it.

On the resolution in favour of adopting a system of universal notation being put to the Congress, it was carried unanimously.

The PRESIDENT said it remained to settle the second point, which system should be adopted. There were three proposals before the Congress, those of Messrs. Bégault, Léon Marie, and King.

M. GERKRATH thought that the English system should be adopted.

M. LÉON MARIE said that the resolution he had proposed would defer for the longest time the full settlement of the question. Nevertheless, it could be amended. As a transitional measure the Congress might recommend the English system, leaving it to a Commission to submit a system of notation to the next Congress.

Mr. MACAULAY supported the adoption of the English system as proposed by M. Bégault.

The PRESIDENT read the text of the amended proposal of M. Léon Marie, namely :

The Congress decides that the question shall be discussed at the next Congress. In the meantime recommends the use of the notation of the Institute of Actuaries of London.

M. GOEMAERE thought that this proposal would be the most reasonable solution of the question. It would satisfy those who preferred the English system, and time would be given before the next Congress to elaborate a system acceptable to all actuaries.

M. RAFFMANN supported the English system of notation.

The PRESIDENT thought the following proposals would be unanimously agreed to, namely :

1. The notation of the Institute of Actuaries shall be employed in preference by actuaries of all countries.

2. Such modifications of it as may be found necessary shall be considered at future International Congresses.

Thus formulated, the proposals would not insist on the use of the English system, but only recommend it : and they were of a kind, he thought, to give satisfaction to all.

These resolutions were put to the vote, and carried unanimously.

*Report of MR. ISRAEL C. PIERSON, Ph. D. Cor. memb. I.A.F.,
Actuary of the "Washington Life Insurance Company",
Secretary of the Actuarial Society of America, New York.*

The Council of the Actuarial Society of America requested me to prepare a brief article on the subject of "The Arrangement of the Commutation Columns." This topic comes naturally under the head of one of the questions in the programme which the Committee of Organization of the International Congress of Actuaries presented for the consideration of the members of the Congress: "La nécessité d'une notation universelle."

Doubtless all will agree that it is extremely desirable to have a universal system of notation, to be employed in the Actuarial profession,

and to be used by the several Institutes and Societies in their text-books, journals and publications. That it is needed does not require any argument, although there may be a difference of opinion or choice as to the significance of certain symbols and the manner of employing them. Under this division of the subjects for discussion it seems very important to consider the proper position of the numbers in the **N** and **S** columns of the Commutation Columns. There are two systems in general use:

1. That of Dr. William Farr, in which

$$D_x = v^x l_x.$$

$$N_x = D_x + D_{x+1} + D_{x+2} + \text{etc.}$$

$$S_x = N_x + N_{x+1} + N_{x+2} + \text{etc.}$$

$$C_x = v^{x+1} d_x.$$

$$M_x = C_x + C_{x+1} + C_{x+2} + \text{etc.}$$

$$R_x = M_x + M_{x+1} + M_{x+2} + \text{etc.}$$

sometimes called the *initial* form.

2. That of Mr. Griffith Davies:

$$D_x = v^x l_x.$$

$$N_x = D_{x+1} + D_{x+2} + D_{x+3} + \text{etc.}$$

$$S_x = N_x + N_{x+1} + N_{x+2} + \text{etc.}$$

$$C_x = v^{x+1} d_x.$$

$$M_x = C_x + C_{x+1} + C_{x+2} + \text{etc.}$$

$$R_x = M_x + M_{x+1} + M_{x+2} + \text{etc.}$$

sometimes called the *terminal* form.

The difference between the two systems is apparent at once on inspection.

The actuarial world is greatly indebted to Frederick Hendriks, Esq., for his "Memoir of the Early History of Auxiliary Tables for the Computation of Life Contingencies", the first article of the first volume of the Journal of the Institute of Actuaries. He has given a record of the invention of the Commutation method made by two men independently, in separate countries, but nearly simultaneously; the first, Johann Nicolaus Tetens, a Dano-German "highly educated professor of mathematics"; the second, George Barrett, an English "self-educated country farmer." The work of Tetens was published in 1785. (*) Barrett's first conception of his system occurred in 1786 as an independent discovery, for it appears that, although Barrett did not make public his method until 1811, he had been working at it for twenty-five years. Tetens based his first "auxiliary columns" upon the number living at each age, and made

$$C_x = v^x l_x, \text{ and } E_x = C_x + C_{x+1} + C_{x+2} + \text{etc.}$$

Barrett based his two columns also on the number living at each age, and made $A_x = l_x(1+i)^{\omega-x}$, and $B_x = A_x + A_{x+1} + A_{x+2} + \text{etc.}$

(*) Correspondence between Mr. George Barrett and Mr. Francis Baily. Professor De Morgan, *J.I.A.*, iv, 185.

In the two cases the methods are different, but the ideas and mathematical principles are the same.

Mr. Griffith Davies extended and increased the utility of the Commutation Columns, but he changed the method of the formation of the second column. Exactly why he made the change does not clearly appear. It has been suggested that he did it to make the values of "*a*" symmetrical.

Dr. Wm. Farr in his English Life Tables, and Mr. David Chisholm in his works, restored the arrangement of the Commutation Columns to the Initial form as devised by the two independent inventors:

$$\begin{array}{ccccccc} \text{TETEN'S SYSTEM} & & \text{BARRETT'S SYSTEM} & & \text{DAVIES' SYSTEM} & & \text{FARR'S SYSTEM} \\ \frac{E_x}{C_x} & = & \frac{B_x}{A_x} & = & \frac{N_{x-1}}{D_x} & = & \frac{N_x}{D_x} = 1 + a_x. \end{array}$$

In 1890, it was resolved by the Actuarial Society of America to adopt the symbols of the Text-book of the Institute of Actuaries, with the exception that the letters denoting Commutation Columns, according to the system of Dr. Farr, customarily followed by American Actuaries, be printed in full-face type, and to make the additions noted below.

$$k_x = \frac{d_x}{l_{x+1}} = \frac{C_x}{D_{x+1}}; u_x = \frac{l_x}{l_{x+1}} (1+i) = \frac{D_x}{D_{x+1}}; S = \text{Sum insured.}$$

$K = \text{Cost of insurance} = q(S - V)$; $\Pi = \text{Single Premium (alternative for } A)$.

The Gothic letters, **D**, **N**, etc., were selected to make it evident at sight what they stand for respectively, as distinguished from the Roman letters, *D*, *N*, etc., or the Italic letters *D*, *N*, etc., which are used in the system of Mr. Griffith Davies.

$$D_x = v^x l_x; N_x = D_x + D_{x+1} + \text{etc.}; S_x = N_x + N_{x+1} + \text{etc.}$$

$$C_x = v^{x+1} d_x; M_x = C_x + C_{x+1} + \text{etc.}; R_x = M_x + M_{x+1} + \text{etc.}$$

As has been remarked by the author of the Institute of Actuaries' Text-book, Part II, in the practical application of the science of life contingencies premiums (which are annuities due, " $1+a$ ") are of much more frequent occurrence than annuities "*a*."

Statistics extracted from latest published reports, so far as they are available, exhibit the comparative relations between the financial transactions in life insurance and in annuities during the last year:

COMPANIES	RECEIPTS FOR PREMIUMS ON LIFE INSURANCE	PAYMENTS FOR ANNUITIES
*United States	\$180,334,000	\$2,335,000
Canada	5,850,000	39,000
Great Britain	83,115,000	6,403,000
Continent of Europe, excepting France	73,181,000	4,321,000

(*) 31 December 1894. Companies of the United States, Assets \$1,060,000,000. Amount of insurance in force, \$4,660,000,000.

In the United States the life insurance companies are obliged to return to the Insurance Commissioners of the several States, annually, on 31 December, a detailed report of receipts, disbursements, assets and liabilities. Some of the commissioners require the data of all the insurance and annuity contracts outstanding, and make a valuation of these liabilities. The Insurance Commissioner of the State of Massachusetts, to whose department the largest number of such reports is made, says that to make valuations of the policies of the companies, it is necessary to use about seven hundred different formulæ for the varying forms of insurance, and about twenty-five different formulæ for the several annuity forms.

In a single case, the New York Life Insurance Company, which has outstanding a larger number of annuity contracts than all the other American companies together, the number of varying formulæ employed for computing premiums and reserves on insurances is 260, while of annuity formulæ there are twelve different varieties.

The statistics and statements which have just been given exhibit these ratios :

LIFE INSURANCE RELATIVE TO ANNUITY TRANSACTIONS :

COMPANIES

United States	77 to 1
Canada	150 to 1
Great Britain	13 to 1
Continent of Europe, excepting France	17 to 1

LIFE INSURANCE RELATIVE TO ANNUITY FORMULÆ USED.

State of Massachusetts	28 to 1
New York Life Insurance Company	22 to 1

The Initial or Farr System has been used in America in all the Life Insurance publications containing Commutation Columns and formulæ, and in all the manuscript work in the offices, although the Gothic letters have not been employed until within a few years. The principal works are mentioned in Note A.

Mr. George King, in the Institute of Actuaries' Text-book, says: (*) "Since, in the practical application of the science of life contingencies, premiums are of much more frequent occurrence than annuities, Davies' alteration in the summation of the columns has the effect of destroying the symmetry of the great majority of the formulas, and is therefore of doubtful benefit." With this opinion many other eminent English Actuaries agree. See Note B.

It is evident:

1. That the prevailing occasions on which the Commutation Tables are employed are those for computations to ascertain the premiums required for life contingencies, annuities due. These computations are made simply and symmetrically if the second column is constructed according to the initial form.

2. That the general opinion is that the summation of the second column of the Commutation Tables, as made originally by Tetens and Barrett and restored by Farr, is the natural one and is mathematically correct.

(*) Institute of Actuaries' Text-book, p. 109.

But the Davies, or Terminal form, is used generally by Actuaries outside of the United States, and is the rule for the Journal of the Institute of Actuaries.

Mr. King says that as Davies' "method of summation has been almost universally adopted, we shall accept it" for the Text-book.

The Actuarial Society of America has adopted the notation of the Institute of Actuaries, having added somewhat to it. They are prepared to agree to an improvement or extension of this system in any way that may be thought best, but at the present time recommend the acceptance of this system with the addition of symbols for the Initial form of the Commutation Columns.

It is proposed to distinguish the Commutation Columns of the Farr or Initial system by the Gothic letters, **D**, **N**, etc., and those of the Davies, or Terminal system, by the Roman or Italic letters, *D*, *N*, etc. Then $\mathbf{D}_x = D_x$; $\mathbf{N}_x = N_x - 1$ etc.

It is advisable to secure a system which will not inconveniently disturb existing tables, formulæ and custom.

Dr. Thomas B. Sprague, in connection with his select tables, proposes to adopt the Initial form, and suggests an open letter, *e.g.*, **II**, to indicate the numbers in that column. His suggestion meets with general approval, but since the Gothic letter **N**, etc., is more easily written than the open letter **II**, etc., it seems best to use Gothic letter.

This important question, "the necessity of a universal notation", has been brought forward very opportunely by the Committee of Organization. It is to be hoped that a system will be approved by this Congress, or in the near future, which general opinion, custom and experience show to be the best to be used in practice, and to be employed in the future in publications in the field of the science of life insurance, whatever the language may be.

NOTE A.

Massachusetts Reports on Life Insurance, 1859-1865. Mr. Elizur Wright, 1865.

New York Insurance Department Reports, 1864-1868.

Valuation Tables, published by the New York Insurance Department.

Agents' Monetary Life and Valuation Tables, by David Parks Fackler, A.M., Ex-President of the Actuarial Society of America, 70 pages. Insurance Monitor, N. Y., 1870.

Principles and Practice of Life Insurance, by Mr. Nathan Willey. First edition published in 1872. The sixth edition, 413 pages, prepared in 1892, by two members of the Actuarial Society of America. New text by Mr. Henry Worthington Smith, A.M., LL.B. All tables revised, with the co-operation of Mr. John Tatlock, Jr., A.M., F.R.A.S., Assistant Actuary of the Mutual Life Insurance Company. The Spectator Company, N.Y., publishers.

Life Tables, with appropriate formulas for the solution of questions pertaining to Life Contingencies, by Lewis C. Lawton and Benjamin Griffen, of the Mutual Life Insurance Company of New York. Second edition, 481 pages. Published by the Company in 1873.

System and Tables of Life Insurance: a treatise developed from the experience and records of thirty American Life Offices, under the direction of a committee of Actuaries, among whom were Messrs. Sheppard Homans, George W. Phillips, D. Parks Fackler, Emory McClintock, Wm. D. Whiting, Charlton T. Lewis and James Weir Mason, members of the Actuarial Society of America. By Levi W. Meech, Actuary in charge, 551 pages. Published in 1881.

Experience of the Washington Life Insurance Company. Published in 1889. Papers and Transactions of the Actuarial Society of America.

NOTE B.

Mr. Griffith Davies, to whom science owes improved adaptions of Barrett's method, committed one mistake, which has led to confusion; he raised the number in columns **N** and **S** one place, and thus adopted an anomalous arrangement of the columns of the same table. Entertaining the highest respect for Griffith Davies, and desirous to retain his forms, I had to choose between the alternative of perpetuating his fortuitous derangement of the natural order of the columns, to the confusion of future students, or of embarrassing, by change of symbols, those who had already grown accustomed to the derangement. But I decided in favor of the natural arrangement of the columns, which has been ever since adhered to, and found to work satisfactorily, and I retained the letters **N** and **S**.—See pp. exix. and exx., English Life Tables, Dr. Wm. Farr.

It is difficult to conceive the amount of confusion which is avoided by taking $N_x = D_x + D_{x+1} \dots$, precisely as $M_x = C_x + C_{x+1} \dots$; and also by taking $a_x = \frac{N_x}{D_x}$, which Professor De Morgan calls "an annuity-due."—English Life Table, p. 157.

Mr. Davies, in his **D** and **N** columns, preferred to shift the latter one line downwards, so that his N_x always corresponds with D_{x-1} or N_{x+1} with D_x . No reason can be assigned for this arbitrary change, unless it be, that he wished the expression for finding an annuity to be always $\frac{N_x}{D_x}$, and not, as it properly

ought to be $\frac{N_{x+1}}{D_x}$.—H. Fillipowski, Editor of "The Doctrine of Life Annuities and Assurances, etc., by Francis Baily."

Although the **N**'s are here stated, according to Jones' arrangement of the column, I greatly prefer to use those of Mr. Chisholm. Joshua Milne, *J.L.A.*, viii, p. 118.

Which of these two systems is the better? By mathematical analogy, no doubt the Initial system would be preferred Again, for the computer's convenience, I should also be inclined to prefer the Initial system, on the principle that in life assurance, which much exceeds annuity business in amount, the annuity-due a premium is of more importance than the simple annuity.—Prof. De Morgan, *J.L.A.*, x, pp. 304, 305.

DISCUSSION.

THE PRESIDENT (Mr. A. J. Finlaison, C.B.) said the Institute was under an obligation to Mr. King for the vigorous and scholarly translation of M. Bégault's paper. The paper was a talented and powerful address, advocating the system of notation promulgated by the Institute of Actuaries. He ventured to think that it would prove of permanent use as a dictionary of the notation for works already in print, and would act as a valuable guide to the student in the notation of the Institute itself. The paper was considered an important one at the Congress, and had tended to add to the estimation in which the Institute of Actuaries was held on the Continent.

Mr. H. W. MANLY said M. Bégault had been the principal translator of the Text-Book of the Institute into French. He, therefore, had become familiar with the Institute notation, and had learnt to appreciate its adaptability to every status which might require to be symbolized. He believed it was generally known that it had been the intention of the French Actuaries to propose that

the whole subject be deferred until a dictionary of notations used in the various countries had been compiled, and it very much surprised them that M. Bégault had taken the subject out of their hands by writing his paper and compiling the dictionary itself. The proposition in question, therefore, necessarily fell to the ground. Notwithstanding the resolution finally come to at Brussels, it would be seen that there had been a general feeling at the Congress that it would be desirable to adopt a universal notation, and that the English would be the best to adopt. The Institute of Actuaries was very much indebted to M. Bégault, not only for introducing our Text-Book on the Continent, but for the production of the paper now under discussion.

MR. J. CHISHOLM said he rose to refer to a small point dealing with one symbol only, which, however, revived memories of battles fought long ago, but still undecided. The Commutation Tables adopted by the Institute differed from those in use in America, and he wished to emphasize, now that they were on the point of adopting a universal notation, that they should do all in their power to prevent the further use of a double system, such as was in vogue at the present time, and he hoped that no further works would be published on the Commutation system other than in the notation adopted by the Institute. He did not think there was a pin's point to choose between them, but it was desirable that under all circumstances the same symbol should have the same meaning, and seeing that Griffith Davies's system had been adopted by the Institute he thought that all should use it.

THE PRESIDENT, having proposed a vote of thanks to Mr. King, which was unanimously accorded,

MR. G. KING, in reply, acknowledged the vote of thanks, and said, with reference to Mr. Chisholm's remarks, with all deference, he thought they were a little beside the point. They did not relate to notation, but to the system of columns to be adopted, and he did not see why both forms of columns should not be used concurrently. It would be right to retain the ordinary N to represent Davies's form of column known as the terminal form, and a different symbol should be adopted for the initial form of N which had been used by Mr. Chisholm, sen., and the late Dr. Farr, and which was in use in America. As a matter of fact, the form of N printed in open type, had already been used in Great Britain to represent the initial form and had been adopted by the Institute. He thought the open N much more suitable than the thick N proposed by the American Actuaries, because it was difficult, in writing, to distinguish the thick N from the ordinary letter. He was pleased to see how loyally their French colleagues adopted the notation of the Institute of Actuaries after the resolution of the Congress had been carried. For a system of notation to be good, it must have at least five characteristics. It must be easy to remember; it must be easy to teach; it must be easy to write; it must be easy to print, and it must readily admit of additions. He thought all those five characteristics belonged to the British system. He was not surprised that their French friends found a little difficulty at first in understanding the Institute notation, for he must confess in the volume, "Institute of Actuaries' Life Tables", where the notation was first

promulgated, no attempt was made to arrange it in any order of principle. He had ventured to alter that in the Text-Book. He had not given there a long array of symbols, but the principles on which those symbols were ranged, with illustrations; and he believed such a method was clearer than merely giving a list of symbols; and now that their French friends had the Text-Book translated into their own language, he had no doubt they would see the simplicity of the British system. It was founded on a few broad principles easy to remember, and therefore it was easy to teach. He thought M. Quiquet was entirely mistaken in thinking there was any difficulty in teaching our system, and he spoke from a large experience. The question of writing was an important one, and it was very desirable that the symbols should be easy of formation. He had tried various systems, and he had found the British by far the most simple. As to the printing, that was also comparatively easy. They had had no difficulty in the printing of the Text-Books and Journals, and there was also no difficulty in printing the Text-Book in Brussels. That this was not the case as regards some of the other systems, had been demonstrated in rather an amusing manner, Messrs. Layton not having some of the type required for the tables appended to the paper, M. Bégault was sending some over. With regard to the last of the five characteristics he had mentioned, namely, the ease of adding to the notation, the notation in the Text-Book had developed to a certain extent from that of the Institute of Actuaries' Life Tables. For instance, symbols for increasing benefits had been introduced, the principle for the symbols remaining identical. A very important addition, also, had been made by Dr. Sprague, and adopted by the Institute, for indicating "selection." One or two incidental points had been mentioned at the discussion of M. Bégault's paper at the Congress. For instance, as to the use of v for the present value of 1 discounted. He would be sorry to drop that symbol, as it was very convenient, and he ventured to add that there was a theoretical difference between v and $(1+i)^{-1}$, although they had of necessity the same numerical value. If he might take an illustration from music, it was something like the difference between A sharp and B flat, and every musician knew that, although these were represented by the same note on the piano, they had, nevertheless a very different meaning in musical composition.

THE INSTITUTE OF ACTUARIES.

EXAMINATIONS OF THE INSTITUTE, APRIL 1896.

EXAMINATION FOR ADMISSION TO THE CLASS OF ASSOCIATE
(PART I).*Examiner*—PROF. S. L. LONEY, M.A.*First Paper.*

1. State the rule for the decimalization of a sum of money, correct to three places of decimals—*i.e.*, for writing down at sight any sum of money in terms of pounds and decimals of a pound. Illustrate by expressing £487. 17s. 10½d. as a decimal, and also by writing down the value of £33·693 in pounds, shillings, pence, and farthings.

2. Having given that $\frac{1}{a+b} + \frac{1}{c} = \frac{1}{a} + \frac{1}{b+c}$, prove that

$$b(a-c)(a+b+c)=0.$$

3. Solve the equations:

$$(1) \quad \frac{x}{3} + \frac{y}{2} = \frac{x}{2} - \frac{y}{3} = 2\frac{1}{2};$$

and (2) $\sqrt{x(a+b-x)} + \sqrt{a(b-a+x)} + \sqrt{b(a+x-b)} = 0.$

4. Insert 5 harmonic means between 5 and 10.

If a , b , and c be three positive quantities in harmonical progression, prove that $a^2 + c^2 > 2b^2$.

5. There are two numbers, each consisting of two digits, and the digits of the second are those of the first but reversed; the product of these numbers is 8,722; if the first number be divided by the second, the quotient will be 1 with a remainder consisting of one figure only. Find the numbers.

6. In the expansion of $(1+x)^n$ by the Binomial Theorem, n being integral, prove that the sum of the even coefficients and the sum of the odd coefficients are each equal to 2^{n-1} .

Resolve $\frac{x^2+3x+1}{(x-1)(x-2)(x-3)}$ into partial fractions, and find the coefficient of x^n in the expansion by the Binomial Theorem, x being less than unity.

7. Find the amount of £P in n years, reckoning compound interest at the rate of £ r per unit per annum.

What does the formula become if the interest be convertible half-yearly?

8. A bond of £100, on which the dividend has just been paid and which has 20 years to run before it is paid off at par, is bought by a man at such a price that he will on his whole investment just receive 4 per-cent on the money he invests. If the bond bear interest at 5 per-cent, paid half-yearly, find, correct to the nearest penny, the amount he gives for the bond.

Given $\log 1.02 = .0086002$, and

$\log 45,289 = 4.655992$.

9. If a man buys equal amounts of a $3\frac{1}{2}$ per-cent stock and a $4\frac{1}{3}$ per-cent stock, his income is £1,880. If he invest so that his income is the same from each kind of stock, his income is £1,872. Find what his income will be if he invests half his capital in each of the two stocks.

10. Prove that integral values can always be found to satisfy the equation $ax - by = c$, provided that a and b be prime to one another.

Solve in positive integers the equation $3x - 4y = 11$.

11. There are $(m+n)$ coins in a bag which are either sixpences or shillings, and n of them are known to be sixpences; of the remaining m coins any number of sixpences is equally probable: prove that the chance of drawing a sixpence is $\frac{2n+m}{2n+2m}$.

12. If one side of a triangle be produced, prove that the exterior angle is equal to the sum of the two interior and opposite angles.

Give the corollaries to this proposition, and find the size of the angle of a regular decagon.

Second Paper.

1. What Books are necessary in a Life Assurance Office in which to record commission payable to Agents?

2. The beam of a defective balance is horizontal when the weight in one scale is one-fourteenth more than the weight in the other. A tradesman placing a pound weight alternately in the two scales of the balance, sells what he thinks to be two pounds of a certain commodity. What weight does he actually give his customer?

3. I have a certain sum of money with which to buy a certain number of nuts, and I find that, if I buy at the rate of 40 a penny, I shall spend 5*d.* too much, whilst if I buy at the rate of 50 a penny I shall spend 10*d.* too little. How much have I to spend?

4. Prove that one root of the equation $ax^2+2bx+c=0$ is equal to one root of the equation $a'x^2+2b'x+c'=0$, if

$$(ac'-a'e)^2=4(bc'-b'e)(ab'-a'b).$$

5. If ${}_nC_r$ denote the number of combinations of n things taken r at a time, prove that ${}_nC_r={}_nC_{n-r}$, and that ${}_nC_r+{}_nC_{r-1}={}_{n+1}C_r$.

Find the number of ways in which n persons may be arranged round a round table so that of three given persons no two may sit together.

6. Sum to n terms the series

$$(1) \quad 1^2+2^2+3^2+\dots+n^2,$$

$$\text{and} \quad (2) \quad 1.3.7+2.4.9+3.5.11+\dots$$

7. A and B travel on the same road towards London, and B is 25 miles behind A. At the fiftieth milestone from London, A overtook a flock of geese which was proceeding at the rate of three miles in two hours. B overtook the same flock of geese at the forty-fifth milestone. At what rate was B travelling?

8. Define a logarithm, and prove that

$$\log_a m = \log_b m \times \log_a b.$$

Given that $\log_{10} e = .434294\dots$, make use of the logarithmic series to find, correct to four places of decimals, the values of $\log_{10} 2$ and $\log_{10} 3$.

9. Find the present value of a perpetual annuity of £P, the first payment to be made at the end of n years from the present time.

Two persons share between them a perpetual annuity of £841; one of them is to receive the first and second instalments, the fifth and sixth instalments, and so on; the other is to receive the third and fourth, the seventh and eighth, and so on: prove that, in order to make this arrangement fair, the first man should pay £410 to the second man. Interest is reckoned at 5 per-cent per annum, and the first payment of the annuity is due at the end of a year from the present time.

10. The sum of two positive integers is 80: find the chance that their product is greater than 600.

11. An event must have happened from one of n mutually exclusive causes, the antecedent probabilities of which are P_1, P_2, \dots, P_n , respectively; the probabilities that, when these causes exist, the event happens are respectively p_1, p_2, \dots, p_n . Prove that, on any occasion on which the event happens, the probability that it was caused by the r th cause is

$$\frac{P_r p_r}{P_1 p_1 + P_2 p_2 + \dots + P_n p_n}.$$

12. A straight line touches a circle, and from the point of contact a straight line is drawn to cut the circle: prove that the angle which the cutting line makes with the touching line is equal to the angle in the alternate segment.

EXAMINATION FOR ADMISSION TO THE CLASS OF ASSOCIATE,
(PART II).

Examiners—Messrs. G. J. LIDSTONE, W. P. PHELPS, M.A., and
S. G. WARNER.

First Paper.

1. Obtain a formula for determining approximately the rate of interest in an annuity-certain, and apply it to the following case (having given that $a_{\overline{20}} = 14.877$):

$$a_{\overline{20}} = 14.462.$$

2. By a well-known formula,

$${}_nV_x = (P_{x+n} - P_x)(1 + a_{x+n}).$$

Obtain an analogous formula for a fixed-term policy, payable in 20 years from the date of its issue and effected at an annual premium, which has been 10 years in force.

3. Prove that $Q^1_{xy} = \frac{1}{2} \left(1 - \frac{e_{x:y-1}}{p_{y-1}} + \frac{e_{x-1:y}}{p_{x-1}} \right)$.

If the probability that x will die before z is .1996,

..	..	x	both y and z is .1610,
..	..	y	before z is .2990,
..	..	y	before both x and z is .2602,

find the values of the following probabilities:

- (1) That the survivor of x and y will die before z .
- (2) That x will die before z , y having died first.

4. Interpret and give verbal explanations of the following expressions:

(a) $v(1 + {}_{n-1}a_x) - {}_na_x$

(b) $vi(A_x + A_x^2 + A_x^3 \dots ad inf.)$

(c) $1 - \frac{P_x}{P_{x+n}}$.

5. Describe the practical processes by which, having given the value of q_x for each age, you would proceed to form a table of the following functions:

$$l_x, d_x, e_x, D_x, M_x.$$

6. In a stationary community supported by 5,000 annual births, each member, on attaining the age of 20, makes a payment of £20, and contributes £1 at the end of each succeeding year until, and inclusive of, the sixtieth birthday; receiving thereafter an annuity of £15 payable at the end of each year. In respect of each contributor who dies before receiving the first payment of £15, a payment of £5 is made. Find expressions for (a) the number of contributors (b) the annual receipts (c) the total yearly annuity-payment, and (d) the annual death claims.

7. Show how, by the aid of Gauss's logarithms, to construct a table of the values of a_x which shall at the same time give the logarithms of D_x and N_x .

8. Obtain a formula for an annual whole-life premium, a compound quinquennial reversionary bonus of £1. 5s. per-cent per annum, and an interim bonus of £1 per-cent per annum, being anticipated and applied in reduction thereof.

9. Explain, in their bearing upon a life policy, the terms "Chose in action", "Donatio mortis causa", "Interpleader", and "Novation."

10. Explain why with-profit policies under annual premiums do not admit of being drawn payable to order or bearer, in the same way as bills of exchange and promissory notes.

11. Discuss the question whether it is proper for a proprietary life office to invest any part of its funds in the purchase of its own shares.

12. What points would you consider in determining the eligibility as an investment for a life office of an American Railway Mortgage Bond issue? How would that eligibility be affected, in your opinion, by an arrangement for redemption by annual drawings?

Second Paper

1. Find the equated time of payment of n sums, $X_1 X_2 \dots X_n$, due respectively at the end of $t_1 t_2 \dots t_n$ years. Show that the error involved in the common approximation is always in favour of the debtor.

2. Prove that the most probable number of deaths within a year among m lives all aged x is r , when r is the greatest integer in $(m+1)q_x$. What would be the amount of the expected claims in the succeeding year among 712 lives, all of the same age, each insured for £700, on the assumption that out of 83,304 lives of that age 819 die within a year?

3. Find the probabilities that, in the t th year from the present time,

- (a) A life now aged x will die, having survived a life now aged y by at least m years, and a life now aged z by at least n years.
- (b) The last survivor of the three lives will die.
- (c) The life z will die, leaving x and y surviving.

4. Interpret the following symbols :

$$A_{xy:\overline{2}}^{(t)} \quad A_{x:y(\overline{t})}^{\overline{31}} \quad (NP)^x \quad {}_x t P A_x | (\overline{n})$$

5. Show how to obtain the "expected deaths" and "expected claims" in one year, for policies

- (a) payable on first death of two lives.
 (b) " " last " " (both alive).
 (c) " " one life pre-deceasing another.

What approximate methods would you use in practice?

6. Show that

$$P_x + d_n V_x = P_{x+n}(1 - nV_x),$$

and give a verbal interpretation of this result.

Having given that, at 4 per-cent interest, $P_{20:\overline{40}} = .01615$ and $A_{40:\overline{20}} = .51078$, find ${}_{20}V_{20:\overline{40}}$ and ${}_{20}(FP)_{20:\overline{40}}$.

7. Describe fully the method you would adopt in forming select mortality tables, for whole life assurances, from the experience of a life office; discussing in this connection the relative merits of "policy" and "calendar" years. Explain what is meant by "the method of final series."

8. Find in commutation symbols the office annual premium for the deferred annuity represented by ${}_n|\ddot{a}_x^{(m)}$; all premiums paid to be returned should x die before y and within n years. (Assume that the office premium, π , is derived from the net premium, P , by the formula $\pi = 1.1P + .015$).

9. Explain the difference between the prospective and retrospective methods of valuing a policy. Express ${}_tV_{x:\overline{n}}$ in both forms and demonstrate their identity.

10. How would you construct and verify a table of the values of ${}_t p_{xy}$? For what purpose would such a table be required?

11. What are the provisions of "The Customs and Inland Revenue Act, 1888", with reference to the stamping of assignments of policies?

A assigned a policy to B, but B omitted to have the deed stamped within the time allowed. In the absence of any other dealings with the policy, do you consider that it would be competent for A to execute another assignment to B and so avoid the stamping penalty?

12. A Life Assurance Company buys 5 per-cent Debenture Bonds, redeemable at par on a fixed date, at a price calculated to yield $3\frac{3}{4}$ per-cent on the investment. How would you deal in the books with the accrued interest (if any) included in the purchase price?

Would you carry the whole of the half-yearly dividends into the Company's accounts as Interest? If not, what course would you follow?

Give your reasons in each case.

EXAMINATION FOR ADMISSION TO THE CLASS OF FELLOW
(PART III, SECTION A).

Examiners—MESSRS. M. N. ADLER, H. W. MANLY, J. SORLEY,
G. F. HARDY, G. TODD, and L. M. SIMON.

First Paper.

1. What are the principal distinctions between the positions of an executor and an administrator?

Define the following:

- (a) Specific legacy.
- (b) Demonstrative legacy.
- (c) Administrator *de bonis non*.

2. What is the meaning of

- (a) A fee simple.
- (b) A base fee.
- (c) A vested remainder.
- (d) A contingent remainder.
- (e) A reversion.

And what is the difference between a joint tenancy and a tenancy in common?

3. What additional powers are given with respect to life policies by "The Married Women's Property Act, 1882", which were not conferred by the Act of 1870?

Sketch a clause to be inserted in a policy effected by a man for the benefit of his wife and children, defining the trusts.

4. A claim has arisen under a policy which has been assigned by A (the assured) to B by way of mortgage, and notice given to the office. On satisfaction of B's debt the policy was handed back (with the mortgage deed) by B to A, but no re-assignment was executed and no notice was given to the office. The policy was afterwards assigned absolutely by A to C, and further assigned by C to D, the claimant. No notice of these assignments was given to the office until after the death of the assured. The three deeds of assignment are unstamped. What requirements should the office make before paying the claim?

Is it material to ascertain whether any and which of the parties were domiciled in England or a foreign country?

5. Describe the various steps to be taken in transferring the business of a proprietary life assurance company to a mutual society.

6. Describe shortly the several annual returns required to be made by a life assurance company incorporated under the Companies' Acts.

7. Explain what is meant by

- (a) The old and new Sinking Funds.
- (b) The Consolidated Fund.
- (c) Local Loans Stock.
- (d) Inscribed Stocks.

How are transfers of inscribed stocks usually effected?

8. What are the chief requirements of a suitable monetary standard of value? It is asserted by some that the present gold standard of this country is wanting in some of these requirements. What is your opinion on this subject?

9. Show how the export of gold from this country affects the reserve of the Bank of England; and explain the means by which the Bank can effectually check the efflux.

10. State the principal influences affecting rates of exchange. Explain what is meant by specie point, and how it differs from the Mint par of exchange.

11. What difference in the security, if any, is there between a cumulative preference stock ranking before the ordinary stock in a distribution of assets, a simple debenture stock, a mortgage debenture stock with a floating charge, and a mortgage debenture stock with a specific charge?

12. A company proposes to raise money by the issue of irredeemable debenture stock, and the alternative propositions are made to the directors (a) the issue of £90,000 4 per-cent debenture stock at par, and (b) the issue of £80,000 $4\frac{1}{2}$ per-cent debenture stock at a premium of $12\frac{1}{2}$ per-cent. State the main practical differences of the two methods (1) from the point of view of the company, and (2) from the point of view of the investors.

Second Paper.

✓ 1. Given the values of a function u_x when $x=0, 5, 10$, &c. Investigate a formula for the interpolation of intermediate terms, for integral values of the variable.

✓ Complete the table of premiums between ages 40 and 45, given the following:

40	.	.	.	£2 13 10
45	.	.	.	3 2 4
50	.	.	.	3 14 10
55	.	.	.	4 13 10

✓ 2. Deduce an expression for the value of $\Delta^n u_x$ in terms of the successive values of u_x ; and show how to expand $\Delta^n u_x v_x$ in a series of ascending differences of v_x .

✓ 3. Demonstrate Lubbock's formula of summation, and show how it may be employed to obtain the value of a_{30} from the values of D_{30}, D_{40}, D_{50} , &c.

✓ 4. Show that the series

$$u_x - u_{x+1} + u_{x+2} - \&c.,$$

may be transformed into the series

$$\frac{1}{2} \left(u_{x-\frac{1}{2}} - \frac{\Delta^2 u_{x-\frac{3}{2}}}{8} + \frac{1.3}{2} \frac{\Delta^4 u_{x-\frac{5}{2}}}{64} - \&c. \right),$$

and find approximately the value of the series

$$\frac{1}{5} - \frac{1}{6} + \frac{1}{8} - \frac{1}{11} + \&c.$$

5. It is desired to ascertain approximately the value of $A_{x:\overline{10}|}^2$. Show how you would practically arrange the work in a columnar form with a view to the application of a summation formula.

6. Describe Makeham's method of graduation as applied to the construction of a mortality table based on a limited experience; and discuss the advantages of the method.

7. Discuss the method used by Milne in the construction of the Carlisle table.

8. Criticize the several investigations which have been made, on the basis of the Institute experience, of the value of annuities at the date of entrance, with reference to their suitability for use in framing tables of office premium rates.

9. It is desired to ascertain the rates of mortality, secession, and sickness, in a sick benefit society giving full pay for the first six months, half pay for the second six months, and quarter pay after twelve months of continuous sickness. Design a card for use in extracting the experience from the books of the Society; and show how the results should be tabulated in order to exhibit the required rates according to length of membership, giving the headings of the various columns in the necessary schedules.

10. What are the necessary conditions with regard to (1) population statistics, and (2) the statistics of a life assurance company, for the formation of a satisfactory mortality table? Discuss the applicability of the 'Theory of Errors' to a mortality experience.

11. Discuss the "Final Series" method of adjusting a mortality experience with respect to its utility or otherwise.

12. What do you consider the respective advantages or disadvantages of basing a mortality table upon the observed experience of lives, policies, or sums assured?

EXAMINATION FOR ADMISSION TO THE CLASS OF FELLOW
(PART III, SECTION B).

Examiners—MESSRS. M. N. ADLER, H. W. MANLY, J. SORLEY,
G. F. HARDY, G. TODD, and L. M. SIMON.

First Paper.

1. Discuss the advisability of adopting a very low rate of interest in calculating reserves.

Your answer should have reference, among other considerations, to the question of loading, the effect of using the Combined Mortality Tables, and the result on the surplus under different conditions of business.

2. State the arguments for and against an allowance in the valuation of the liabilities of a life office for initial expenditure.

3. Describe the principle upon which assessment assurance companies are theoretically based, and the causes which generally lead to their failure in practice. How does the system differ from assurance by a fixed ascending scale of premiums?

4. An office distributes its surplus with reference to the sources from which such surplus is derived. How would you allocate the surplus from each source, and what precautions would you adopt to minimize the risk of violent fluctuation in the relative rates of bonus on policies of different durations at successive distributions?

5. An assurance company distributes its surplus as a cash percentage on the premiums paid since the previous valuation. What conditions should hold, in your opinion, to render this method fairly equitable; and, assuming an option be given at each division of receiving bonus in cash, reduction of premium or reversionary addition, upon what basis would you determine the alternative amounts?

6. Describe the method you would employ, for the purpose of valuation, in classifying a large number of endowment assurances taken out at different dates and for widely varying terms. What annuity would you use in valuing the premiums, and what adjustments would you make to meet the cases of half-yearly or quarterly premiums and unequal distribution of dates of entry?

7. A life assurance company invests part of its funds in the purchase of policies issued by other companies.

What are the special points which require consideration in determining whether to bid for a policy offered at public sale; and state on what basis you would fix the price.

8. A, aged 50 next birthday, has a life-interest in a leasehold ground rent, amply secured, of 1 per annum, having 30 years to run. How would you value, as an investment,

(a) The life-interest of A?

(b) The reversion on the death of A?

9. The profits of a life assurance company are distributed by way of a compound reversionary bonus. The directors are considering the propriety of introducing a scale of premiums reduced by the anticipation of future bonuses. Sketch a report embodying your views on the subject. On what basis would you construct a table of rates if such a scheme were adopted?

10. A person aged x , now healthy, desires to have the option of effecting a policy on his own life at the death of another person now aged y , at the ordinary rate of premium for his age at the time the option is exercised. How would you calculate the single premium required for the grant of such an option?

11. Give expressions for the value of the following:

$$\frac{d}{dt} a_{xy}; \quad \frac{d}{dx} a_{xy},$$

and show that if $l_x = k s^x g^{c^x}$, then $c^y \frac{d}{dx} a_{xy} = c^x \frac{d}{dy} a_{xy}$.

12. Express the following benefits in the form of definite integrals:

$$I\bar{a}_x; Q_{xyz}^1; \bar{a}_{yz:x}^1; \bar{A}_{xyz}^2.$$

Second Paper.

1. A, aged 30, is entitled on the death of B, aged 60, to freehold property in the City of London valued at £30,000, and he is desirous of obtaining an immediate annuity during the joint lifetime of himself and B on the security of his reversionary interest.

How would you calculate the annuity that an office could safely allow to A

1. If the debt is to accumulate at 5 per-cent compound interest until B's death?
2. If it is to be repayable by a fixed charge on the reversion?

2. A is absolutely entitled to a reversion expectant on the death of the life tenant, a lady aged 50, to a share of trust-funds, at present invested in

- (a) $2\frac{3}{4}$ per-cent Consols,
- (b) Leasehold property, having about 35 years of the time unexpired,
- (c) Bank shares,

and desires to dispose of his interest.

For the purpose of valuing the reversion, how would you estimate the value to be placed upon the three classes of investment respectively?

What *special* points would have to be considered as to the leasehold property and the bank shares?

3. State the different ways in which the purchase of a reversion may be treated in the books of a life office. Which method do you consider most suitable for an office making an annual valuation?

4. A, aged 70, is life-tenant of a landed estate, and B, his eldest son, aged 30, is tenant in tail. The estate yields a *net* income of £10,000 per annum, and B (who is married and has 4 sons) desires to raise money on his reversion.

Describe fully the usual methods of raising money from an insurance office in such a case if:

- (a) £5,000 is required.
- (b) £30,000 is required.
- (c) £100,000 is required.

What precautions are usually adopted by the purchaser or mortgagee of base fees? Can you suggest any other method which would render these securities marketable?

5. Discuss the question of the rating-up of under-average lives with special reference to the point whether the same addition should be made to age for ordinary whole-life, endowment and short term assurances.

6. Upon what bases, in respect of mortality, interest and loadings, would you compute tables of annual premiums for

- (a) Short-term assurances?
- (b) Contingent survivorship assurances?

7. How would you calculate the annual premium to be charged for what is termed a 5 per-cent Debenture Policy—that is to say, an annuity-certain of 5 for 20 years, dating from the death of the assured, and the payment of 100 at the expiry of the 20 years?

8. Explain briefly the criteria for maxima and minima of functions, and show that the numbers dying in a mortality table in a given small interval of time, after attaining age x , will be a maximum or minimum when the force of mortality is equal to the square root of its differential coefficient.

9. Find an expression for the value of $\bar{A}_{xy:\frac{1}{2}}$ in terms of the forces of mortality and the joint annuities. How would you obtain a numerical result in practice?

The Candidate having handed in his answers to the foregoing questions, will be supplied with the following books:—"Institute of Actuaries' Life Tables"; a Premium Conversion Table (Orchard or Rothery and Ryan); Gray, Smith, and Orchard's Survivorship Assurance Premiums; and a book of logarithms; so that he may give numerical solutions to the following questions. The books of Tables are supplied for the sake of convenience, and it must not be inferred that they are held to be suitable. The Candidate is requested to state, in answering Questions 22 and 23, what mortality tables and rates of interest he would consider most suitable for adoption in each case in practice.

10. A, aged 50, has a life-interest in, and B the absolute reversion on A's death to, a fund consisting of

- £5,000 Consols, standing at 107;
- £5,000 Canada 4 per-cent Registered Stock, redeemable in 1910, standing at 112;
- A freehold house producing a net income of £150, which can be sold for £3,000.

It is desired to apportion the fund by mutual consent. At what do you estimate their respective shares?

11. By his will a man bequeathed £50,000 Consols to his son A for life, with remainder to A's issue, whom failing, to his grandchildren, being children of his daughters, B and C, *per stirpes*.

A and his wife are both alive and in good health, aged respectively 70 and 65, without issue. B has died, leaving three children. C has died, leaving one son, now of age, who offers his reversionary interest for sale.

Fix the price an assurance office could give for it, showing the process of valuation in full.

12. Distinguish between three kinds of United States Railway Bonds: (1) currency, (2) gold dollar, (3) sterling. What is the objection to currency bonds as an investment?

With regard to classes (1) and (2), why is there always a difference between the London and New York prices?

A 6 per-cent currency bond, due in April 1930, is bought in London at the price of 120; assuming the rate of exchange £4.88 to remain constant throughout, what is the return on the investment?

PROCEEDINGS OF THE INSTITUTE.—SESSION 1895-96.

First Ordinary Meeting, 25 November 1895.

The first ordinary meeting of the session 1895-96 was held at the Hall of the Institute, on the 25th day of November 1895.

The President (MR. ALEX. J. FINLAISON) in the Chair.

The President read an address on the International Congress of Actuaries held at Brussels in September 1895.

Second Ordinary Meeting, 30 December 1895.

The President (MR. ALEX. J. FINLAISON) in the Chair.

The following gentlemen were elected:

As Associates:

Israel Coriell Pierson, Ph.D.
Andrew James Stiven, F.F.A.

As Corresponding Members:

Léon Marie.
Jacques Martin-Dupray.
Albert Quiquet.

A paper "On some Methods of Grouping Policies for the purpose of Valuation", was read by the author, Mr. Fredk. Schooling.

The following gentlemen took part in the discussion:—Messrs. R. P. Hardy, E. Woods, T. G. Ackland, H. W. Manly, J. A. Robertson, J. H. Barnes, and the President.

Third Ordinary Meeting, 27 January 1896.

The President (MR. ALEX. J. FINLAISON) in the Chair.

A paper "On the System of Bonus Distribution to Policyholders as a Percentage per annum for the Valuation Period on the Sum Assured or on the Sum Assured and Existing Bonuses, considered in relation to some Recent Influences on Life Assurance Finance", was read by the author, Mr. H. W. Andras.

The following gentlemen took part in the discussion:—Messrs. A. B. Adlard, G. F. Hardy, J. Chisholm, T. G. Ackland, and the President.

Fourth Ordinary Meeting, 24 February 1896.

The President (Mr. ALEX. J. FINLAISON) in the Chair.

The Messenger Prize Essay, "On the Books and Forms to be used in Scheduling the Particulars of the Risks of a Life Assurance Company under its Assurance and Annuity Contracts, for Periodical or Interim Valuations, Distribution of Surplus, and for Investigation of the Rates of Mortality, Surrender, and Lapse", was read by the author, Mr. James Chatham.

The following gentlemen took part in the discussion:—Messrs. R. P. Hardy, C. D. Higham, J. Chisholm, and the President.

Fifth Ordinary Meeting, 30 March 1896.

The President (Mr. ALEX. J. FINLAISON) in the Chair.

Mons. Paul Guieysse, President of the Institut des Actuaires Français, was elected a Corresponding Member.

Mr. George King read his translation of Mons. Bégault's paper, "On the necessity for a Universal Notation", together with an abstract of the discussion thereon.

A discussion followed, in which the following gentlemen took part:—Messrs. H. W. Manly, J. Chisholm, and the President.

Sixth Ordinary Meeting, 27 April 1896.

The President (Mr. ALEX. J. FINLAISON) in the Chair.

Mr. G. A. Brown, of the Clerical, Medical and General Life Assurance Company, was elected an Associate.

A paper entitled "(1) An Investigation of some of the Methods for deducing the Rates of Mortality, and of Withdrawal, in Years of Duration; with (2) the Application of such Methods to the computation of the Rates experienced, and the Special Benefits granted, by Clerks' Associations", was read by the author, Mr. T. G. Ackland.

The following gentlemen took part in the discussion:—Messrs. H. W. Manly, G. H. Ryan, R. Todhunter, J. Meikle, and the President.

The Forty-Ninth Annual General Meeting, 1 June 1896.

The President (Mr. A. J. FINLAISON) in the Chair.

The proceedings at the Annual General Meeting will be found on page 59.

REPORT, 1895-96.

The Council have pleasure to report to the members upon the progress of the Institute during the session of 1895-96, the forty-eighth year that it has been in existence.

The increase in the number of members in the year has been 13, as compared with 41 in that which preceded it. At the end of the year in which the Institute was incorporated by the Royal Charter the number of

members was 434, while five years later, at 31 March 1890, it was 601. Since that time it has grown as follows:

At 31 March 1891	to 620,
„ 1892	„ 645,
„ 1893	„ 674,
„ 1894	„ 734,
„ 1895	„ 775,
„ 1896	„ 788.

The following schedule shows the additions, changes, and losses in the membership, which have occurred during the year ending 31 March last.

Schedule of Membership, 31 March 1896.

	Honorary Members	Fellows	Associates	Students	Corres- ponding Members	Total
i. Number of Members in each class on 31 March 1895 .	2	173	221	366	13	775
ii. Withdrawals by						
(1) Death	1	3	...	1	73
(2) Resignation	1	5	32	4	
(3) Default in pay- ment of Sub- scriptions	1	3	22	...	
iii. AdditionstoMembership	2	170	210	312	8	702
(1) By Election	2	...	4	86
(2) By Order of Council	78	...	
(3) By Re-instatement	2	...	
iv. Transfers	2	170	212	392	12	788
(1) By Examination:						
<i>from Associates</i>	6
<i>to Fellows</i>	6
(2) By Examination:	2	176	206	392	12	788
<i>from Students</i>	1
<i>to Fellows</i>	1
(3) By Examination:	2	177	206	391	12	788
<i>from Students</i>	17
<i>to Associates</i>	17
v. Number of Members in each Class on 31 March 1896 .	2	177	223	374	12	788

The Council have, with great regret, to report the loss by death during the year of one Fellow, Mr. H. J. Puckle; three Associates, Messrs. D. J. Alexander, S. Kirkness, and A. J. Stiven; and a Corresponding Member, M. Leon Mahillon, of Brussels.

The Accounts for the year show that the total funds on 31 March last amounted to £5,874. 2s. 2d., being an increase during the year of £268. 13s. 5d.

The Annual Subscriptions, together with admission and other fees, amounted to £1,571. 17s. 0d., being about equal to those of the previous year.

The total Income for the year was £2,056. 13s. 8d., and the total Expenditure £1,788. 0s. 3d. Of this latter sum it will be seen that sums amounting to £484. 13s. 1d. have been expended on the following objects, namely: The preparation of an Index to Vols. xxi-xxx of the *Journal*; the Mortality Experience Investigation now in progress; the publication of the Government Annuity Tables; and the preliminary work connected with the preparation of tables based upon Dr. Sprague's Select Mortality Experience investigations.

The Revenue Account and Balance Sheet are given herewith (p. 58).

The stock in hand of the Institute publications on 31 March was as follows:

No. of Copies	Description of Work
85	<i>Text-Book</i> , Part I.
312	" II.
62	Mortality Experience Tables.
859	Government Joint-Life Annuity Tables.
527	Logarithm Cards.
389	Messenger Prize Essay (Friendly Societies).
512	Index to 10 Vols.
73	" to 20 "
9,653	Parts of <i>Journal</i> .

The following papers were submitted at the sessional meetings of the Institute, namely:

- 25 *November* 1895.—"An address by the President, Mr. Alex. J. Finlaison, C.B., on the International Congress of Actuaries, held in Brussels in September 1895."
- 30 *December* 1895.—"On some methods of grouping Policies for the purpose of Valuation"—Mr. Frederick Schooling.
- 27 *January* 1896.—"On the system of Bonus Distribution to Policyholders as a percentage per annum for the valuation period on the sum assured, or on the sum assured and existing bonuses, considered in relation to some recent influences on Life Assurance Finance"—Mr. H. W. Andras.
- 24 *February* 1896.—Mr. James Chatham's Messenger Prize Essay "On the books and forms to be used in scheduling the particulars of the risks of a Life Assurance Company under its Assurance and Annuity contracts, for periodical or interim valuations, Distribution of Surplus, and for Investigation of the Rates of Mortality, Surrender and Lapse"—The Author.

30 *March* 1896.—“A translation of Mons. Bégault’s paper on the necessity for a Universal Notation”; together with an abstract of the discussion thereon—Mr. George King.

27 *April* 1896.—“(1) An investigation of some of the methods for deducing the Rates of Mortality, and of Withdrawal, in years of duration; with (2) the application of such methods to the computation of the rates experienced, and the special benefits granted, by Clerks’ Associations”—Mr. T. G. Ackland.

For the Examinations held in the United Kingdom on 17, 18, 20, and 21 April last, 154 candidates presented themselves, namely:

62	for Part	I.
52	„	„ II.
18	„	„ III, Section A.
22	„	„ III, „ B.

Of these the following numbers were successful:

35	in Part	I.
17	„	„ II.
9	„	„ III, Section A.
10	„	„ III, „ B.

The following are the successful candidates, the names in each class being arranged alphabetically.

PART I.

Examiner—PROF. S. L. LONEY, M.A.

Supervisors—MESSRS. T. E. YOUNG and C. D. HIGHAM.

Class I:

Maxwell Hicks.

Class II:

W. H. Brown.
C. Mack.

C. A. Penny.
H. S. Saunders.

Class III:

C. H. Ashley.
F. W. Briggs.
H. N. Briggs.
Harold Brown.
G. Gillies.
D. L. Hatten.
A. T. Heslop.
E. F. Horn.
R. F. Jones.
W. M. Jones.
C. W. Kenchington.
H. de C. McArthur.
H. M. Mansel-Jones.
G. F. H. Morgan.
E. W. Newnham.

H. J. P. Oakley.
C. W. Panton.
J. P. Parry.
C. E. Pearce.
G. P. A. Richards.
B. A. Roodenburch.
P. Schouten.
L. G. Smith.
R. P. Smith.
R. A. C. Thomas.
G. A. Vokins.
S. V. Windett.
A. H. Woolmer.
W. J. Yeldham.
A. S. Young.

PART II.

Examiners—MESSRS. G. J. LIDSTONE, W. P. PHELPS, M.A., and
S. G. WARNER.

Class I:

None.

Class II:

H. H. Austin.
H. E. W. Lutt.
J. C. Macmillan.

A. Moorhouse.
H. M. Trouncer.
A. T. Winter.

Class III:

H. T. Adlard.
F. Appleton.
C. R. V. Courtts.
O. H. Covington.
W. I. Featherstonehaugh.

W. H. Hodgson.
P. H. Merfield.
R. P. Parker.
A. H. Raisin.
E. B. Wilkinson.

H. C. Wyley.

PART III.

Examiners—MESSRS. M. N. ADLER, H. W. MANLY, J. SORLEY,
G. F. HARDY, GEO. TODD, and L. M. SIMON.

SECTION A.

Class I:

None.

Class II:

*A. Levine.
†J. N. Lewis.

G. F. Robinson.
L. A. Wintle.

Class III:

J. M. Allen.
*J. A. Archer.

H. W. Glover.
G. E. May.

J. Holliday.

SECTION B.

Class I:

None.

Class II:

†H. J. Baker.
O. Kentish.

*A. Levine.
H. Moir.

Class III:

*J. A. Archer.
K. W. Elder.
†J. F. Moran.

R. G. Salmon.
†W. A. Sim.
†L. Stahlschmidt.

Those marked (*) passed in both sections.

Those marked (†) have now completed the examination for the Class of Fellow.

In the Colonies the Examination entries numbered 50, as under:

For Part	I, 21.
„	II, 17.
„	III, Section A, 6.
„	III, „ B, 6.

The results of the Colonial Examinations will be duly announced.

A prize of the value of 25 guineas from the Messenger Legacy Fund has been awarded to Mr. James Chatham, F.I.A., for his Essay on the subject chosen by the Council. The Essay was read by the author at the sessional meeting on the 24th February 1896.

The tables calculated by Dr. Sprague on the basis of his Select Mortality Experience Investigations, which he has kindly placed at the disposal of the Institute, have been considerably added to and rendered complete under the supervision of Mr. H. C. Thiselton, who has devoted much time and labour to the work. The tables, together with an introduction by Dr. Sprague, will very shortly be ready for publication.

The Index to the ten volumes of the *Journal* from the twentieth to, and inclusive of, the thirtieth of the series has been completed in the most thorough and exhaustive manner under the superintendence of Mr. G. H. Ryan, the Honorary Editor; and a copy of the work, which forms an invaluable auxiliary to the *Journal*, has been sent to every Member of the Institute.

The Council have to announce with great regret that Mr. Ryan, owing to the pressure of official business, has reluctantly felt compelled to resign the position of Honorary Editor of the *Journal*, which he has filled with marked ability during the last three years, following on an experience of three years as Assistant Editor. The question of the appointment of his successor is under the consideration of the Council.

The first International Congress of Actuaries was held in Brussels in September last, under the Presidency of M. Mahillon.

All the chief countries of Europe, the United States of America, Australia, South Africa and Japan, were represented, and the Institute by the President, Mr. A. J. Finlaison, C.B., Mr. H. W. Manly, Mr. George King, Mr. J. Chisholm and Mr. G. H. Ryan, while the Faculty of Actuaries sent Dr. Sprague and Mr. Meikle.

The Members of the Institute will have learnt with much regret of the sad death of M. Mahillon, to whose untiring energy the International Congress in great measure owed its foundation, and whose ability and courtesy as President added greatly to the success of the gathering.

It was resolved to resume the Congress in London in the spring of 1898.

A complimentary dinner was given by the Members of the Institute to Mr. A. H. Bailey in December last on his retirement after many years of Insurance practice. Mr. Bailey has been a member and staunch supporter of the Institute almost from its foundation in 1848.

In connection with the new Mortality Experience Investigation by the Institute, in conjunction with the Faculty of Actuaries, the Council have to report that the majority of the contributing Offices have already sent in the cards recording their experience.

The Council hope the remaining Offices will all send in their cards within the next few months, but, in order that the work of completing, sorting and classifying the cards need not be delayed, arrangements are being made that it should be promptly proceeded with.

The Council is again much indebted to the honorary Examiners, who have bestowed great care and labour upon the important work intrusted to them.

PROCEEDINGS AT THE ANNUAL GENERAL MEETING.

The Annual General Meeting of the members was held at Staple Inn Hall, on Monday, 1st June, the President, Mr. A. J. Finlaison, C.B., in the chair.

The Report of the Council (given on p. 52) having been read,

The PRESIDENT, in proposing the adoption of the Report, said: The number of members had been more than maintained, notwithstanding the inevitable losses they had to deplore. In the year which ended on the 31st March last, the Institute had lost one Fellow, by the death of Mr. H. J. Puckle, and since that date, he regretted to add, that Mr. William King, a member of long standing, and Dr. William Smith, one of the few remaining original Fellows, had died. They had also to mourn the sad premature death of a talented corresponding member, Mr. Léon Mahillon. The accounts indicated that the Institute prospered, while they showed that the Council had spent nearly £1,800 in the course of the year. Of this amount, about £360 was incurred for the *Journal*, £220 for the New Mortality investigation, £215 for tables published by the Institute, £35 for the library, and £26. 5s. for prizes. The expenditure on nearly all these objects would have been far greater had it not been for the honorary services that had been generously given by many of the members, in connection with these and other matters. The papers read during the past Session, had formed a happy combination of theoretical and practical subjects. The standard of the examinations had been fully maintained; and nearly the same proportion of candidates had succeeded in passing as in the previous year. Dr. Sprague's Select Mortality Tables, which he had generously placed at the disposal of the Institute, had been prepared under the able supervision of Mr. Thiselton, and would shortly be ready for publication—Dr. Sprague having contributed a most valuable introduction. An Index to the ten volumes of the *Journal* (ending with the thirtieth volume), which has been prepared by the honorary editor, Mr. Ryan, had been issued to the members, who, he was confident, would duly appreciate the great care and labour bestowed upon its production. He regretted to announce that Mr. Ryan's many engagements had compelled him reluctantly to resign the onerous post of Editor of the *Journal*. With regard to the New Mortality investigation, he was pleased to report that there were only a few offices who had not yet furnished the necessary cards, and it was hoped that these would now very shortly be sent in. The Council had under consideration a scheme for the supervision of the work, which he thought would prove very successful.

The PRESIDENT then formally moved: "That the Report and Accounts be received and adopted."

Mr. R. P. HARDY, in seconding the motion, said the amount and quality of the work accomplished in the year showed no diminution either in volume or in value. Amongst the workers of the year, he thought he was entitled to claim for Mr. Chatham, who had won the Messenger prize offered by the Council, the place of honour. The Report made two announcements, which would be received with profound regret. The first was the retirement of Mr. Ryan from the editorship of the *Journal*. The *Journal* had never stood higher than it did now, nor had its contents ever been more varied or more useful. Mr. Ryan had left another memorial in

the new Index to the *Journal*, every line of which bore testimony to the knowledge and care brought to bear in its preparation. The other announcement was one that came particularly home to him—he referred to the retirement from active life of his old friend Mr. Bailey, who, he rejoiced to say, was still bodily with them, and still in heart and spirit with the Institute.

The resolution was unanimously adopted.

Messrs. Milner and Todhunter, having been appointed Scrutineers, a ballot was taken for the election of President, Vice-Presidents, Council, and Officers for the ensuing year. The Scrutineers reported that the following gentlemen, recommended by the Council, had been unanimously elected:

President.

THOMAS EMLEY YOUNG, B.A.

Vice-Presidents.

GEORGE KING.

HENRY COCKBURN.

HENRY WILLIAM MANLY.

WILLIAM HUGHES.

Council.

ALFRED BARTON ADLARD.

AUGUSTUS HENDRIKS.

MARCUS NATHAN ADLER, M.A.

CHARLES DANIEL HIGHAM.

HENRY WALSHINGHAM ANDRAS.

GEORGE KING.

*THOMAS G. C. BROWNE.

FRANCIS LAING.

ARTHUR FRANCIS BURRIDGE.

HENRY WILLIAM MANLY.

JAMES CHISHOLM.

*GEOFFREY MARKS.

HENRY COCKBURN.

LOUIS MICHAEL SIMON.

FRANCIS E. COLENSO, M.A.

THOMAS BOND SPRAGUE, M.A.

*GEORGE STEPHEN CRISFORD.

WILLIAM SUTTON, M.A.

*ROBERT CROSS.

HERBERT CECIL THISELTON.

*JOHN JAS. WALKER DETCHAR.

SAMUEL GEORGE WARNER.

ALEX. JOHN FINLAISON, C.B.

ERNEST WOODS.

NIEL BALLINGAL GUNN.

FRANK BERTRAND WYATT.

GEORGE FRANCIS HARDY.

*WM. JOS. HUTCHINGS WHITTALL.

*WILLIAM HUGHES.

THOMAS EMLEY YOUNG.

* New Members of Council.

Treasurer.

JAMES CHISHOLM.

Honorary Secretaries.

FRANK BERTRAND WYATT.

| ARTHUR FRANCIS BURRIDGE.

Mr. T. E. YOUNG returned thanks on behalf of the Council, Vice-Presidents and Officers. He also thanked the meeting for having elected him to the high office of President, and said, he would employ any capacity for service he possessed for the Institute and its members, seeking in every way to preserve untarnished the reputation of their body which had been so excellently maintained by his predecessor.

Mr. REA proposed, as auditors for the ensuing year, the re-election of Messrs. Holt and Jellicoe, and the election of Mr. H. R. Harding, in the place of Mr. Moore, who retired in accordance with their rule.

Mr. W. S. HIGHAM seconded the resolution, which was carried.

Mr. T. C. DEWEY proposed a vote of thanks to the President, Vice-Presidents, Council, Officers and Examiners, for their services during the past year.

Mr. L. F. HOVIL seconded the motion, which was cordially agreed to.

The PRESIDENT briefly acknowledged the vote on his own behalf, and on behalf of the Vice-Presidents, Council, Treasurer, and other Officers.

On the motion of MR. T. G. ACKLAND, seconded by Mr. J. H. Barnes, a vote of thanks was given to Messrs. Moore, Holt and Jellicoe, for their services as Auditors during the past year.

The PRESIDENT then announced that the meeting was adjourned to 30th November next.

COLONIAL EXAMINATIONS.

Examinations were held on 17 and 18 April 1896, at Sydney, Melbourne, Adelaide, Wellington, Montreal, and Toronto, with the following results:

PART I.

Twenty-one Candidates sent in their names, of whom twenty presented themselves, and thirteen passed as follows:

Class I:

Diamond, G. F. (Sydney).
Little, J. F. (Sydney).
Reid, E. E. (Toronto).
Stuckey, E. J. (Adelaide).
Wilson, J. S. (Melbourne).

Class II:

Cooper, N. C. (Sydney).
Macaulay, T. B. (Montreal).
Mills, T. P. (Sydney).
Pownall, H. W. (Sydney).

Class III:

Hallman, M. S. (Toronto).
Kember, O. G. (Wellington).
Rudd, A. J. (Melbourne).
Walker, D. E. (Sydney).

PART II.

Seventeen Candidates sent in their names, of whom fifteen presented themselves, and seven passed as follows:

Class II:

Harris, F. J. (Sydney).
Martin, S. G. (Wellington).

Class III:

Jobson, A. (Melbourne).
Sanderson, F. (Montreal).
Shlager, J. (Melbourne).
Stuckey, J. J. (Adelaide).
Wickens, C. H. (Melbourne).

PART III (SECTION A).

Six Candidates sent in their names, of whom four presented themselves, and two passed as follows:

Class II:

* Henderson, R. (Montreal).

Class III:

Bradshaw, T. (Toronto).

PART III (SECTION B).

Six Candidates sent in their names, of whom three presented themselves, and one passed, namely:

* *Class II:*—Henderson, R. (Montreal).

* Passed in both sections.

Additions to the Library.

THE following works have been added to the Library since the publication of the *Journal* for October 1895 :

*By whom presented
(when not purchased).*

Accountants and Anditors, Society of.
List of Members and Library Catalogue, 1895 and 1896. *The Society.*

Actuarial Society of Edinburgh.
Transactions of the, Vol. 3, Nos. 10, 11, 12 and 13. *The Society.*

Annals of the Actuaries' Club.
A. F. Burridge.

Associated Scottish Life Offices.
Resolutions and Proceedings, 1895. *Hy. Cockburn.*

Aubrey (W. H. S.) LL.D.
Stock Exchange Investments:
The Theory; Methods; Practice; and Results. *The Author.*

Australian Mutual Provident Society.
Forty-seventh Annual Report, 1896. *The Society.*

Bailey (Thomas).
Records of Longevity: with an introductory discourse
on Vital Statistics, 1857. *Purchased.*

Belgium.
Compte Rendu des Operations et de la Situation, de
la Caisse Générale d'Epargne et de Retraite,
1895. *Belgian Government.*
Premier Congrès International d'Actuaires tenu a
Bruxelles, 2 au 6 Septembre, 1896. Documents. *M. A.M. Bégault.*

Black (Morris A.).
The Assurance of Diseased and Doubtful Lives on a
new principle. 1861. *G. H. Ryan.*

Bödiker (Dr. T.).
Die Arbeiterversicherung in den Europäischen Staaten.
Leipzig, 1895. *Purchased.*

Bowser (Wilfred A.).
Valuation and Other Tables. *The Author.*

Canada Life Assurance Co.
Mortality Experience for the Years 1847 to 1893.
Two copies. *The President of the C. L. A. Co.*

Clare (George).
A Money Market Primer, and Key to the Exchanges. *Purchased.*

Coglan (T. A.).
Statistics of the Seven Colonies of Australasia, 1861-
1894.
The Wealth and Progress of New South Wales,
1894. *The Government Statistician.*

*By whom presented
(when not purchased).*

Eldridge (G. D.).

Life Insurance: The System, one; The Methods, many. *The Author.*

Equitable Life Assurance Society.

Report on the Valuations of the, 1896. Two copies. *H. W. Manly.*

Faculty of Actuaries.

List of Members, 1895 and 1896. *Hy. Cockburn.*

Finlaison (John).

State and Progress of The Life Annuity Sinking Fund, 1829-1835. Showing the loss involved through the use of the Northampton Table (Manuscript). *A. J. Finlaison.*

Accounts relative to The Aged and Select Nominees, together with Alphabetical Index and Data, arranged for obtaining Mortality Table of Female Annuitants (Manuscript). *A. J. Finlaison.*

Tables giving results of two early investigations (Manuscript). *A. J. Finlaison.*

Tables of Single Life 4 per-cent Annuities, Half Ages, and Joint Life (Two Males) for all combinations of ages, 0 to 93 (Manuscript). *A. J. Finlaison.*

Tables of Annuities on Two Joint Lives—interest $4\frac{1}{2}$ per-cent (Manuscript). *A. J. Finlaison.*

Tables of Joint Life Annuities for all Ages, founded upon the observations of 1840—interest 4 per-cent: (1) Two Males; (2) Two Females (Manuscript). *A. J. Finlaison.*

Tables of Joint Life Annuities for all Ages, founded upon the observations of 1840—interest 4 per-cent: (1) Male and Female (Male the elder); (2) Male and Female (Female the elder) (Manuscript). *A. J. Finlaison.*

Tables of Annuities on Two Joint Lives—interest 5 per-cent; also Valuation of Leases held on the tenure of lives within the Diocese of Gloucester (Manuscript). *A. J. Finlaison.*

France.

Bulletin trimestriel de L'Institut des Actuaire Français. *The Institute.*

Tables de Mortalité du Comité des Compagnies D'Assurances a primes fixes sur la vie. *The Committee.*

Fuller (Frank Baden) B.A.

The Law of Friendly Societies, 1896. *Purchased.*

Germany.

Zeitschrift des Königlich Preussischen Statistischen Bureau, 1895-6. *German Government.*

Gordon (J. W.).

Mathematical Tables. *The Insurance Spectator of London.*

Hewat (Archibald).

Widows' and Pension Funds, 1896. *Purchased.*

*By whom presented
(when not purchased).*

Holland.

Jaarboekje van de Vereeniging voor Levensverzekering.
1896.

*The Algemeene Maatschappij van
Levensverzekering en Lijfrente.*

Van Leven en Sterven het Verleden en Heden der
Levensverzekering. By J. Van Schevichaven.

The Author.

Van Rekeningh in Spelen van Geluck.

Christian Huygens.

Algemeene Maatschappij.

Vereeniging voor Levensverzekering te Amsterdam.

Publications of the.

Leonard Wolterbeek.

Ditto

ditto.

A. G. Mackenzie.

Vereeniging van Wiskundige Adviseurs bij Nederlandsche
Maatschappijen voor Levensverzekering. Publica-
tions of the.

The Society.

Verslag over het jaar 1895, het Drie en Dertigste
Boekjaar, Uitgebracht door het bestuur der
Nationale Levensverzekering — Bank aan de
Algemeene Vergadering Aandeelhouders.

Anonymous.

Institute of Actuaries' Club.

Record and Origin of the.

T. E. Young.

Institute of Chartered Accountants in England and Wales.

Charter of Incorporation, Bye-Laws, Report and
Accounts, and List of Members, 1895 and 1896.

The Institute.

Ireland, Insurance Institute of.

Report 1895-96.

The Institute.

Laurent (H.).

Théorie et Pratique des Assurances sur la vie.

Purchased.

Leslie (George).

The Rates of Mortality in New Zealand.

The Author.

Macleod (Henry Dunning) M.A.

The History of Economics. London 1896.

Purchased.

Manchester Insurance Institute.

Reports 1893-4, and 1894-5.

The Insurance Institute.

Mental and Physical Conditions of Childhood.

Report on the Scientific Study of the.

Anonymous.

Monilaws (W. M.).

The Surplus Funds of Life Assurance Offices. How
arrived at and how disposed of, 1895. Eighth
Year of Publication.

Purchased.

Mutual Life Insurance Company of New York.

Examination of the.

Hy. Cockburn.

Mutual Life Underwriters.

Proceedings of the Twentieth Annual Session of the
National Convention of.

G. D. Eldridge.

*By whom presented
(when purchased).*

Official Year Book of the Scientific and Learned Societies
of Great Britain and Ireland. 1896.

Purchased.

Old Age Pensions.

A Description of the Principal Schemes, and some New
Suggestions.

Hq. Cockburn.

Parliamentary Papers.

Colonies.

Canada.

Reports of the Superintendent of Insurance
for 1891, 1892, 1893 and 1894.

T. B. Macaulay.

New South Wales.

*The Government
Statistician.*

Statistical Register, 1894 and previous years.

New Zealand.

Official Year Book for 1895.

N. Z. Government.

Statistics of the Colony of, for 1894.

N. Z. Government.

Friendly Societies.

Nineteenth Annual Report by the Registrar of

The Registrar.

South Australia.

The First Report by the Public Actuary
relating to Friendly Societies, 1888-1892.

*The Public
Actuary of S. A.
Agent-General
for Victoria.*

Victoria.

Statistical Register of the Colony of, 1893.

The 16th and 17th Annual Reports of the
Government Statist in Connexion with
Friendly Societies.

*The Government
Statist.*

Western Australia.

The Friendly Societies Acts and Regulations,
1896.

*The Registrar of
Friendly Societies.*

Friendly Societies Bill, 1896.

Purchased.

Life Assurance Companies (payment into Court) Bill.

Hq. Cockburn.

Life Assurance Companies Returns, 1895.

The Board of Trade.

Periodicals.

Institute of Bankers' Journal.

The Institute.

" " Index to vols. 1-15,
1879-1894.

G. H. Ruan.

Insurance Record, 1895.

The Editor.

Insurance Register, 1896.

C. & E. Layton.

Insurance Spectator of London, 1895.

The Editor.

London Mathematical Society's Journal.

The Society.

Post Magazine.

The Editor.

Post Magazine Almanac.

The Editor.

Royal Statistical Society's Journal.

The Society.

Zeitschrift für Versicherungs-Recht und Wissenschaft

Bd. 1. Heft 2, 3, and 4, and B1. 11. Heft 1.

The Editor.

Phelps (James T.).

Life Insurance Savings.

The Author.

Pixley (Francis W.).

Auditors: Their Duties and Responsibilities. 7th Ed.
1896.

Purchased.

Royal Astronomical Society. 1892-95.

Memoirs of the.

The Society.

*By whom presented
(when not purchased).*

Royal Exchange Assurance.
An Historical Sketch.

Hy. Cockburn.

Strahan (J. A.).
A General View of the Law of Property.

Purchased.

Streeter (G. T. P.) B.A.
Metric System of Weights and Measures.

The Author.

Streeter (T. E.).
The Elements of Practical Book-keeping for use in
Schools. London, 1896.

The Author.

Sweden.
Über Zuschlagsprämien und einige damit Zusammen-
hängende Fragen. by Dr. Hans Tisllius. Two Copies.

The Author.

Switzerland.
Denkschrift über die Höhe der finanziellen Belastung.
Berne, 1895.
Mémoire sur la Charge Financière des Caisses contres
les Maladies. Berne, 1895.
Recherches Techniques sur L'Assurance Mutuelle contre
les Accidents. Berne 1895.
Versicherungstechnische Untersuchungen über die nach
dem Entwurfe zu einem Bundesgesetze einzurich-
tende eidgenössische Unfallversicherung. Berne,
1895.
Rapport du Bureau Fédéral des assurances sur les Entre-
prises Privées en Matière d'Assurances en Suisse
en 1894.
Handbuch der Zinseszins, Renten, Anleihen, und Obli-
gationen—Rechnung von V. Baerlocher.

Swiss Government.

G. H. Ryan.

**Value of Family History and Personal condition in estimating
a Liability to Consumption.**

*The Mutual Life In-
surance Co. of N. Y.*

Victoria, the Insurance Institute of.
Report of Proceedings. Session, 1895.

The Institute.

Watt (Peter).
Progress and present state of the Science of Life
Assurance, with Thermometrical Tables. Also,
Observations on Health Insurance, &c. Edinburgh.

Aug. Hendriks.

Weskett, (John).
A complete digest of the Theory, Laws, and Practice of
Insurance. London, 1781.

Purchased.

Yorkshire, Insurance Institute of.
Report 1894-5.

The Institute.

Young, (T. E.).
La Loi Allemande D'Assurance contre L'Invalidité et la
Vieillesse. Two Copies. Translated from the English
by G. H. Adan.

M. G. H. Adan.

The following duplicate copies of works already in the Library have also been added :

*By whom presented
(when not purchased).*

Ackland (T. G.) and G. F. Hardy.

Graduated Exercises and Examples. Two Copies.

Purchased.

Babbage (Charles).

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- (I). *An Investigation of some of the Methods for deducing the Rates of Mortality, and of Withdrawal, in Years of Duration; with (II) the Application of such Methods to the computation of the Rates experienced, and the Special Benefits granted, by Clerks' Associations.* By THOMAS G. ACKLAND, *Fellow of the Institute of Actuaries.*

[Read before the Institute, 27th April, 1896.]

THE investigation of the experience, and valuation of the liabilities, of what are known as Clerks' Associations, present some features of special interest; and I have thought that it might be useful to discuss and explain some of the methods that have been employed in analyzing the data, and in deducing therefrom the tables of money values appropriate for the valuation of the risks.

In dealing with this subject I have had occasion to investigate the most suitable methods of deducing the numbers exposed to risk, and the rates of mortality and of withdrawal, as affected by the ages of the subscribing members, and by the duration of their membership respectively; and I propose, in the first part of the present paper, to discuss, in some detail, such of the methods suggested for dealing with the experience amongst assured lives, as may seem to attain, in whole or in part, the desired objects. I shall investigate the formulæ appropriate to give effect to these methods in a convenient tabular form; and shall illustrate their practical working, and compare their results, by an analysis of a portion of the experience of a Clerks' Association, the data of which I have recently investigated. I shall then seek to show the adaptability of the several methods and formulæ suggested to an investigation of lives insured in an assurance office, and the distinctive characteristics of such an experience; also, how the methods are modified in the case where the period of observation is limited by calendar years and by policy years respectively.

In the second portion of this paper, I propose to state the general characteristics of Clerks' Associations, as to contribution, benefit, and the like; and to investigate the methods of computing the values of their varying benefits at death; also of deducing the rate, and computing the value, of the special benefit during non-employment, granted by such associations; with due allowance throughout for the effect of withdrawals or secessions upon the values ascertained, and generally upon the valuation reserves.

(1). INVESTIGATION OF METHODS FOR DEDUCING THE RATES OF MORTALITY AND OF WITHDRAWAL IN YEARS OF DURATION.

(A). PERIOD OF OBSERVATION LIMITED BY CALENDAR YEARS.

ILLUSTRATIVE EXPERIENCE.

It will be sufficient for the present to state (reserving further details for the second portion of this paper) that the data here employed for purposes of illustration formed part of the experience of a Clerks' Association during the period of five complete calendar years, 1888 to 1892 inclusive; that the age at entry recorded on the cards constituting the experience was in all cases the "office age" upon which the member's subscriptions at entry were based; that the subscriptions were throughout payable, at monthly intervals, upon the first day of each calendar month; and that the number of withdrawals or secessions was heavy, especially in the early years of duration.

With a view to investigating the rates of mortality and of withdrawal, as well as the rate of "non-employment", as affected by the age of the member, and by his duration, it was decided to tabulate the facts, and deduce the experience throughout, according to office ages at entry and years of duration. By this means, although at the expense of some additional labour, the material was readily available for ascertaining, in respect of each particular rate investigated, whether it was in the main a function of the age, or of the year of duration, or of both age and duration; and the material could be readily combined, in such a way as to give effect to the conclusions in this respect, as indicated by the experience.

The data available, in the particular case here dealt with, were, however, evidently not sufficiently extensive to permit of trustworthy results, as tabulated for each successive age at entry, and for each year of duration; and it was therefore determined to group the entry ages quinquennially, and to consider each group as representing the experience of the central age at entry in the group. Thus, cases entering at office ages 18, 19, 20, 21, and 22, were all classed together as entrants at "Central Age at Entry (20)", and so on; and, assuming that the numbers entering, and the rates obtaining, on either side of the central age, did not greatly differ, or so differed as to introduce compensating errors, there was evidently no material departure from the truth involved in this assumption.

The formulæ employed, and presently to be developed, are,

however, applicable equally to the case of individual ages at entry; and where the experience is sufficiently extensive (as, for example, in the New Experience of the Institute of Actuaries), the natural and preferable course would be to tabulate the data, and deduce the results, for each separate age at entry.

The individual years of duration were, in all cases, scheduled and dealt with separately, and the duration being entered upon each card, the material was available for ascertaining, in each quinquennial group of entry-ages, and in each separate year of duration, the precise incidence of deaths, withdrawals, and of the benefit during non-employment, with a view to deducing the true rates of mortality, withdrawal, and benefit, respectively.

The several methods which have been proposed for dealing with a life experience according to "policy years", or years of duration, were all carefully referred to. These methods have been very usefully summarized and discussed by Mr. Ryan (*J.I.A.*, xxvi, 256), Mr. Chatham (*J.I.A.*, xxix, 81), and Mr. Whittall (*J.I.A.*, xxxi, 161); while some of them have been illustrated in a practical form by Dr. Sprague (*J.I.A.*, xxxi, 205) and Mr. Meikle (*J.I.A.*, xxxi, 229). None of the methods suggested, as applied by the above writers, appeared, however, in all respects to meet the case here investigated; and I have thought that it would be of interest to deal fully with their special application to this particular case, in the hope that a further discussion of these several methods might elucidate more fully their comparative advantages.

The methods which I have selected for illustration and comparison are—(1) the Exact Duration Method; (2) the Mean Duration Method; and (3) the Nearest Duration Method; each method being so applied as strictly to preserve the incidence of the cases throughout in their true years of duration.

I should have liked also to include in this investigation the special method suggested by Mr. G. F. Hardy and the late Mr. Rothery (*J.I.A.*, xxvii, 165), which may perhaps appropriately be described as the "Mean Age Method"; as well as that proposed by Mr. G. King (*J.I.A.*, xxvii, 218), which might be styled the "Nearest Age Method"; as also the somewhat similar, but in some respects inferior method, called by Dr. Sprague the "Final Age Method" (*J.I.A.*, xxxi, 215); and especially as these methods are all very simple and facile in their operations. It appeared to me, however, that, although these methods were doubtless admirably adapted for the purposes designed by their respective authors, they certainly do not tabulate the cases

exposed to risk, and deduce the rates experienced, both of mortality and withdrawal, strictly in years of duration. This has been shewn, as it appears to me, clearly, by Mr. Whittall (*J.I.A.*, xxxi, 182-6); and I was, therefore, unable to include them as suitable for my present purpose. A further reason for their exclusion was, that they all involved data based upon the years of birth, or the birthdays, of the lives included; and, in the particular experience here selected for illustration of the selected methods, I had throughout no information whatever as to the dates of birth of the lives.

EXACT DURATION METHOD.

The durations of the cases were entered on the cards constituting the experience (1) as at the commencement of the period of observation, where the case was then in force; (2) as at the close of the period of observation, where the case was then existing; (3) as at exit during the period of observation, by death or by withdrawal. The duration was obtained in class (1) by taking the difference between the date of entry and January 1888; in class (2) by taking the difference between the date of entry and January 1893; and in class (3) by taking the difference between the date of entry and the date of exit. As all subscriptions were due on the first day of a calendar month, the durations were thus truly stated in integral months, excepting only in the cases of death, where the duration was stated with a possible error not exceeding a month (and averaging a fortnight) in any particular case. It may be added that this error would have been reduced by one-half, if the date of death had been entered on the cards, or computed for the purpose of ascertaining the duration, as the first day of the month *nearest* to the actual date of death*.

The durations were, however, recorded on the cards, not in years and months, but in years and equivalent decimals, correct to one decimal place. Thus, actual durations of 3 years and 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11 months were entered as 3·1, 3·2, 3·3, 3·3, 3·4, 3·5, 3·6, 3·7, 3·7, 3·8, 3·9. The fractional durations, thus expressed decimally, were found to be much more convenient, in practical use, than if expressed in years and months, being more easily cast and aggregated by inspection, and adapting themselves much more readily to the subsequent processes. Upon the assumption, which was found to be fully justified in this particular experience, that members

* If this modification be introduced, care must, however, be taken that the cases of death are throughout located in their true years of duration.

are equally likely to enter under observation, or to withdraw, in any month of the year, the decimal expressions will be found to give an average result in which the balance of errors is equal; the aggregate of the decimal expressions for the successive months of duration

$$(0 + \cdot 1 + \cdot 2 + \cdot 3 + \cdot 3 + \cdot 4 + \cdot 5 + \cdot 6 + \cdot 7 + \cdot 7 + \cdot 8 + \cdot 9),$$

and of the actual months of duration

$$(0 + 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 + 11),$$

being both equal to 5.5 years or 66 months.

While this plan was found to work well, and to introduce no perceptible error, in the particular experience here investigated, it is probable that, in the case of ordinary policy investigations, where the assurances will have a strong tendency to terminate at or near to quarterly intervals in the year of duration, the current quarter at exit, expressed decimally as .25, .50, .75, or 1.00, would give, on the whole, more satisfactory results.

The durations having thus been recorded upon the cards, they were first sorted into quinquennial groups of ages at entry, all cases entering at office ages 18 to 22 inclusive being grouped together as "Central Age at Entry (20)"; entrants between ages 23 to 27 inclusive as "Central Age (25)"; and so on, up to central age (45), which included the highest age at entry admissible under the rules. There were thus six groups of central ages at entry.

Taking now the group of cases entering at office ages 18 to 22 inclusive, or at the central age (20), which group I shall throughout employ to illustrate the methods here followed, the cards constituting the group were sorted into "survivors," in force on 1 January 1888, and "new entrants" coming on the books during the five years 1888-92 inclusive. The new entrants were counted, and their total number recorded. The "survivors" were then sorted, according to the curtate duration at entry on observation, as recorded on the cards, and so that all cases, for example, whose recorded duration at the commencement of the period ranged from 5.1 to 6.0 inclusive, were grouped together as of "curtate" duration 5. The number of survivors in each year of duration was then counted, and recorded in a tabular form; and the aggregate fractional exposure of the cases in each year of duration as recorded on the cards was cast by inspection, and tabulated against the number of corresponding cases.

The operation of casting the decimal figures, disregarding the integers, is rapid and easy; the only point to note being that, according to the method of classification here suggested, an

exact duration of 6·0 (for instance) is considered as of "curtate" duration 5 and "fractional" exposure 1·0; but this will be found in practice to give rise to no difficulty whatever.

It may be added that, in respect of the survivors at the commencement, as well as of the cases existing at the close, of the observation, it seems to be quite immaterial whether an integral duration of (say) 6·0 is classified among cases having six years duration and upwards (and counted as 0) or among cases having five years' duration and upwards (and counted as 1). In the case, however, of withdrawals and deaths, it seems to be clear that they can only be properly treated by considering them as cases of withdrawal (or of death) occurring at the end of the sixth year of membership, and therefore classifying them among emergents having five years' duration and upwards. This will be evident if consideration be given to the withdrawals, for instance, at the end of twelve months' duration, which should clearly, as it appears to me, be tabulated with the withdrawals occurring in the first year, for the purpose of correctly deducing the rate of withdrawal in that year.

The sorting would have been more symmetrical and a little more facile in this respect if all cases of fractional duration had been entered upon the cards at the next higher integer, or what may be called the "current year of duration" (leaving, of course, integral durations unaltered); and in this case the fractional period would have had to be separately dealt with by way of deduction. This appears to have been the plan most frequently followed in published investigations; but, after careful consideration, I arrived at the conclusion that the disadvantages of this plan of deduction would on the whole be greater than the slight difficulty involved in the special treatment of the cases of integral duration. To provide against any risk of possible error in the tabulations, a sort of "danger-signal" may be put up, by underlining the '0 in red ink upon the cards, in all cases of integral duration, as a reminder to the clerk that they are each to be counted for a full unit.

The number of original entrants, and the number and aggregate fractional duration at entry, of the "survivors" in each year of duration, having been thus recorded, the cards were combined, and re-sorted into cases *existing* at the close of the period of observation; cases *withdrawing* during the period; and cases *dying* during the period. The cards in each of these three groups were then sorted according to their "curtate" durations (for the existing) at the close of the observation, or (for withdrawals

and deaths) at exit; and the number of cases, and their aggregate fractional exposures, were then tabulated in their appropriate years of duration.

The following Schedule (A) shews the form in which the cases entering at "central age at entry (20)", and their fractional durations at entry and at exit, were tabulated, as a result of the above processes of sorting and grouping.

It will be remarked that the aggregate fractional exposures, as deduced from the cards, represents in the case of the "survivors" that portion of the current year of duration already expired at the commencement of the period of observation; and in the case of the "existing," "withdrawals," and "deaths," that portion of the current year of duration over which the cases were actually at risk during the period of observation.

It will also be observed that the fractional exposure of the death cases, as well as of the withdrawals, is, in this investigation, terminated at the date of exit, whether by death or withdrawal, and not continued until the end of the year of duration current at death. This was done advisedly, as the numbers exposed to risk, so arrived at, formed at the same time a suitable denominator for computing the rate of allowance during unemployment, and a convenient basis for deducing, by a simple modification, the denominators appropriate for the calculation of the rates of mortality, and of withdrawal, respectively. The formulæ adopted were also, throughout, more symmetrical and convenient upon this basis.

I now proceed to state the formulæ for deducing the number exposed to risk, and the rates of mortality and withdrawal. A definition of the leading symbols employed throughout this paper may first be given.

Let $\bar{E}_{[x]+t}$ = the number exposed to risk, in respect of cases entering at "office age" x , during the $(t+1)$ th year of duration; the bar over the E indicating that the number exposed is computed up to the actual cessation of the risk, whether by death, withdrawal, or close of the period of observation;* and let

$E_{[x]+t}$ = the number exposed to the risk of death and $(wE)_{[x]+t}$ = the number exposed to the risk of withdrawal during the $(t+1)$ th year of duration; where the cases of death and of withdrawal are respectively given a full year's exposure in the year of duration current at exit.

* The function $\bar{E}_{[x]+t}$ really represents the number exposed [in the $(t+1)$ th year] to risk of death or withdrawal; and would be appropriately employed in calculations (1) of benefits the continuance of which depends upon the member's being alive and in full membership (such as the annuity by which the members' subscriptions would be valued) (2) of benefits which would necessarily cease on the occurrence of either death or withdrawal (such as an allowance during sickness or non-employment).

SCHEDULE (A).—CENTRAL AGE AT ENTRY (20).

Table of the Numbers Surviving, Existing, Withdrawing, and Dying, in Years of Duration; with Fractional Exposure of the Survivors, as at Entry on Observation, and of the Existing, Withdrawals and Deaths, as at Close of Observation, or Exit.

Curtate Duration ⁻	SURVIVORS		EXISTING		WITHDRAWALS		DEATHS		Curtate Duration ⁻
	Cases	Fractional Exposure	Cases	Fractional Exposure	Cases	Fractional Exposure	Cases	Fractional Exposure	
t	$s_{[x]+t}$	$s'_{[x]+t}$	$e_{[x]+t}$	$e'_{[x]+t}$	$w_{[x]-t}$	$w'_{[x]-t}$	$d_{[x]-t}$	$d'_{[x]-t}$	t
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
0	137	81.8	82	44.2	51	24.4	1	0.5	0
1	101	56.5	83	43.3	47	27.1	5	3.6	1
2	132	73.2	7	6.1	44	25.5	3	1.3	2
3	120	65.8	72	40.1	45	20.2	1	0.5	3
4	129	73.9	75	43.1	55	25.3	3	1.3	4
5	101	56.3	69	41.1	44	25.3	3	1.1	5
6	77	37.8	62	37.9	33	18.7	6
7	89	53.6	74	43.4	23	13.5	4	2.3	7
8	87	49.2	82	45.5	27	16.0	5	2.8	8
9	76	41.2	89	51.1	16	7.9	1	0.3	9
10	64	37.7	69	39.6	19	8.8	3	1.8	10
11	48	24.0	48	24.2	20	10.2	6	3.3	11
12	15	7.6	65	39.9	11	6.6	2	1.0	12
13	19	11.1	69	37.7	13	6.5	2	0.3	13
14	19	11.0	59	34.7	11	6.0	1	0.6	14
15	10	7.2	46	28.0	2	0.7	2	1.1	15
16	13	7.7	35	17.7	5	3.2	16
17	15	9.6	11	6.5	3	2.3	17
18	10	3.8	13	6.5	3	1.1	18
19	4	3.0	16	9.7	1	0.8	19
20	9	7.2	7	5.1	3	1.8	20
21	10	6.6	11	7.1	1	0.6	1	0.2	21
22	15	8.1	10	6.0	22
23	16	6.6	9	3.5	1	0.2	23
24	10	6.5	4	3.0	24
25	7	3.0	9	7.2	1	0.1	1	0.6	25
26	10	6.6	1	0.8	1	0.5	26
27	12	5.6	1	0.9	27
28	14	5.6	1	0.9	28
29	8	5.8	2	0.3	29
30	5	2.6	30
	1,333	753.0	1,225	698.4	481	254.5	48	24.3	

* Integral durations of $(t+1)$ years being treated throughout as of "curtate" duration (t) , and "fractional exposure" (1.0).

NOTE.—The above experience represents a small portion only of the available data, the cases entering at the grouped entry ages 18 to 22 inclusive having been selected solely for the purpose of illustrating the different methods employed.

Also, let $q_{[x]+t}$ be the annual rate of mortality; and
 $(wq)_{[x]+t}$ the annual rate of withdrawal, in the $(t+1)$ th year of duration.

Let $s_{[x]+t}$ = the *survivors* in force at the commencement of the period of observation, having a duration exceeding t , but not exceeding $(t+1)$ years;
 $n_{[x]}$ = the *new entrants* at "office age" x , during the period of observation;
 $e_{[x]+t}$ = the cases *existing* at the close of the period of observation, having a duration exceeding t , but not exceeding $(t+1)$ years;
 $w_{[x]+t}$ = the *withdrawals* during the period of observation, having a duration at exit exceeding t , but not exceeding $(t+1)$ years;
 $d_{[x]+t}$ = the *deaths* during the period of observation, having a duration at death exceeding t , but not exceeding $(t+1)$ years.

Also, let $s'_{[x]+t}$ = the exact fractional exposure of the $s_{[x]+t}$ survivors, computed from the commencement of the $(t+1)$ th year of duration, up to the commencement of the period of observation;
 $e'_{[x]+t}$ = the exact fractional exposure of the $e_{[x]+t}$ cases existing, computed from the commencement of $(t+1)$ th year of duration, up to the close of the period of observation;
 $w'_{[x]+t}$ = the exact fractional exposure of the $w_{[x]+t}$ cases of withdrawal, computed from the commencement of the $(t+1)$ th year of duration, up to the date of withdrawal;
 $d'_{[x]+t}$ = the actual fractional exposure of the $d_{[x]+t}$ cases of death, computed from the commencement of the $(t+1)$ th year of duration, up to the date of death;

Also, let $(e + w + d) = f$,

that is, the *total decrement* in the year of duration, in respect of cases existing, withdrawing, and dying; and let

$$(s - f) = g,$$

that is, the *net movement* in the year of duration, among cases surviving, existing, withdrawing, and dying; and similarly, let

$$(e' + w' + d') = f'$$

$$(s' - f') = g'$$

where f' and g' represent the aggregate fractional exposures of the f and g cases respectively.

Then we have (*see* Appendix A):

$$\bar{E}_{[x]} = n_{[x]} + g_{[x]} - g'_{[x]} \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

$$E_{[x]+t} = \bar{E}_{[x]+t-1} + g_{[x]+t} - \Delta g'_{[x]+t-1} \quad . \quad . \quad . \quad (3)^*$$

formulae by which the numbers exposed to risk in the first year, and those in successive years, can be continuously computed.

$$\text{Also,} \quad \bar{E}_{[x]+t} = n_{[x]} + \Sigma g' - g'_{[x]+t} \quad . \quad . \quad . \quad . \quad (4)$$

an alternative formula for deducing the numbers exposed to risk by summation, in a convenient tabular form.

The appended Schedule (B) shows the arrangement of the data, and the computations for deducing, in respect of the group of entrants at "central age at entry (20)", the numbers exposed to risk, and the rates experienced, in successive years of duration. The cases surviving, existing, withdrawing, and dying, are entered respectively in columns (2), (3), (4) and (5); and under them are placed the corresponding fractional exposures of the cases, printed in a distinctive type. Column (6) gives the value of the total decrement $f(=e+w+d)$, and the corresponding fractional exposures $f'(=e'+w'+d')$; and column (7) gives the value of the net movement of cases $g(=s-f)$. In columns (8) to (10) the numbers exposed to risk are deduced by the summation formula (4); the values of (g) being continuously summed and added to the number of original entrants (n), in column (8); while the value of $g'(=s'-f')$ entered in column (9) is deducted from the values in column (8), and the result, entered in column (10), gives the value of the number exposed to risk $\bar{E}_{[x]+t}$. In columns (11) to (13) the alternative formula (3) is employed, the value of $\Delta g'_{[x]+t-1}$ being entered in column (11) and deducted from the values of $g_{[x]+t}$ in column (7), and the difference entered in column (12); the numbers exposed to risk being then obtained by continuous addition in column (13), starting with the value of $n_{[x]}$. In column (14) the numbers exposed to the risk of death are obtained by the formula

$$E_{[x]+t} = \bar{E}_{[x]+t} + (d-d')_{[x]+t},$$

and the numbers exposed to the risk of withdrawal in column (16) by the formula

$$(wE)_{[x]+t} = \bar{E}_{[x]+t} + (w-w')_{[x]+t} \text{ (see Appendix C).}$$

*The formulae are numbered consecutively in the Appendices; and to avoid confusion, the same numbering has been employed in those formulae cited in the text.

SCHEDULE (B).—OBSERVATION EXTENDING OVER
Table shewing alternative methods of deducing the Numbers Exposed to duration, and with exact Fractional

Curtate Duration	Survivors	Existing	Withdrawals	Deaths	Total Decrement	Net Movement	SUMMATION METHOD		
t	$s_{[x]+t}$ $s'_{[x]+t}$	$e_{[x]+t}$ $e'_{[x]+t}$	$w_{[x]+t}$ $w'_{[x]+t}$	$d_{[x]+t}$ $d'_{[x]+t}$	$f_{[x]+t}$ $f'_{[x]+t}$	$g_{[x]+t}$ $= (s-f)$	$n_{[x]}$ $+ \Sigma e'(g)$	$g'_{[x]+t}$ $= (s'-f')$	$E_{[x]+t}$ $= (S)-(H)$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
0	187 81'8	82 44'2	51 24'4	1 0'5	184 69'1	+ 3	$n_{[x]}=421$ 424	— 12'7	411'3
1	101 56'5	83 43'3	47 27'1	5 3'6	185 74'0	— 34	390	— 17'5	407'5
2	132 73'2	7 6'1	44 25'5	3 1'3	54 32'9	— 78	468	+ 40'3	427'7
3	129 65'8	72 40'1	45 20'2	1 0'5	118 60'8	— 2	470	— 5'0	465'0
4	129 73'9	75 43'1	55 25'3	3 1'3	133 69'7	— 4	466	+ 4'2	461'8
5	101 56'3	69 41'1	44 25'3	3 1'1	116 67'5	— 15	451	— 11'2	462'2
6	77 37'8	62 37'9	33 18'7	—	95 56'6	— 18	433	— 18'8	451'8
7	89 53'6	74 43'4	23 13'5	4 2'3	101 59'2	— 12	421	— 5'6	426'6
8	87 49'2	82 45'5	27 16'0	5 2'8	114 64'3	— 27	394	— 15'1	409'1
9	76 44'2	89 51'1	16 7'9	1 0'3	106 59'3	— 30	364	— 15'1	379'1
10	64 37'7	69 39'6	19 8'8	3 1'8	91 50'2	— 27	337	— 12'5	349'5
11	48 24'0	48 24'2	20 10'2	6 3'3	74 37'7	— 26	311	— 13'7	324'7
12	15 7'6	65 39'9	11 6'6	2 1'0	78 47'5	— 63	248	— 39'9	287'9
13	19 11'1	60 37'7	13 6'5	2 0'3	84 44'5	— 65	183	— 33'4	216'4
14	19 11'0	59 34'7	11 6'0	1 0'6	71 41'3	— 52	131	— 30'3	161'3
15	10 7'2	46 28'0	2 0'7	2 1'1	50 29'8	— 40	91	— 22'6	113'6
16	13 7'7	35 17'7	5 3'2	..	40 20'9	— 27	64	— 13'2	77'2
17	15 9'6	11 6'5	3 2'3	..	14 8'8	+ 1	65	+ 0'8	64'2
18	10 3'8	13 6'5	3 1'1	..	16 7'6	— 6	59	— 3'8	62'8
19	4 3'0	16 9'7	1 0'8	..	17 10'5	— 13	46	— 7'5	53'5
20	9 7'2	7 5'1	3 1'8	..	10 6'9	— 1	45	+ 0'3	44'7
21	10 6'6	11 7'1	1 0'6	1 0'2	13 7'9	— 3	42	— 1'3	43'3
22	15 8'1	10 6'0	10 6'0	+ 5	47	+ 2'1	44'9
23	16 6'6	9 3'5	1 0'2	..	10 3'7	+ 6	53	+ 2'9	50'1
24	10 6'5	4 3'0	4 3'0	+ 6	50	+ 3'5	55'5
25	7 3'0	9 7'2	1 0'1	1 0'6	11 7'9	— 4	55	— 4'9	59'9
26	..	10 6'6	1 0'8	1 0'5	12 7'9	— 12	43	— 7'9	50'9
27	..	12 5'6	..	1 0'9	13 6'5	— 13	30	— 6'5	36'5
28	..	14 5'6	1 0'9	..	15 6'5	— 15	15	— 6'5	21'5
29	..	8 5'8	..	2 0'3	10 6'1	— 10	5	— 6'1	11'1
30	..	5 2'6	5 2'6	— 5	..	— 2'6	2'6
	1,333 753'0	1,225 698'4	481 254'5	48 24'3	1,754 977'2	— 421	6,210	..	6,434'2
						— 224'2	..

FIVE CALENDAR YEARS.—EXACT DURATION METHOD.—SCHEDULE (B).
Risk, and the Rates, of Mortality and of Withdrawal, in true years of Exposures.—Central Age at Entry (20).

CONTINUOUS METHOD			MORTALITY		WITHDRAWAL		Currate Duration
$\Delta g'_{[x]+t-1}$	$-\Delta g'_{[x]+t-1}$	$\bar{E}_{[x]+t}$	Exposed $= \bar{E} + (d - d')$	Rate $\frac{Q'_{[x]+t}}{d} = \bar{E}$	Exposed $= \bar{E} + (w - w')$	Rate $\frac{(w'Q')_{[x]+t}}{w} = \bar{E}$	
(11)	(12)	(13) $\mu^{(2)}_{[x]} = 421$	(14)	(15)	(16)	(17)	(18)
+ 12.7	- 9.7	411.3	411.8	.00243	437.9	.1165	0
- 30.2	- 3.8	407.5	408.9	.01223	427.4	.1100	1
+ 57.8	- 20.2	427.7	429.4	.00609	446.2	.0986	2
- 35.3	- 37.3	465.9	465.5	.00215	480.8	.0919	3
- 0.8	- 3.2	461.8	463.5	.00647	491.5	.1119	4
- 15.4	+ 0.4	462.2	464.1	.00646	480.9	.0915	5
- 7.6	- 10.4	451.8	451.8	..	466.1	.0708	6
+ 13.2	- 25.2	426.6	428.3	.00634	436.1	.0527	7
- 9.5	- 17.5	460.1	411.3	.01216	420.1	.0643	8
0	- 30.0	379.1	379.8	.00263	387.2	.0413	9
+ 2.6	- 29.6	349.5	350.7	.00856	359.7	.0528	10
- 1.2	- 24.8	324.7	327.4	.01833	334.5	.0598	11
- 26.2	- 36.8	287.9	288.9	.00692	292.3	.0376	12
+ 6.5	- 71.5	216.4	218.1	.00917	222.9	.0583	13
+ 3.1	- 55.1	161.3	161.7	.00618	166.3	.0662	14
+ 7.7	- 47.7	113.6	114.5	.01747	114.9	.0174	15
+ 9.4	- 35.4	77.2	77.2	..	79.0	.0633	16
+ 14.0	- 13.0	64.2	64.2	..	64.9	.0462	17
- 4.6	- 1.4	62.8	62.8	..	64.7	.0464	18
- 3.7	- 9.3	53.5	53.5	..	53.7	.0186	19
+ 7.8	- 8.8	44.7	44.7	..	45.9	.0654	20
- 1.6	- 1.4	43.3	44.1	.02268	43.7	.0229	21
+ 3.4	- 1.6	44.9	44.9	..	44.9	..	22
+ 0.8	+ 5.2	50.1	50.1	..	50.9	.0197	23
+ 0.6	+ 5.4	55.5	55.5	..	55.5	..	24
- 8.4	+ 4.4	59.9	60.3	.01658	60.8	.0165	25
- 3.0	- 9.0	50.9	51.4	.01945	51.1	.0196	26
+ 1.4	- 14.4	36.5	36.6	.02732	36.5	..	27
0	- 15.0	21.5	21.5	..	21.6	.0463	28
+ 0.4	- 10.4	11.1	12.8	.15625	11.1	..	29
+ 3.5	- 8.5	2.6	2.6	..	2.6	..	30
..	..	6,434.2	6,457.9	..	6,660.7	..	
- 2.6	- 418.4	

Finally, in columns (15) and (17) the rates of mortality and of withdrawal are deduced by the respective formulæ

$$q_{[x]+t} = \frac{d_{[x]+t}}{E_{[x]+t}}$$

and

$$(wq)_{[x]+t} = \frac{w_{[x]+t}}{(wE)_{[x]+t}}$$

In practice, one or other of the two methods set out in columns (8) to (10), and (11) to (13), would alone be adopted, thus reducing the labour and the number of columns involved; and it will probably be found preferable to adopt the continuous formula shown in columns (11) to (13), and to employ the summation formula in verification of the results at any stage, thus

$$\begin{aligned}\bar{E}_{[20]+10} &= n_{[20]} + \sum_0^{10} (g) - g'_{[20]+10} \\ &= 421 - 84 + 12.5 \\ &= 349.5\end{aligned}$$

$$\begin{aligned}\bar{E}_{[20]+20} &= n_{[20]} + \sum_0^{20} (g) - g'_{[20]+20} \\ &= 421 - 376 - 0.3 \\ &= 44.7\end{aligned}$$

$$\begin{aligned}\bar{E}_{[20]+30} &= n_{[20]} + \sum_0^{30} (g) - g'_{[20]+30} \\ &= 421 - 421 + 2.6 \\ &= 2.6\end{aligned}$$

I have, however, preferred to set out the full process under both methods, partly to illustrate the two operations, and partly to secure verification throughout.

The rates of mortality as set out in column (15), being based upon only 48 deaths, are, of course, very irregular, and I need hardly say that they are only computed here for purposes of illustration and comparison, and are not presented as representing results practically available. The withdrawals are 481 in number, and the rates of withdrawal, as set out in column (17), show a smoother progression in years of duration.

Bearing in mind that the Exact Duration Method gives effect to the precise exposures of the cases, and deduces the rates, both of mortality and of withdrawal, strictly as experienced in each successive year of duration, it may, I think, be considered as not unduly laborious for the valuable results obtained. Under this method no assumptions whatever are made, either as regards the ages attained, or as to the average epochs of entry or of exit, in the several years of duration.

In comparing the extent of the Schedule (B) with other tabular statements which have been published, it must be borne in mind that in the appended schedule the numbers exposed to risk and the rates experienced are deduced both for mortality and withdrawal.

MEAN DURATION METHOD.

If we consider that upon the average the fractional exposure of the survivors, in the years of duration current at the commencement of the period of observation, and of the cases existing, withdrawing, and dying, in the years of duration current at the close of the period, or at exit, is approximately equal to half a year, formulæ (1) and (3) will become [*see* Appendix (A)]

$$\bar{E}_{[x]} = n_{[x]} + \frac{g_{[x]}}{2} \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

$$\bar{E}_{[x]+t} = \bar{E}_{[x]+t-1} + \frac{1}{2}(g_{[x]+t-1} + g_{[x]+t}) \quad . \quad . \quad . \quad (6)$$

a very simple and convenient formula for computing the numbers exposed to risk by a continuous operation.

Similarly, formula (4) will become, upon the above assumptions,

$$E_{[x]+t} = n_{[x]} + \sum_0^t (g) - \frac{1}{2}(g_{[x]+t}) \quad . \quad . \quad . \quad . \quad (7)$$

a formula by which the number exposed to risk can be obtained by tabular summation, or which can be applied as a useful check upon the results obtained by formula (6).

In this case it is not, of course, necessary to record the exact duration of the cases upon the cards, but only, in the case of the "survivors", the "curtate" duration at entry; and in the case of the existing, withdrawing, and dying, the "curtate" duration at the close of the observation, or at exit; cases of integral duration of $(t+1)$ years being treated, as before, as of "curtate" duration (t) . The cards of the "survivors" must then be sorted and tabulated according to the duration at entry, and those of the cases existing, withdrawing, and dying, according to the duration at exit, as recorded upon the cards.

The appended Schedule (C) shows the process followed in computing the number exposed to risk (\bar{E}) by the two formulæ (6) and (7), also the values of E and (wE) deduced from the formulæ

$$E_{[x]+t} = \bar{E}_{[x]+t} + \frac{d}{2}$$

$$\text{and } (wE)_{[x]+t} = \bar{E}_{[x]+t} + \frac{w}{2} \quad (\text{See Appendix C}).$$

SCHEDULE (C).—OBSERVATION EXTENDING OVER

Table shewing alternative methods of deducing the Numbers Exposed to duration, and with mean or average Fractional

Curtate Dura- tion	Survivors	Existing	With- drawals	Deaths	Total De- crement	Net Movement	SUMMATION METHOD		
t	$s_{[x]+t}$	$e_{[x]+t}$	$w_{[x]+t}$	$d_{[x]+t}$	$f_{[x]+t}$	$g_{[x]+t} = (s-f)$	$n_{[x]} + \sum_0^t (g)$	$-\frac{g_{[x]+t}}{2}$	$\bar{E}_{[x]+t}$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
0	137	82	51	1	134	+ 3	$n_{[x]}=421$ 424	- 1.5	422.5
1	101	83	47	5	135	-34	390	+ 17.0	407.0
2	132	7	44	3	54	+ 78	468	- 39.0	429.0
3	120	72	45	1	118	+ 2	470	- 1.0	469.0
4	129	75	55	3	133	- 4	466	+ 2.0	468.0
5	101	69	44	3	116	-15	451	+ 7.5	458.5
6	77	62	33	...	95	-18	433	+ 9.0	442.0
7	89	74	23	4	101	-12	421	+ 6.0	427.0
8	87	82	27	5	114	-27	394	+ 13.5	407.5
9	76	89	16	1	106	-30	364	+ 15.0	379.0
10	64	69	19	3	91	-27	337	+ 13.5	350.5
11	48	48	20	6	74	-26	311	+ 13.0	324.0
12	15	65	11	2	78	-63	248	+ 31.5	279.5
13	19	69	13	2	84	-65	183	+ 32.5	215.5
14	19	59	11	1	71	-52	131	+ 26.0	157.0
15	10	46	2	2	50	-40	91	+ 20.0	111.0
16	13	35	5	...	40	-27	64	+ 13.5	77.5
17	15	11	3	...	14	+ 1	65	- 0.5	64.5
18	10	13	3	...	16	- 6	59	+ 3.0	62.0
19	4	16	1	...	17	-13	46	+ 6.5	52.5
20	9	7	3	...	10	- 1	45	+ 0.5	45.5
21	10	11	1	1	13	- 3	42	+ 1.5	43.5
22	15	10	10	+ 5	47	- 2.5	44.5
23	16	9	1	...	10	+ 6	53	- 3.0	50.0
24	10	4	4	+ 6	59	- 3.0	56.0
25	7	9	1	1	11	- 4	55	+ 2.0	57.0
26	...	10	1	1	12	-12	43	+ 6.0	49.0
27	...	12	...	1	13	-13	30	+ 6.5	36.5
28	...	14	1	...	15	-15	15	+ 7.5	22.5
29	...	8	...	2	10	-10	5	+ 5.0	10.0
30	...	5	5	- 5	...	+ 2.5	2.5
	1,333	1,225	481	48	1,754	-421	6,210	210.5	6420.5

FIVE CALENDAR YEARS.—MEAN DURATION METHOD.—SCHEDULE (C).

Risk, and the Rates, of Mortality and of Withdrawal, in true years of Exposures.—Central Age at Entry (20).

CONTINUOUS METHOD		MORTALITY		WITHDRAWAL		Curate Duration
$\frac{g_{[x]+t-1} + g_{[x]+t}}{2}$	$\bar{E}_{[x]+t}$	Exposed $E_{[x]+t}$ $= \bar{E} + \frac{d}{2}$	Rate $q_{[x]+t}$ $= \frac{d}{E}$	Exposed $(wE)_{[x]+t}$ $= \bar{E} + \frac{w}{2}$	Rate $(wq)_{[x]+t}$ $= \frac{w}{wE}$	
(11)	(12)	(13)	(14)	(15)	(16)	(17)
	$n_{[x]} = 421$					
+ 1.5	422.5	423.0	.00237	448.0	.1138	.0
- 15.5	407.0	409.5	.01221	430.5	.1092	1
+ 22.0	429.0	430.5	.00697	451.0	.0982	2
+ 40.0	469.0	469.5	.00213	491.5	.0917	3
- 1.0	468.0	469.5	.00639	495.5	.1110	4
- 9.5	458.5	460.0	.00652	480.5	.0915	5
- 16.5	442.0	442.0	...	458.5	.0719	6
- 15.0	427.0	429.0	.00932	438.5	.0524	7
- 19.5	407.5	410.0	.01220	421.0	.0641	8
- 28.5	379.0	379.5	.00264	387.0	.0413	9
- 28.5	350.5	352.0	.00852	360.0	.0528	10
- 26.5	324.0	327.0	.01835	334.0	.0599	11
- 44.5	279.5	280.5	.00713	285.0	.0386	12
- 64.0	215.5	216.5	.00924	222.0	.0586	13
- 58.5	157.0	157.5	.00635	162.5	.0679	14
- 46.0	111.0	112.0	.01786	112.0	.0179	15
- 33.5	77.5	77.5	...	80.0	.0625	16
- 13.0	64.5	64.5	...	66.0	.0455	17
- 2.5	62.0	62.0	...	63.5	.0472	18
- 9.5	52.5	52.5	...	53.0	.0189	19
- 7.0	45.5	45.5	...	47.0	.0638	20
- 2.0	43.5	44.0	.02273	44.0	.0227	21
+ 1.0	44.5	44.5	...	44.5	...	22
+ 5.5	50.0	50.0	...	50.5	.0198	23
+ 6.0	56.0	56.0	...	56.0	...	24
+ 1.0	57.0	57.5	.01739	57.5	.0174	25
- 8.0	49.0	49.5	.02020	49.5	.0202	26
- 12.5	36.5	37.0	.02703	36.5	...	27
- 14.0	22.5	22.5	...	23.0	.0435	28
- 12.5	10.0	11.0	.18182	10.0	...	29
- 7.5	2.5	2.5	...	2.5	...	30
- 2.5						
- 421	6,420.5	6,444.5	...	6,661.0	...	

The values of $q_{[x]+t}$ and $(wq)_{[x]+t}$, as deduced from these numbers exposed to risk, are also appended. Here, again, the columns (8) to (10) can be dispensed with, and the summation formula employed simply for purposes of verification at suitable intervals.

This method has the advantages of being very simple and rapid in working, of avoiding, to a great extent, the employment of fractions, and, at the same time, of preserving the incidence of the cases strictly in their appropriate years of duration. It treats all cases of entry, of emergence, and of existence, as occurring in the middle of the year of duration: and one effect of this is that "surviving" cases entering and emerging in the same year of duration are altogether eliminated from the experience. Thus, a case entering upon the period of observation at a duration of 2.1 years, and emerging at a duration of 2.9 years, is considered as entering upon observation at 2.5 years, and emerging at 2.5 years. As some compensation for this, a case entering (for instance) at 2.9 years and emerging at 3.1 years is considered as entering at 2.5 and emerging at 3.5, and as under observation for a full year. Upon the whole, however, the method deals with the fractional exposures fairly at average values; and in the case of an experience such as that here investigated, gives, as will be seen, values for the numbers exposed to risk, and for the rates of mortality and of withdrawal, in successive years of duration, which agree very closely with those deduced by the Exact Duration Method.

NEAREST DURATION METHOD.

This is the method which has been illustrated by Dr. Sprague (*J.I.A.*, xxxi, 208-12) in schedules arranged according to ages at entry and years of duration. It has, I think, been somewhat hastily assumed that this method necessarily involves a "mixing-up of the policy years" (*J.I.A.*, xxxi, 309-315), and that, therefore, although possessing manifest and acknowledged advantages by way of simplicity of arrangement and facility of computation, it is unsuitable for an investigation (such as the present) which aims at deducing the true experience of each year of duration. It is mainly with a view to a further discussion of this question that I here introduce this method; and I shall illustrate its application by the same partial experience which has served to illustrate the Exact Duration Method and the Mean Duration Method.

The Nearest Duration Method proceeds upon the assumption that all cases entering on the period of observation do so either at the commencement or at the end of the year of duration

current at entry; that all cases emerging during the period do so either at the commencement or at the end of the year of duration current at exit; and, consequently, that all cases existing at the close of the period have then completed integral years of duration. These assumptions involve the reference of all cases of fractional exposure to the beginning or to the end of the year of duration current at entry, or at exit: and in carrying this into effect, the *nearest* boundary of the year of duration is in all cases adopted. Thus, cases having a duration at entry (or at exit) of 6.1 to 6.4 years inclusive, would be considered as entering (or emerging) at an integral duration of 6 years; cases having durations of 6.6 to 7.0 inclusive would be considered as entering (or emerging) at an integral duration of 7 years; and cases entering (or emerging) precisely mid-way, or at 6.5 years, would be alternately classed as entrants (or as emergents) at integral durations of 6, and of 7 years. The operation, so far, deals solely with the amount of fractional exposure within each separate year of duration, and adopts convenient and average assumptions of equivalent integral durations, *but always so as strictly to preserve the incidence of the cases in their true years of duration.*

In carrying this method out in practice, the cards of the "survivors" at the commencement of the period of observation would be entered up with the nearest integral duration at entry, according to the above plan; the cards of the cases "existing" would similarly be entered up with the nearest integral duration at the close of the period; and the cards of the cases of withdrawal would be entered up with the nearest integral duration at withdrawal. As regards the deaths, a different course would be followed, in an investigation intended to deduce the rate of mortality; for, in order to give each case of death a full year's exposure in the year of death, the duration entered upon the cards must be the year of duration then current (that is, the curtate duration + 1) and not the nearest integral duration.

The cards are then sorted into original entrants, and "survivors"; the latter being then sorted, counted, and tabulated, according to the integral durations at entry, as recorded upon the cards; and the cards then combined, and re-sorted into cases existing, withdrawals, and deaths; and these again sorted, counted, and tabulated according to their recorded integral durations at exit.

The tabulation takes the form set out in Schedule (D).

SCHEDULE (D).—OBSERVATION EXTENDING OVER

Table shewing methods of deducing the Numbers Exposed to Risk, and the Fractional Exposures being taken to the nearest integer. (This method the Rate of Withdrawal).—Central Age at Entry (20).

Duration	Survivors	Existing	Withdrawals	Deaths	Total Decrement	Net Movement	MORTALITY	
							Exposed	Rate
t	$\left. \begin{matrix} (bs)_{[x]+t-1} \\ + (as)_{[x]+t} \end{matrix} \right\} \\ = s_{[x]+t}$	$\left. \begin{matrix} (be)_{[x]+t-1} \\ + (ae)_{[x]+t} \end{matrix} \right\} \\ = e_{[x]+t}$	$\left. \begin{matrix} (bw)_{[x]+t-1} \\ + (aw)_{[x]+t} \end{matrix} \right\} \\ = w_{[x]+t}$	$d_{[x]+t-1}$	$(e + w + d) \\ = f_{[x]+t}$	$(s - f) \\ = g_{[x]+t}$	$n_{[x]} + \sum_{i=1}^t (g_i) \\ = E_{[x]+t}$	$\frac{d_{[x]+t}}{E_{[x]+t}} \\ = q_{[x]+t}$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
0	56	40	32	...	72	- 16	$n_{[x]}=421$ 405	·00247
1	128	78	43	1	122	+ 6	411	·01217
2	114	47	42	5	94	+ 20	431	·00696
3	132	42	54	3	99	+ 33	464	·00216
4	123	70	51	1	122	+ 1	465	·00645
5	114	70	40	3	113	+ 1	466	·00644
6	95	67	40	3	110	- 15	451	...
7	72	68	24	...	92	- 20	431	·00926
8	93	82	29	4	115	- 22	409	·01222
9	80	85	24	5	114	- 34	375	·00267
10	69	77	18	1	96	- 27	348	·00862
11	64	63	21	3	87	- 23	325	·01846
12	30	50	12	6	68	- 38	287	·00697
13	16	73	12	2	87	- 71	216	·00926
14	17	61	12	2	75	- 58	158	·00633
15	14	49	7	1	57	- 43	115	·01739
16	11	50	1	2	53	- 42	73	...
17	15	19	4	...	23	- 8	65	...
18	16	14	4	...	18	- 2	63	...
19	3	11	2	...	13	- 10	53	...
20	6	11	2	...	13	- 7	46	...
21	10	8	2	...	10	0	46	·02174
22	15	14	1	1	16	- 1	45	...
23	17	11	1	...	12	+ 5	50	...
24	11	3	3	+ 8	58	...
25	10	6	6	+ 4	62	·01613
26	2	10	1	1	12	- 10	52	·01923
27	...	15	1	1	17	- 17	35	·02857
28	...	13	...	1	14	- 14	21	...
29	...	8	1	...	9	- 9	12	·16667
30	...	8	...	2	10	- 10	2	...
31	...	2	2	- 2
	1,333	1,225	481	48	1,754	- 421	6,440	...

FIVE CALENDAR YEARS.—NEAREST DURATION METHOD.—SCHEDULE (D).

Rates (a) of Mortality. (b) of Withdrawal, in true years of duration; the involves a re-sorting throughout of the Withdrawals and Deaths, to deduce

With- drawals	Deaths	Total Decrement	Net Movement	WITHDRAWAL		Dura- tion
				Exposed	Rate	
$v_{[x]-t-1}$	$\begin{matrix} (bd)_{[x]-t-1} \\ + (ad)_{[x]-t} \\ = d_{[x]-t} \end{matrix}$	$\begin{matrix} (c + w + d) \\ = f'_{[x]-t} \end{matrix}$	$\begin{matrix} (s - f') \\ = g'_{[x]-t} \end{matrix}$	$\begin{matrix} n_{[x]} \\ + \sum_{t=1}^n (g') \\ = (wE)_{[x]-t} \end{matrix}$	$\begin{matrix} w \\ wE \\ = (wQ)_{[x]-t} \end{matrix}$	t
(10)	(11)	(12)	(13)	(14)	(15)	(16)
				$n_{[x]} = 421$		
...	1	41	+ 15	436	·1170	0
51	1	130	- 2	434	·1083	1
47	6	100	+ 14	448	·0982	2
44	2	88	+ 44	492	·0915	3
45	3	118	- 5	497	·1107	4
55	2	127	- 13	484	·0909	5
44	1	112	- 17	467	·0707	6
33	1	102	- 30	437	·0526	7
23	6	111	- 18	419	·0644	8
27	3	115	- 35	384	·0417	9
16	1	94	- 25	359	·0529	10
19	6	88	- 24	335	·0597	11
20	4	74	- 44	291	·0378	12
11	2	86	- 70	221	·0588	13
13	0	74	- 57	164	·0671	14
11	2	62	- 48	116	·0172	15
2	1	53	- 42	74	·0676	16
5	...	24	- 9	65	·0462	17
3	...	17	- 1	64	·0469	18
3	...	14	- 11	53	·0189	19
1	...	12	- 6	47	·0638	20
3	1	12	- 2	45	·0222	21
1	...	15	0	45	...	22
...	...	11	- 6	51	·0196	23
1	...	4	+ 7	58	...	24
...	...	6	- 4	62	·0161	25
1	2	13	- 11	51	·0196	26
1	...	16	- 16	35	...	27
...	1	14	- 14	21	·0476	28
1	2	11	- 11	10	...	29
...	...	8	- 5	2	...	30
...	...	2	- 2	31
481	48	1,754	- 421	6,667		

I shall throughout employ the convenient symbols

(*as*), (*ae*), (*aw*), and (*ad*)

to indicate the cases surviving, existing, withdrawing, and dying, which are, by the Nearest Duration Method, referred to the *beginning* of the year of duration current at entry or at exit; and the symbols

(*bs*), (*be*), (*bw*), and (*bd*)

to indicate the cases referred to the *end* of the year of duration current at entry or at exit; the sums of these quantities being, of course, equal to

s, *e*, *w*, and *d*

in any given year of duration.

Thus, by the Nearest Duration Method, the number of "survivors" tabulated against duration *t*, which I shall call $s_{[x]+t}$, is equal to

$$[(bs)_{[x]+t-1} + (as)_{[x]+t}] = s_{[x]+t}$$

and similarly with the cases existing (*e*), and withdrawing (*w*); as shewn in the headings of columns (2), (3), and (4), of Schedule (D).

The facts being entered in columns (2) to (5), the sum of the numbers existing, withdrawing, and dying, gives the value of (*f*) in column (6); and deducting these values from the number of survivors in column (2), we deduce the values of (*g*) in column (7). Summing these values continuously, and including throughout the number of original entrants (n_x), we arrive in column (8) at the numbers exposed to the risk of death in each year of duration. Finally, dividing the value of $E_{[x]+t}$ (in column 8) into that of $d_{[x]+t}$ (on the line below) in column (5), we arrive at the value of $q_{[x]+t}$, the rate of mortality in the (*t*+1)th year of duration, in column (9). See Appendix (A), [Schedule (D).]

It will be seen that I have, in the tabular arrangement of Schedule (D), attempted to improve and simplify the form as given by Dr. Sprague (*J.I.A.*, xxxi, 212), by setting the values of $E_{[x]+t}$ and of $q_{[x]+t}$ (instead of those of $E_{[x]+t-1}$ and $q_{[x]+t-1}$) opposite the duration *t*. The numbers in the column headed "deaths" must necessarily be the successive values of $d_{[x]+t-1}$; but there is no practical difficulty in dividing the value of $E_{[x]+t}$ in column (8) into that of $d_{[x]+t}$ on the line below in column (5); and the headings of the columns are throughout more symmetrical, as now submitted. The same

remarks will apply to the columns (10) to (15) (which I shall presently refer to) for deducing the numbers exposed to risk, and the rate of withdrawal.

The values of $q_{[x]+t}$ thus computed agree closely with those deduced by the Exact Duration and the Mean Duration Methods, in Schedules (B) and (C) respectively; and there is, so far, no "mixing-up" of the years of duration. It may at first sight appear that the methods of sorting and tabulation do, in fact, mix up the years of duration; for, as regards the cases surviving, existing, and withdrawing, the cases having durations (at entry or at exit) between $(t-\cdot5)$ and t years, are throughout combined with those having durations between t and $(t+\cdot5)$ years. Further consideration will, however, shew that it is quite immaterial, *for the purpose of deducing the numbers exposed to the risk of death*, whether we consider the survivors, combined as above, as entering at the end of the t th or at the beginning of the $(t+1)$ th year of duration; and, similarly, it is quite immaterial whether we consider the cases existing and withdrawing as emerging at the end of the t th, or at the beginning of the $(t+1)$ th year. The deaths are, in all cases, tabulated in their true years of duration; and the rates of mortality are thus deduced without any overlapping of those years.

When, however, it is desired to deduce the rate of *withdrawal* in years of duration, it is quite clear that the facts as tabulated in the first eight columns of Schedule (D) will not give the desired result. For the true estimation of the rate of withdrawal, it is necessary to give the cases of withdrawal a full year's exposure in the year of exit, and to give the death-cases only their true (or estimated) exposure up to the date of death. In column (4) of Schedule (D), however, the withdrawals are exposed only up to their (estimated) date of exit; while in column (5) the death-cases have a full year's exposure. Further, the number of withdrawals in column (4) includes all cases withdrawing between durations $(t-\cdot5)$ and $(t+\cdot5)$;^{*} and it is clear that if these values be employed as a numerator in deducing the

^{*} It would thus appear that the ratio of withdrawals as deduced from the values in column (4) would give some approximation to the *force of withdrawal*; and this suggestion, which appears to have been originally made by Mr. G. King, is referred to by Mr. Ryan (*J.L.A.*, xxxi, 310). I cannot, however, trace the original reference, attributed to Mr. King, in the pages of the *Journal*. The divisor $E_{[x]+t}$, in column (8), is not appropriate for deducing the true force of withdrawal (as is pointed out by Mr. Ryan, *loc. cit.*); and I have been unable, from the values given in columns (2) to (8) of Schedule (D), to deduce any expression which would give a true (or very approximate) representation of the rate of withdrawal in successive years of duration.

rates of withdrawal, the rate deduced will not truly represent that obtaining either in the t th or $(t+1)$ th year of duration; and that the withdrawals of adjacent years of durations would be improperly combined in the expression for the rate of withdrawal so deduced.

It will, moreover, be seen that the facts as tabulated in columns (4) and (5) of the Schedule, give no means of introducing the necessary corrections in the amount of exposure, nor of scheduling the cases in their true years of duration.

Under this method, then, the only course available for accurately deducing the rates of withdrawal appears to be that of *re-sorting throughout the whole of the cases of withdrawal and death*, the former according to the year of duration current at exit (that is, the curtate duration $+1$), the latter according to the nearest integral duration at death. This is, of course, a laborious process. I have set out in columns (10) and (11) of Schedule (D), the results of this re-sorting.

Following, then, precisely the same course as in deducing the rate of mortality, we arrive at the modified values of \mathbf{f}' ($=e+w+d$) in column (12), and of \mathbf{g}' ($=s-\mathbf{f}'$) in column (13); then summing the values of \mathbf{g}' as before, and adding in the number of original entrants ($n_{[x]}$), we arrive in column (14), at the numbers exposed to the risk of withdrawal; and dividing ($w_{[x]+t}$) in column (10) by $(wE)_{[x]+t}$ in column (14), we deduce the true rate of withdrawal $(wq)_{[x]+t}$ in column (15).

These rates will be found to agree closely with those deduced in Schedules (B) and (C), by the Exact Duration and the Mean Duration Methods respectively.

By this second process of sorting and tabulation (which, as will be seen, nearly doubles the work involved) the rate of withdrawal can, then, be deduced in true years of duration.

Let us now investigate the constituent parts of the values tabulated in columns (4) and (10), and in columns (5) and (11) of Schedule (D), in which the withdrawals and deaths are set out in the forms required respectively for deducing the rate of mortality and of withdrawal.

We have, for the withdrawals in column (4)

$$[(bw)_{[x]+t-1} + (aw)_{[x]+t}] = w_{[x]+t},$$

and for the withdrawals in column (10)

$$[(aw)_{[x]+t-1} + (bw)_{[x]+t-1}] = w_{[x]+t-1}.$$

Similarly, for the deaths in column (5), we have

$$[(ad)_{[x]+t-1} + (bd)_{[x]+t-1}] = d_{[x]+t-1},$$

and for the deaths in column (11)

$$[(bd)_{[x]+t-1} + (ad)_{[x]+t}] = d_{[x]+t}.$$

These alternative expressions for the cases of withdrawal and of death can therefore be deduced by any process which shall set out separately the values of

$$(aw), (bw), (ad), (bd)$$

in successive years of duration. In other words, if the withdrawals and deaths are sorted *according to half-years of actual duration at exit*, the above expressions can be readily deduced.

In Schedule (E) I have thus sorted and tabulated the withdrawals and deaths, so that in column (4) are given the values of (aw) , and (below them) of (bw) , in each year of duration; while in column (5) are given the values of (ad) and of (bd) , in each year of duration. The sorting into half-years at exit involves very little additional labour, for in arriving at the nearest integral duration, the half-year of exit necessarily comes under observation. Thus, the cards at nearest integral duration (t) are made up of two groups; (1) cases with durations from $(t - \cdot 5)$ to t years inclusive; and (2) cases with durations from t to $(t + \cdot 5)$ inclusive; and if the operator, instead of combining these groups, keeps them separate throughout, the cards are at once sorted in half-years of duration. Where the exact duration has not been already recorded upon the cards, they may be conveniently marked as follows:—

Duration over 7 years and less than $7\frac{1}{2}$ years	
(with one-half of the cases of exact duration $7\frac{1}{2}$ years).	7 +
Duration over $7\frac{1}{2}$ years, up to and including 8 years	
(with one-half of the cases of exact duration $7\frac{1}{2}$ years).	8 -

The values of (s) and (e) in columns (2) and (3) agree with those given in Schedule (D), and those of (f) in column (6) of Schedule (E) are arrived at by summing the values in columns (3), (4), and (5), thus:—

$$e_{[x]+t} + \left\{ \begin{array}{l} (bw)_{[x]+t-1} \\ (aw)_{[x]+t} \end{array} \right\} + \left\{ \begin{array}{l} (bd)_{[x]+t-1} \\ (ad)_{[x]+t} \end{array} \right\} = (c + w + d)_{[x]+t} = f_{[x]+t}$$

SCHEDULE (E).—OBSERVATION EXTENDING OVER
Table shewing methods of deducing the Numbers Exposed to Risk, and the Fractional Exposures being taken to the nearest integer. (Cases of Central Age at Entry (20)).

Dura- tion	Survivors	Existing	Withdrawals	Deaths	Total Decrement	Net Movement	$\frac{n_{[x]}}{+ \Sigma_0^t(g)}$
t	$\left. \begin{array}{l} (bs)_{[x]+t-1} \\ + (as)_{[x]+t} \\ = s_{[x]+t} \end{array} \right\}$	$\left. \begin{array}{l} (be)_{[x]+t-1} \\ + (ae)_{[x]+t} \\ = e_{[x]+t} \end{array} \right\}$	$\left. \begin{array}{l} (aw)_{[x]+t} \\ (bw)_{[x]+t} \end{array} \right\}$	$\left. \begin{array}{l} (ad)_{[x]+t} \\ (bd)_{[x]+t} \end{array} \right\}$	$f_{[x]+t}$	$g_{[x]+t} - f_{[x]+t} = (s - f)$	$\bar{E}_{[x]+t}$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
0	56	40	32	1	73	-17	$n_{[x]} = 421$
1	128	78	19	0			404
			24	1	122	+ 6	410
			23	4			
2	114	47	19	2	95	+19	429
			25	1			
3	132	42	29	1	98	+34	463
			16	0			
4	123	70	35	3	124	- 1	462
			20	0			
5	114	70	20	2	112	+ 2	464
			24	1			
6	95	67	16	..	108	-13	451
			17	..			
7	72	68	7	1	93	-21	430
			16	3			
8	93	82	13	3	117	-24	406
			14	2			
9	80	85	10	1	112	-32	374
			6	0			
10	69	77	12	1	96	-27	347
			7	2			
11	64	63	14	4	90	-26	321
			6	2			
12	30	50	6	2	66	-36	285
			5	0			
13	16	73	7	2	87	-71	214
			6	0			
14	17	61	6	0	73	-56	158
			5	1			
15	14	49	2	1	58	-44	114
			0	1			
16	11	50	1	..	52	-41	73
			4	..			
17	15	19	0	..	23	- 8	65
			3	..			
18	16	14	1	..	15	- 2	63
			2	..			
19	3	11	0	..	13	-10	53
			1	..			
20	6	11	1	..	13	- 7	46
			2	..			
21	10	8	0	1	11	- 1	45
			1	0			
22	15	14	15	0	45
					
23	17	11	1	..	12	+ 5	50
			0	..			
24	11	3	3	+ 8	58
					
25	10	6	0	0	6	+ 4	62
			1	1			
26	2	10	0	1	13	-11	51
			1	0			
27	..	15	..	0	16	-16	35
			..	1			
28	..	13	0	..	14	-14	21
			1	..			
29	..	8	..	2	11	-11	10
			..	0			
30	..	8	8	- 8	2
					
31	..	2	2	- 2	..
	1,333	1,225	256	29	1,754	-421	6,411
			225	19			

FIVE CALENDAR YEARS.—NEAREST DURATION METHOD.—SCHEDULE (E).

Rates, of Mortality and of Withdrawal, in true years of duration; the Withdrawal and Death sorted in half-years of duration at exit.—

MORTALITY			WITHDRAWALS			Duration
Exposed	Deaths	Rate	Exposed	Withdrawals	Rate	
$\frac{E_{[x]+t}}{= \bar{E} + (ad)}$	$d_{[x]+t}$	$\frac{q_{[x]+t}}{d} = \frac{d}{\bar{E}}$	$\frac{(wE)_{[x]+t}}{= \bar{E} + (aw)}$	$w_{[x]+t}$	$\frac{wq_{[x]+t}}{= \frac{w}{wE}}$	t
(9)	(10)	(11)	(12)	(13)	(14)	(15)
405	1	·00247	436	51	·1170	0
411	5	·01217	434	47	·1083	1
431	3	·00696	448	44	·0982	2
464	1	·00216	492	45	·0915	3
465	3	·00645	497	55	·1107	4
466	3	·00644	484	44	·0909	5
451	467	33	·0707	6
431	4	·00926	437	23	·0526	7
409	5	·01222	419	27	·0644	8
375	1	·00267	384	16	·0417	9
348	3	·00862	359	19	·0529	10
325	6	·01846	335	20	·0597	11
287	2	·00697	291	11	·0378	12
216	2	·00926	221	13	·0588	13
158	1	·00633	164	11	·0671	14
115	2	·01739	116	2	·0172	15
73	74	5	·0676	16
65	65	3	·0462	17
63	64	3	·0469	18
53	53	1	·0189	19
46	47	3	·0638	20
46	1	·02174	45	1	·0222	21
45	45	22
50	51	1	·0196	23
58	58	24
62	1	·01613	62	1	·0161	25
52	1	·01923	51	1	·0196	26
35	1	·02857	35	27
21	21	1	·0476	28
12	2	·16667	10	29
2	2	30
..	31
6,440	48	..	6,667	481	..	

as indicated by the numbers coupled by brackets in the Schedule. In column (7) the values of $g(=s-f)$ are set out, and in column (8) these values are summed vertically, including the number of original entrants $n_{[x]}$. The values thus arrived at are those of $\bar{E}_{[x]+t}$, the numbers exposed to risk, computed up to the date of death or withdrawal, the formula being (*see* Appendix A)

$$\bar{E}_{[x]+t} = n_{[x]} + \Sigma_0^t(g) \quad . \quad . \quad . \quad . \quad (11)$$

The numbers exposed to the risk of death and of withdrawal respectively are then deduced by the formulæ

$$\begin{aligned} E_{[x]+t} &= \bar{E}_{[x]+t} + (ad)_{[x]+t} \\ (wE)_{[x]+t} &= \bar{E}_{[x]+t} + (aw)_{[x]+t} \quad (\textit{see Appendix C}). \end{aligned}$$

These numbers are tabulated in columns (9) and (12) respectively. In columns (10) and (13) I have set out (for the sake of clearness) the values of $d_{[x]+t}$ and of $w_{[x]+t}$, which represent simply the sums of the adjacent values tabulated in pairs in columns (5) and (4) respectively; and in columns (11) and (14) are deduced the values of $q_{[x]+t}$ and of $(wq)_{[x]+t}$. These rates, as well as the numbers exposed to risk in columns (9) and (12), are identical throughout with those given in Schedule (D).

It will be remarked that the experience dealt with in both schedules differs from that investigated by Dr. Sprague (*J.I.A.*, xxxi, 212) by including a body of survivors (s) in force at the commencement of the period of observation, instead of tracing the assurances from their original entry. The introduction of the survivors presents no special feature of difficulty. It must, however, be noted that the survivors who have, at entry on the period of observation, a duration of less than six months, will, by the rule of nearest duration, be classed as of "Duration 0" at entry, and will thus practically be included among the cases entering as new assurances during the period of observation. Thus, there were, in the experience here dealt with, 56 cases of survivors of less than six months' duration at the commencement of the period of observation. These 56 cases were included with the 421 cases of new entrants during the period, thus raising the total to 477.

On the other hand, there were, at the close of the period of observation, 40 cases existing with a duration of less than six months, and therefore classed as existing at "Duration 0": these 40 cases were altogether eliminated from the experience, thus

practically reducing the new entrants to 437. Of these, 32 withdrew, and one died, within six months of entry (that is at "Nearest Duration 0"); and the number exposed to risk in the first year of duration ($E_{[x]}$) was thus finally $437 - (32 + 1) = 404$.

I have, for the sake of clearness, stated these operations at length: but it is to be remarked that they are, throughout, deduced directly by the formulæ and methods adopted in Schedule (E), and involve *no exceptional treatment whatever in the first year of duration*. In this respect I venture to hope that they will be found to compare favourably with the alternative methods adopted by Dr. Sprague (*loc. cit.*); and I submit the method, illustrated in Schedule (E), as a ready and practical means of deducing the numbers exposed to risk, and the rates of mortality and of withdrawal in true years of duration, according to the Nearest Duration Method.

I have given in Appendix (A) an algebraical analysis of the Nearest Duration Method as thus applied in deducing the numbers exposed to risk, and the rates of death and of withdrawal. It is there shewn that, while, by the Exact Duration Method,

$$\bar{E}_{[x]+t} = n_{[x]} + \sum_0^t (g) - g'_{[x]+t} \quad . \quad . \quad . \quad (4)$$

by the Nearest Duration Method,

$$\bar{E}_{[x]+t} = n_{[x]} + \sum_0^t (g) \quad . \quad . \quad . \quad . \quad (11)$$

$$= n_{[x]} + \sum_0^t (g) - (bg)_{[x]+t} \quad . \quad . \quad (12)$$

The expressions $n_{[x]}$ and $\sum_0^t (g)$ are identical by the two formulæ, and $\sum_0^t (g)$, in formula (4), is equal to

$$\sum_0^t \{s - (e + w + d)\}$$

and represents the number exposed to risk in the $(t+1)$ th year, upon the assumption that the "survivors" are exposed to a *full year's risk* in that year, and that the cases "existing" and emerging are exposed to *no risk* in that year. The expression in the Exact Duration formula (4) above

$$-g'_{[x]+t} = (e'_{[x]+t} + w'_{[x]+t} + d'_{[x]+t} - s'_{[x]+t})$$

modifies and corrects the expression $\sum_0^t (g)$ by *adding the true fractional exposures* of the cases "existing" and emerging in the $(t+1)$ th year, and *deducting the complementary fractional exposures* of the cases "surviving" in that year, and thus deduces the true value of

$$\bar{E}_{[x]+t} = \sum_0^t (g) - g'_{[x]+t}$$

Turning now to the Nearest Duration formula (12) above, we have the corrective expression $-(bg)_{[x]+t}$, which is equal to

$$(be)_{[x]+t} + (bw)_{[x]+t} + (bd)_{[x]+t} - (bs)_{[x]+t}$$

and this expression similarly modifies the value of $\Sigma_0^t(g)$, by adding the integral durations assumed to be approximately equal to the true fractional exposures of the cases "existing" and emerging in the $(t+1)$ th year, and deducting the integral durations assumed to be approximately equal to the complementary fractional exposures of the cases "surviving" in that year.

The difference in the value of $E_{[x]+t}$ by the two methods is thus represented solely by the error introduced in computing the fractional exposures in the $(t+1)$ th year at their nearest integral value; and there is no overlapping of the years of duration at any stage of the processes followed, as set out in Schedule (E).

COMPARISON OF AGGREGATE NUMBERS EXPOSED TO RISK.

Comparing now the numbers exposed to risk of death and of withdrawal, as deduced in Schedules (B), (C), and (D) or (E), it will be seen that the aggregate numbers are :

	$\Sigma(E)$	$\Sigma(wE)$
Exact Duration Method . . .	6,457.9	6,660.7
Mean Duration Method . . .	6,444.5	6,661.0
Nearest Duration Method . . .	6,440.	6,667.

The values in individual years of duration, both of the numbers exposed to risk and of the rates of mortality and withdrawal, are also practically identical throughout; and it is thus abundantly evident that, *in the investigation of this particular experience*, each of the three methods will give accurate and trustworthy results.

I now proceed to consider how far these conclusions will be modified, when consideration is given to the special characteristics of the experience here employed for purposes of illustration, and of that obtaining amongst assured lives generally.

(To be continued.)

NOTE.—The remainder of Part I of this paper, and Part II, with the Appendices, will be continued in the next number of the *Journal*.

JOURNAL

OF THE

INSTITUTE OF ACTUARIES.

Opening Address by the President.

The Nature and History of Actuarial Work as exemplifying the Mode of Development and the Methods of Science. A Presidential Address delivered before the Institute of Actuaries on the 30th of November 1896, by T. E. YOUNG, B.A., PRESIDENT.

I. INTRODUCTION.

IN pursuing the traditional practice of introducing myself officially by an Inaugural Address, I desire unaffectedly to state that any conventionality, which this course may have acquired from custom, is merged and lost in a deep feeling of grateful recognition of your generous confidence and goodwill.

With the historic associations which cluster around our famous Institute,—associations admirably blending the prosecution of Scientific Method with Practical Research: both the Experimenta Lucifera and Fructifera of Bacon,—to belong to this Fraternity of Experts as a student should now prove an inspiration; to reach official position and ampler scope of devotion to its interests is a signal distinction; while the attainment of its Headship, by the kindly suffrages of Colleagues, not merely constitutes the “consummate flower” of a genuine ambition, but is also suggestive, at the moment of realisation, of humbling reflections that personal service in the past, however earnestly

intended, had not assumed a more valid proportion to the crowning honour of so high a trust.

Although the limits of a Presidential Address are generously wide, and, indeed, somewhat nebulous, certain obvious considerations assign a reasonable range. The introduction of questions involving a distinctly controversial character should wisely be avoided; and a disquisition upon a definite Professional subject would be reft of its stimulating and instructive form by reason of the conventional absence of subsequent discussion. An Introductory Address, moreover, should, if possible, appeal to a larger and more general sphere of thoughts and sympathies than can usually be discovered within the narrower confines of a technical thesis.

In this embarrassed suspense of motive, the conviction finally dawned that the dignified position of the Institute; the commanding status which the Profession has attained after vigorous years of arduous and fruitful toil; the hopes and interests especially of the younger Members, which must ever exercise a pathetic attractiveness for their senior colleagues, who watch with sympathetic thoughtfulness their nascent career: all these reflections concurrently suggested that I might fitly engage your attention in a rapid survey of the Nature and History of Actuarial Work as exemplifying the Mode of Development and the Methods of Science.

And in explicitly claiming at the outset for our Actuarial studies the title and dignity of Science in the ample meaning of that term, I am misled by no unsubstantial ambition or delusive hope, nor do I employ mere language of generality expressive rather of indefinite assumption than assertive of a formal and completed fact. For I propose to ascertain the conception of the Nature and Methods of Science elaborated by acknowledged Masters* who have deduced the Principles of *Science* from a survey of the rational evolution of the *Sciences* themselves, and to exhibit the essential congruity, with whatever secondary limitations may be requisite, which exists between this established Body of philosophical truths and the growth and form of the investigations with which we ourselves are proud to be associated.

This course will also afford me the opportunity of discussing

* Herschel: Discourse on the Study of Natural Philosophy; Herbert Spencer: Essays, Scientific, Political, and Speculative, Vols. 1 and 3; Whewell: *Novum Organon Renovatum*; Jevons: *The Principles of Science*; Mill: *A System of Logic*, Vols. 1 and 2; and Bain: *Logic*: Part II, on Induction.

the aptitudes which constitute the Actuarial character ; the interchange of influence which unceasingly mediates between the Science created and the creating Mind, as the area of dominion widens ; and the appropriate educational training and equipment which the survey may suggest.

II. COMMON KNOWLEDGE AND SCIENCE.

The Criterion of Scientific Knowledge as distinguished from Common or Uncultivated Knowledge cannot be assigned as the possession of an Organised Body of classified facts, deduced inferences, and anticipated results ; nor does it consist in Prevision of consequences based upon observed relations among phenomena ; for these features are essentially discernible in each period of Knowledge as the tentative Prelude and the rigorous Sequel of a common Progress. But the culmination of Ordinary Knowledge into Science is attained when its previsions are not merely *qualitative*, or assertive solely of the *kind* of phenomena to be expected under stated conditions, but reach the completing stage of *quantitative* expression, where anticipated consequences are capable of *measurement* in precise numerical form. In this advance, the Modifications in phenomena produced by operating forces,—for all physical Sciences are exclusively concerned with movements in materials effected by incident Force,—may be too irregular and indeterminate to proceed beyond the sphere of empirical and fragmentary deduction, or may finally, by successive essays of approximation, emerge so clear and definite that, in a felicitous moment as Herschel termed it, the detected relations may even suggest the class of mathematical functions which symbolise their bond of connexion, and thus signalise the advent of the epoch of exactness.

And, without anticipating at this point any detailed exposition, I may summarily and justly appeal to our investigations into phenomena,—Mortality Statistics,—under the operation of the Force of Dissolution*, and to our technical results, as adequate evidence that the history of Actuarial study, from its elementary notions derived from games of chance, has developed gradually into preciser modes which, in later days, have received a vast accession of scope and power by application of the Instrumental Principles of the Differential and Integral Calculus.

* Employing this term as distinctive from the technical phase of the “ Force of Mortality.”

III. THE CLASSIFICATION OF THE SCIENCES.

With this conception of the Nature of Science, we may proceed to a general Scheme of Classification of the Sciences, which will prove sufficient for our purpose without discussion of the theoretical differences which still separate the two philosophical scientists, Mr. Herbert Spencer and Professor Bain.*

The Sciences, then, may be summarily partitioned into—

- (i) ABSTRACT SCIENCES,—such as Logic and Mathematics,—which treat of Time and Space or the *Forms* in which phenomena are cognised, whatever be the Metaphysics we profess regarding the nature of these Notions as Forms of Thought or Forms of Things :
- (ii) ABSTRACT-CONCRETE SCIENCES, like Mechanics, Physics, and Chemistry, which are concerned with *phenomena* in their *elements*, as they are presented under these Universal Forms, and
- (iii) CONCRETE SCIENCES, such as Geology, Biology, and Sociology, whose subject-matter consists of *Aggregates* or Totalities of *phenomena*.

Our own Science is clearly included within the final Department, and we observe that the Principles evolved in the Abstract section, when applied to the phenomena we study, are instrumental in attaining that exactness of expression and mathematical relationship which provide the Scientific form.

Moreover, all Concrete Sciences are concerned both with phenomena and with the manifold Forces which their varying movements embody. Astronomy—to select the exactest Science in this Division—is dominated by the Force of Gravity, whose permanent uniform action is termed a Law. The connotation of this expression is unhappily capable of metaphorical misuse, but the Scientific definition must be remembered that Laws of Nature are not Rules† which phenomena *must* obey, but consist simply of uniformities of co-existence and succession which they have been discovered to display.

In our wildest ambition we cannot range our Science in approximate relationship with a finished system like Astronomy,

* Spencer: The Classification of the Sciences: Essays, Vol. 3.

Bain: Logic: Part I: Deduction: Appendix A.

† Whewell: "Bridgewater" Treatise on Astronomy and General Physics: Introduction, cap. 2.

Spencer: The Study of Sociology, cap. 2.

but I have selected this illustration with a view to showing, by juxtaposition, that the factors in the two regions of research are similar—materials to observe and classify, and a Force whose Law or uniform operation is to be deciphered and interpreted, since every phenomenon is a manifestation of Force under changing modes.* The conception of Force, too, in these compared provinces is precisely identical, for the only possible knowledge of Force is that of Movement in Objective Facts.

The phenomena and forces, again, in the more rigorous systems of Truth, are of a stable character, or subject only to periodic changes which contain implicitly within themselves the re-establishment of prior relations, while our data are modified continually by a combination of Causes, multitudinous, complex, and fluctuating. Consider for a moment how the medical and surgical discoveries of the present generation alone have fundamentally affected our materials. And this example is illustrative further of that interaction of development and expression which is continuously mediating between the different spheres of Scientific Truth, in evidence of which may be cited the significant mode in which Optical discoveries have proved ancillary to Astronomy. The Antiseptic Method of Lister is undoubtedly one of the most momentous discoveries of any age. Many of the accessories which accompanied its introduction have naturally disappeared, but its essential character is permanent in its recognition of the external origination of disease and in the creation of an artificial atmosphere or environment which these exterior pathogenic organisms are incompetent to penetrate. So memorable and distinctive a revolution has thus been effected that it is difficult to express a statistical comparison with the prior state of Surgery with numerical precision. Amputations alone are available as the term of contrast; and here, previous to 1875, a mortality of 22 per-cent was considered to be extremely favourable, while, since the adoption of Antiseptic precautions, the percentage in similar cases is only 2·8. For the signalling character of that method consists in the circumstance that operations of most vital and capital nature; thoracic and abdominal explorations; thrombosis of the cerebral sinuses; forms of intestinal obstruction; and direct treatment of the kidneys, were enterprises which, anterior to Lister's epoch, no professional daring, skill, or

* If indeed Science should not hereafter regard the Universe as simply Matter under changing forms or purely Force in varying modes.

sagacity even attempted to perform. And, parenthetically interposing a remark in relation to Annuitants, the result of my enquiries definitely shows that, independent of the improvement of general conditions affecting longevity, the successful practice of Ovariectomy on Antiseptic principles is becoming largely accountable for the prolonged existence of Annuitants who are chiefly recruited from the class of Females. In this operation, the death-rate, which, prior to 1878, varied from 65 to 87 per-cent, has now been reduced within a range of 7 to 10 per-cent.*

But in what direction are our data thus affected as compared with the practically changeless nature of the materials of the completed Science I have chosen as a distinctive superior? Not in the Methods employed for determining quantitative ratios; but in the modes of incidence of the associated Force; and the nicer discrimination of the degree of its intensity of action at various stages, with the resulting practical application to systems of graduation, rates of premium, and valuations.

And having, for the purpose of vivid comparison, adduced the elaborated Science of Astronomy, it is agreeable to notice, on the other hand, that in relation to other branches included within the Concrete circle, our own investigations present an admirable superiority. In Biology, Political Economy, and Sociology, many valid uniformities have been traced, but it is evident, on examination, that those departments of specialised knowledge do not display that general character of exactness which distinguishes our Science. Dr. Whewell† and Professor Jevons‡ have ingeniously striven,—the former, to illustrate, the latter,

* The statistical information above presented, expresses the experience of one of our principal London Hospitals, and may be accepted, I am assured, as typical of that prevailing in the Metropolitan Hospitals generally.

The subject is so impressive in relation to our data, that a brief reference to the disease of Pulmonary Consumption should not be omitted. The view formerly adopted regarded Phthisis Pulmonalis as essentially originating within the system, though various elements contributed a predisposing force; but the discovery by Dr. Koch and others of the existence of vegetable organisms or bacilli in tubercular growths has materially modified the ancient teaching. With the doctrine of the dependence of Tuberculosis upon vital organisms, which must of necessity be introduced within the system from external sources, attention to exterior agencies may now be deemed to be the effective instrument for protection or mitigation. We are thus confronted with the two factors of (i) a possible constitutional predisposition, or (to speak figuratively) a possible provision of appropriate pabulum for these pathogenic organisms, and (ii) the possible prevention, even where such a diathesis exists, of the intrusion of bacilli by suitable conditions of isolation and physical environment.

† Whewell: *Cambridge Philosophical Transactions* for 1829, 1831, and 1850.

‡ Jevons: *The Theory of Political Economy*.

systematically to prove,—that Political Economy, for example, is capable of deduction and extension by the processes of Algebra and the Calculus, but the more temperate view of Professor Cairns* in opposition to this doctrine seems to be conclusive. The essential materials are analogous in each of the two ranges of observation and thought, but while in our work the adoption of mathematical analysis appears to be naturally congruent, the mathematical relations so employed in Political Economy exhibit merely a graphic form of symbolic illustration of truths elsewhere elicited, void of the capacity of expansion into an organon of inference and research.

In proceeding to more detailed exposition, I merely deal, statistically and historically, with the common elements of our Professional study; and the only novelty I claim is that of so massing and presenting the materials in connexion with the fundamental processes which the development of Science involves as to exhibit, by their grouping and succession, the conformity of our procedure with genuine Scientific Method. With a view to forming an independent judgment, I have found it necessary to peruse once more the original Treatises of our great predecessors,—a laborious enterprise, it is true, but one which has been enriched with ample compensation in the freshness of thought and ingenuity of resource which distinguish the pages of De Moivre, Simpson, Milne, Gompertz, and Davies.

IV. THE BASIS OF SCIENCE.

In the multitudinous variety of the phenomena which Nature presents, with the diverse uniformities of the embodied forces, a primary process is termed the Decomposition or Analysis of Facts with a view to their Classification into Elementary Data which shall possess the characteristics of reality, limitation, clearness of apprehension, and definiteness of ascertainment. This initial step has necessarily preceded the discovery of the Laws of Nature, hitherto revealed. A fundamental distinction, as I have observed, is at once apparent in the circumstance that, in the physical Sciences, the substances and forces constitute stable combinations and fixed and determinate relations, while, in our materials of mortality, we possess the entanglement of these external factors with the phenomena of human beings exhibiting determining features of their own: a complex intermixture of the distinguishing

* Cairns: *The Character and Logical Method of Political Economy.*

facts of Human Nature with the elements of General Nature itself. Even were the complete materials and their uniformities fully displayed to our gaze, the definite goal of exact expression would yet be infinitely distant, since the complicated problem would still appeal to a restricted intellect. For, unhappily, the development of knowledge, and the consequent specialisation which it compels, has not been accompanied by a concurrent enlargement of our powers of insight and causal combination.

Observations of facts, again, as every Science reveals, are indissolubly interwoven with the investigator's personal judgments, presumptions, and almost unconscious interpretations, which require, as far as possible, to be detected and eliminated. This natural confounding of inference with observation is conspicuously evident, to employ a passing example, in the common causal induction between superior longevity and married life. Milne* discussed the problem briefly and inadequately in 1815, but it was reserved to Mr. Herbert Spencer† to prove decisively that the deduction drawn from mortality observations of the Married and Single was capable, on the ground of this psychological limitation, of quite a different interpretation.

These Elementary Facts, again, besides being true and definite, must be *appropriate* for the purposes of the specific enquiries which the Science pursues.

But in the Collection and Classification of facts, as well as their *Colligation*, to adopt Whewell's‡ term, the observer, if a stable foundation is to be secured, must be guided by certain appropriate Mental Conceptions, derived, as John Stuart Mill§ has remarked, either from the groups of Intellectual Notions collected in other Sciences, or obtained by the more arduous and delicate process of eduction from the very observations which they are intended subsequently to connect. It would be futile in Chemical problems to apply merely the conception of Mechanical Force, just as research in Physiology would be nugatory were the notions of Mechanics and Chemistry alone to be employed. In our own Mortality investigations, to anticipate for a moment, it is clear that the conception of Vital Powers is the fitting basis of union; and though, at first sight, this proposition presents the appearance of the ancient fallacy of *Circulus*

* A Treatise on the Valuation of Annuities and Assurances; Vol. 2; Appendix No. 5.

† The Study of Sociology, cap. 5.

‡ Whewell: *Novum Organon Renovatum*, cap. 4.

§ Mill: *A System of Logic*, Vol. I, Lib. 3, cap. 2.

in *Definiendo*, it serves, at all events, the purpose of *defining* the region of Intellectual Notions to which we must appeal, and has been utilised,—baselessly, I deeply regret to feel,—in the hypothesis of Gompertz.

These appropriate scientific conceptions of Analysis and Colligation are generally due, in the first instance, to the native sagacity and tested skill of the investigator, and become clearer and more capable of definite service with the concurrent advance of specialised Intellectual Education, and the clarifying and corrective discipline of Scientific Discussion.

With all these difficulties confronting us, the history of Actuarial work has exhibited a congenial assimilation with established science.

Our British investigator, Dr. Edmund Halley,* expounded for the first time in 1693 the necessity of the Conception of Age, in the preparation of the Breslau Table as a classification of facts. In the compilation of the Bills of Mortality in this Country,—commenced as a late sequel to the General Visitation of Religious Establishments in 1538, and fitfully continued until their formal organisation into the General Register Office of 1836,—the ages were omitted, and it was not until John Smart,† with similar insight, enlarged upon the defect in 1726‡ that, in 1727-1728, this element was included.

The illustrious Grand Pensionary, Johan de Witt,§ again, presented his Report on Annuity calculations to the States-General of Holland and West Friesland, at the earlier date of 1671, in which the results were based upon the ages recorded. Thus, at the earliest period of our systematic history, this governing conception of Age in the tabulation and employment of Statistics was distinctly recognised in consonance with approved Scientific Method.

The second indispensable scientific conception,—that of the *Appropriateness* of the data for the specific investigation,—was also possessed by Halley, though in a confused form. He justly condemned the London records as an unsuitable index of Metropolitan Mortality in consequence of the deranging effect produced

* Philosophical Transactions of the Royal Society: reprinted in the Journal of the Institute of Actuaries, Vol. xviii, pp. 251 and 262.

† Smart: Tables of Interest and Annuities.

‡ In connexion with the employment of these observations for Mortality deductions.

§ Journal of the Institute of Actuaries: Vol. ii, pp. 121, 222: Vol. iii, p. 93. [Vide Note on p. 121.]

by incessant immigration from the Provinces under the pressure of prevailing social and economic conditions of industrial life.* The Breslau observations appeared to him, for specific reasons he assigned, to constitute a more adequate standard, and though his conception of appropriateness was thus vague and loose,—necessarily limited, as it was, by the materials at his command,—we must admire the scientific insight and disciplined ingenuity which he displayed at this great historical epoch of our Science.

A similar recognition of the necessity of Appropriateness was exhibited by de Witt, since it is clear that, for the purpose of determining the values of Life Annuities, he collected his statistics, not from the experience of the general population but, from the Registers of Annuities granted by the States of Holland and West Friesland.

It has proved a most happy stimulus in the rapid and stable progress of our work,—thus avoiding merely tentative principles of which subsequent experience would require the abandonment,—that these two distinguished precursors,—Halley and de Witt,—were possessed,—by mental constitution and facile grasp of mathematical discrimination and scientific analogies,—of the primary conceptions of Age and Appropriateness of data in constructing the foundation of Actuarial knowledge.

A partial illustration of the conception of Suitability is evident also in the formation of the Northampton and Carlisle Tables.

The genuine course was more fully pursued by Dr. Farr in 1843, 1853, and 1864, in the preparation of Tables appropriately expressive of the general mortality of this country; while, fitly consummating the scientific process, the splendid investigation of the Institute of Actuaries now in progress, worthily sequent to its similar enterprise in 1863, should be honoured as culminating the search for suitability of Experience as a basis of deduction in our sphere of work,—the finest example in our annals of the Conception of Appropriateness of data for specific prevision, and competent in this respect to rank with the most notable achievements of corresponding Scientific procedure.

I might also briefly specify the Decomposition of facts in the distinction between the mortality experience of Male and Female lives,—apparently first attempted by De Parcieux in 1746,—nor should the important resulting detection be omitted of the

* This consideration was especially significant in Halley's time, when the population of London, as we may infer from Macaulay's History of England, only approximated to about 500,000 persons.

exceptional rate of mortality at early adult ages in the case of Males,* a valuable addition to our knowledge which composite Tables could not have revealed; and finally I allude to the most significant results,—destined, I conceive, to affect appreciably our practical judgments,—deduced from the analysis and construction of materials under the guiding conception of *Selection*.†

V. HYPOTHESES, AND THE LAWS OF PHENOMENA.

The object of all researches into Nature, and the power that stimulates and directs the enlarging scope of mental conception,—the more rigorous and competent colligation of phenomena,—is the discovery of those Laws or Uniformities of operation which the forces inherent (so to speak) in the materials actually present.

Every scientific advance has been signalised and completed by the ascertainment of the permanent relations latent in phenomena; by the successive subsumption of narrower generalisations under inductions of deeper and wider import; and by the consummate stage of expressing these uniformities within the compact compass of mathematical functions as the advent of the quantitative era of research. For although all physical inductions and deductions involve a purely empirical and approximate character, the phenomena of Nature and the laws they display are exact and minutely regular; the mathematics we have discovered and generalised are implicitly and punctually embodied in the varied Universe; the phenomena of the Heavens and the Earth are but objective symbols of mathematical truth; and the discrepancy between these august exhibitions of mental power and symmetry, and the feeble picture we alone are competent to pourtray, expresses simply the presentation of an infinitely complex problem to the finitely constituted mind.

In the extraction of Laws from phenomena, we may either tentatively and directly assume some apparent order which the facts ostensibly present, and by successive simplifications and modifications endeavour to decipher the Law, or this anticipation

* Journal of the Institute of Actuaries, Vol. xiv, p. 247.

† Journal of the Institute, Vol. i, p. 179; Vol. xv, p. 328; Vol. xx, p. 95; Vol. xxi, p. 229; Vol. xxii, p. 391.

In the Tabulation of the numbers collected, and the deduction of the ratios involved, I might include the Conception of the appropriate mode of operation. The Tables of Halley, Simpson, and Price were based upon the deaths, and the accurate method was first fully pursued in Milne's investigations where the deaths occurring at each age are brought into relation with the corresponding numbers living.

of the uniformity may be guided by some prior Hypothesis* or supposition,—some general conception derived from the accumulations of cognate branches of Science. In this operation, the slightest suggestion of symmetry, the faintest indication of order, the most fugitive hint of definite progression, may prove the acceptable supposition by which to colligate the facts. Indeed, as John Stuart Mill† has observed, the limits of hypothetical assumption in pursuing the quest into natural regularities are simply coincident with the limits of the human imagination itself, provided the hypothesis is harmonious with ascertained Laws of Nature, and admits of affirmation or negation by subsequent appeal to Experience. The dictum of Lord Bacon‡ against the Anticipation of Nature has long been exploded as an authentic Rule in Science; and the history of every Science forms a perpetual and consistent demonstration that this process of Anticipation, with its sequent Verification, has proved the impressive and opulent source whence the large generalisations and imposing accumulations of exact knowledge have been added to the intellectual heritage of the Race. Newton's "Hypotheses non fingo",§ again, only applied to suppositions that rested on no probable basis and violated the system of established truths, for the Law of Gravitation was itself the grandest hypothesis which the genius of man has yet devised, although transcending the grasp of human conception through the absolute indifference of its Force to interposing bodies. The Undulatory Theory of Light, again, shows the deepest perplexities in the way of harmony of conception: the conjunction of an infinite mobility or elasticity in the Ether with a vaster solidity than Steel! Yet it possesses the property of a *Vera Causa*, for not only are the phenomena of Light, Heat, and Radiant Energy explicable on its assumption in analogous congruence with the phenomena of Sound, but Clerk

* Hypotheses or Suppositions are the mental conceptions which, under the tendency of the mind towards generalisation and unity, are provisionally "placed beneath" (as the term etymologically signifies), the apparently disconnected facts as their rational support and explanation. And although scientific language is not precisely determinate upon the point, we may legitimately affirm that an hypothesis, which has received adequate confirmation by recognised scientific tests, may, at that supreme moment, be designated a *Theory*, or an original fact of Nature which the Speculator or Ideal Spectator (for this etymological implication the term "Theory" involves) would, if gifted with commanding power of mental vision, directly observe as an integral constituent of the Physical Scheme.

† Mill: *A System of Logic*, Vol. 2, Lib. 3, cap. 14.

‡ Bacon: *Novum Organum*: Lib. 1, Aph. 26.

§ Newton: *Principia*, Lib. 3.

Maxwell has adduced valid reason for surmise that the complicated manifestations of Electricity and Magnetism constitute but stresses and motions in this marvellous adamantine medium.

An essential condition of a serviceable hypothesis as a probable representation of Nature consists in the possibility of comparison between its calculated consequences and the phenomena actually observed. It was on this ground that the hypothesis of Vortices of Descartes as an explanatory exposition of Celestial Mechanics failed; the conditions of verification could not be rigidly applied: and, in Optics, the rival pretensions of the Undulatory and Corpuscular Hypotheses were definitely decided by the Experimentum Crucis of the relative velocity of light in denser and rarer media.

And a final attribute of a genuine hypothesis resides in the circumstance that, as it reaches its ultimate shape, a deeper and wider simplicity of form is presented, with a consequent facility and extension of application.

In our own sphere a vigorous and persistent search has been instituted to discover the Law of Mortality supposed to be decipherable from our observations. This adventure appears to have started in what Comte described as the theological stage, or that primitive mode of conception, which, in his judgment, constituted the origination of Science. The mystic number 7, derived from Chaldean usage, was invoked in explanation, and septennial periods of life were assumed to be naturally expressive. In an Act of Parliament passed in 1540, it was accepted on that basis, that a single life was equivalent to a lease of 7 years, two lives to a lease of 14 years, and three to a lease of 21 years, thus creating the stereotyped restriction of leases to 7, 14, and 21 years. Even after the publication of Halley's researches in 1693, Acts were passed in 1694 and in 1703, in which the ancient numerical relationship (somewhat modified) was adopted in the assessment of pecuniary values.

We reach, in 1671, the important epoch of de Witt, who assumed that the probability of death continued uniform during various consecutive stages of life, but in each period presented a definite numerical proportion of increase, varying with the interval, to the initial ratio. From researches which have recently been made into the primitive history of Life Contingencies in Holland, I should imagine that these results were found to be approximately presented by the series of numbers living without the introduction of any directing anticipation. It is interesting

to notice that, in the correspondence between de Witt and the scientific Burgomaster, Hudde, in 1671, a provisional supposition is mentioned that, from an early age, the decrements of life are so constituted that, out of 80 young persons alive, about one dies annually until the whole body is extinct.

This anticipation of De Moivre* naturally introduces the hypothesis of the latter, published in 1825. It may be preliminarily remarked that, however incompetent the attempt, De Moivre possessed the merit of treating in reasoned detail of a general uniformity in the natural sequences of the numbers recorded from age to age; while, at the same time, he displayed a sound appreciation of the necessity of verification from various sources, and of the need of congruity between any Mortality law and the character of Natural uniformities already established. His supposition was avowedly designed as the tentative basis of a facile method for computing the values of Life Annuities. With the tact of a mathematician, he at once differenced the numbers contained in Halley's Table, and noting the approximate uniformity of decrements, he assumed, on general and special considerations, that the limiting age might be fixed at 86, with the involved hypothesis which enshrines his name. But the hypothesis was a mere verbal enunciation of the disclosed series of differences; it was extracted from a limited and defectively constituted Table; it possessed, therefore, no natural generality of expression; and its sole recommendation, beyond its aid to calculation, appears to have consisted in its specious aspect of simplicity in consonance with crude notions of physical truth.

The genuinely ambitious attempt to obtain a Law of phenomena which our annals record, and the one which is precisely in harmony with scientific method, is the hypothesis of Gompertz. And had the principle which guided him in his selection of a formula been an authentic induction from physiological data, or countenanced by physiological research, that hypothesis would have justly ranked among the most admirable achievements of Science, having regard to the intrinsic importance of the problem and the manifold complexity of the facts.

Dealing generally, in his preliminary Memoir of 1820,† with the development of functions which decrease with increments of time, he made an impressive advance in his famous Memoir of 1825† with the enunciation, conceived in the true philosophic

* De Moivre: Annuities upon Lives.

† Gompertz: Philosophical Transactions of the Royal Society.

spirit, of the possible co-existence of fortuitous and continuous forces in determining the reduction of vital power. Proceeding at first on the basis of the latter supposition alone, he obtained his celebrated formula expressive of the intensity of mortality, with the deductive formula for the number living at any given age.

Mr. Jellicoe,* as far as I can ascertain, was the first writer to approve the formula on the distinctive ground of its philosophic character, and subsequent authors,† as Gray, De Morgan, Makeham, and Woolhouse, have assigned to it a validity and natural supremacy by reason of the physiological principle on which it was professedly founded. For Gompertz proceeded in the rigorous manner which all eminent discoveries of physical laws disclose. In almost every supreme instance in the Sciences, the resulting mathematical formula, which expressed the bond of connexion between the observed facts, or their representative numbers, has not been derived empirically from the observations themselves, but has been suggested primarily by some appropriate hypothesis respecting the probable nature of the connexion itself. A remarkable example is afforded by the explanation of the Fringes of Shadows in Optics, which defied even the marvellous sagacity and intuitive vision of Newton, who confined himself to direct examination of the phenomena alone, and it was only when the problem was approached under the guidance of the Principle of the Interference of Undulations, by Fresnel, Young, and others, that the hidden law became revealed. It will be remembered, too, that Darwin was directed to Natural Selection and the Survival of the Fittest in Natural History by means of the Theory of Population of Malthus. In a similar scientific spirit, Gompertz pursued the legitimate course of examining the series of numbers living in the light of his assumed principle of the mode and measure of reduction of power in the physical frame. But the principle of selection must be a genuine physical induction as well as appropriate in its nature to the observations; and unless that condition be satisfied its introduction is futile and delusive so far as any natural authority is thereby conferred upon the dependent formula. The mathematical function will only bear the authentic symbol of physical

* *Journal of the Institute of Actuaries*: Vol. iv, p. 199.

† *Journal of the Institute*: Vol. vii, p. 121; Vol. viii, p. 181; Vol. xiii, p. 325; Vol. xv, p. 389.

truth when the hypothesis by which it is imprinted constitutes a valid fact of Nature. Now Gompertz's physical assumption is clearly appropriate to the enquiry; but, unhappily for any natural distinction with which it might endow the formula, it is a purely unfounded speculation which no physiological generalisation, no record of physiological investigation, no acknowledged scientist, past or living, either indicates or confirms. To express a principle in the language of physiology is essentially different from the announcement of physiological truth.

The formula of Gompertz further failed to satisfy the requisite conditions of scientific verification: the disruption of continuity through change of constants marked its want of harmony with Nature; and his subsequent introduction of additional constants* merely burdened the function with cumbrous elements, like the successive accumulation of appendages to the hypothetical mechanism of Epicycles and Eccentrics, with consequent remoteness from the severe simplicity of scientific truth.

I need only refer to Makeham's admirable generalisation† by the inclusion of a function representing the operation of the accidental causes, which expanded the range of the formula while preserving its symmetric form, since the inherent defect in the primal assumption obviously attaches to all extensions of the expression.

Gompertz, though postulating accidental causes of disease, clearly did not anticipate consciously the recent teachings of Bacteriology; but it is nevertheless a striking circumstance that his happy and sagacious divination is coincident with the modern discovery of extraneous forces or vital organisms with their character of fortuitous incidence.

A definite Law of Mortality then would seem to be as shadowy as a dream; and the advent of the happy discoverer,—combining the powers of the Physiologist, the Physicist, and Mathematician,—to form but a baseless hope.

I fear I have lingered out of due proportion upon this fascinating aspect of scientific work, more especially as our Science has unfortunately failed to realise this object of its quest. The high intellectual charm of the subject must win forgiveness, and to this claim I add my genuine admiration of the scientific

* *Journal of the Institute of Actuaries*: Vol. xvi, p. 329.

† *Journal of the Institute*: Vol. viii, p. 301; Vol. xxviii, pp. 152, 185, and 316.

power and philosophic spirit displayed by Gompertz in his supremely able attempt.*

Comte justly insisted that power of Prediction constituted an essential attribute of a valid hypothesis; and the history of Science is richly emblazoned with these felicitous and skillful scientific guesses. Astronomy is laden with memorable examples, of which the subtle mathematical vision of Neptune is not the least; Physical Optics includes the marvellous predictions of Fresnel and Sir W. R. Hamilton in the complicated phenomena of double and conical refraction; in General Physics, Professor James Thomson's prevision of the depression of the melting point of ice under pressure has been finely confirmed; and finally we learn that Clerk Maxwell's famous prediction of the constitution of Saturn's rings has recently been spectroscopically verified.† Had Gompertz's formula rightly expressed the facts of mortality, this scientific requirement would have been excellently exemplified by inclusion of the rule which De Morgan‡ termed the Law of Uniform Seniority.

Beyond the general prediction which a Table of Mortality involves, I can only recall at the moment a prevision uttered by Milne§ in 1815, that a heavier rate of Mortality would be found prevailing in the Upper Classes as compared with the middle Classes of Society, with which he united the hope that specific investigations would be pursued. The memorable researches into the Mortality of Peerage Families in 1861|| fulfilled that hope, but at the same stroke finally shattered the prophetic anticipation.

VI.—THE INSTRUMENTS OF SCIENCE.

The distinctive epochs in Science have generally been marked, and frequently inaugurated, by the invention of some Instrument of Research and Deduction,—the introduction of Instantaneous Radii,

* Reference might also be made to an investigation (Journal of the Institute of Actuaries: Vol. xvii, p. 56) instituted into the law supposed to be exhibited in the ages at which Assurances are effected. Even if this uniformity were established, it would prove to be simply of an empirical character implying no adequate ground for extension beyond the individual or composite experience in which it might be observed.

† By Professor Keeler.

‡ Journal of the Institute of Actuaries: Vol. viii, p. 181.

§ A Treatise on the Valuation of Annuities and Assurances: Introduction, p. 51.

|| Journal of the Institute: Vol. ix, p. 305.

to employ the picturesque phraseology of Bacon.* In the language of Sir Humphrey Davy: "Nothing tends so much to the advancement of knowledge as the application of a new Instrument."

The establishment of the Science of Thermotics may almost be said to have been effected by the invention of the Thermometer; in Chemistry, the nice and delicate adaptation of the Balance inaugurated an era of fruitful discovery; the application of the Microscope has supplied the key to many Biological secrets by its revelation of primitive life-histories in Embryology; while, in more modern times, the significant results obtained from the perfection and skilful use of the Spectroscope have immeasurably widened our knowledge of cosmical spaces, and gathered into an imposing unity of structure the myriad spheres and systems of the Skies. I have selected in our own Science four illustrative Instruments which appear to be deserving of special distinction.

(i) The first to be recorded is the Life Table itself; and it has proved of happiest augury in our history that, from the commencement of research, this instrument was devised in so masterly a form as to facilitate investigation and deduction in the completest mode. Halley's Table† has since been modified only,—so adequate was its construction,—in the separate expression of the decrements of life, first furnished explicitly, but in an inconvenient manner, by Thomas Simpson‡ in 1742.

De Witt did not produce a Table of Mortality, but his calculation of the Value of a Life Annuity as a type of procedure presumes a similar basis.

The honour of origination is distinctly to be accorded to Halley.

(ii) The Columnar or Commutation Method may be honourably specified as a second Instrument. A careful comparison of authorities, and actual computations, definitely assign priority of conception to William Dale,§ who, in 1772, clearly explained the

* *Novum Organum*: Lib. ii: Aph. 39.

† It is curious to notice that the interpretation of this Table has not proved generally evident. Montucla conceived that the number (1000) placed against "age cur^{tu} 1" expressed the births: Daniel Bernouilli understood that the number of infants born was not furnished, and that the 1000 were supposed to attain the age of 1: he accordingly estimated the radix of the Table to be 1300; while Farren considered that the 1000 represented the number of children who were aged one year.

‡ *The Doctrine of Annuities and Reversions*: containing a Table deduced from the London Bills of Mortality.

§ *Journal of the Institute of Actuaries*: Vol. i, p. 15*: "Calculations deduced from First Principles."

principle of the method and furnished correct examples of its use. Had he applied his process from the earliest age instead of age 50 (for his calculations were intended for employment by Pension Societies whose grants commenced at the latter epoch), or had he multiplied his actual figures by a constant factor of interest determined by the initial age he selected, the contents of his columns would have been obtained in the precise form to which we are now accustomed.

Professor Johan Nicolaus Tetens,* of Kiel and Copenhagen, independently devised the same conception in 1786, for, recalculating his results, it is obvious that his column C_x is exactly the same as our column D_x , while his column E_x is precisely similar to the modern N_x , but expressed in the form of N_{x-1} .

Barrett's arrangement,† absolutely independent of that of Tetens, as the process of Tetens was unsuggested by Dale's—was published in 1813, although conceived in 1786: and the method received its final extension by Griffith Davies‡ in 1825. It is constantly occurring in Science that when any Preludial stage of knowledge is verging upon its Historic Epoch, to employ the happy language of Whewell, the magic word of discovery is ready to be uttered by many a well-trained voice; and the history of Science acquires much of its human interest and attractiveness from the simultaneous or promptly successive unison of these disciplined inspirations of mind.

(iii) The third instrument I select is the Continuous Method of Annuities and Assurances.§

Surveying a Table of Mortality in its customary form of mere numerical expression, we gain no clear conception of the full meaning it implies; but project the numbers successively living in the form of a Curve, and at once the suggestion arises of a gradual progression in time. Under the guidance of this conception, we contemplate the race of Man, or an Aggregate of Men, as a Quantity or Fluent, to adopt the language of Newton's Calculus, flowing continuously through time, of which the Fluxion or Differential Co-efficient is the Rate of Flow. Hence, naturally and spontaneously, we perceive, as in an objective spectacle, how minutely the Higher Calculus is adapted to the interpretation;

* Journal of the Institute of Actuaries: Vol. i, p. 2.

† Journal of the Institute, Vol. i, p. 1; Vol. iv, p. 185.

‡ Tables of Life Contingencies.

§ Journal of the Institute: Vol. xv, p. 96.

and when, as employed in our calculations, the factor of interest exhibits the same continuous character, the perfect congruity of this method is not merely observed as a felicitous union of Principle with appropriate Materials, but its very natural assimilation to our work, if I may so express it, affords a sure prophecy of its extended and fruitful use.

(iv) I confine myself to the selection of one other Method or Instrument, which, I think, I may justly include. This is presented under the form of what has been variously described (in somewhat unhappy language) as the Model, or Average, or Representative Office for the exhibition of Valuation results.* I conceive that a complete change was produced in our reasonings, or at least in their general and effective shape, when, from isolated comparison of individual Policy-Values, at different ages and for various durations, we passed to the assemblage of values which the Representative Table displayed, and which alone are practically serviceable in forming an adequate conception of the aggregate effect of various Mortality Tables and Rates of Interest.

VII. THE METHODS EMPLOYED IN THE FORMATION OF SCIENCE.

All methods adopted in the construction of Science are allied in the essential feature of facilitating the detection of the Order of Progression or the Modes of Relation, of which phenomena are the material signs, and their completed aim consists in the discovery of a mathematical function from which results in agreement with the facts of observation may be deduced.

Adapting the technical processes of physical Science, the successive procedure in our investigations, if we pursue a mathematical law, comprises the selection of the Independent Variable, the Construction of a Formula of Expression, and the Determination of the Co-efficients it involves. And in the whole compass of scientific enquiries with which my very restricted knowledge is cognisant, the palmary researches in 1817 of the French physicists, Dulong and Petit,† into the relation between a series of temperatures and the velocity of the cooling of bodies, present, in principle, very curious analogies with the processes we must

* Journal of the Institute of Actuaries: Vol. xiv, p. 249.

† Whewell: Nov. Org. Renov.: Lib. 3, cap. vi.

Balfour Stewart: A Treatise on Heat: Lib. ii, cap. 4.

adopt, and appear to furnish an instructive type of operation if its object be the ascertainment of a Law. The Independent Variable of the temperature was made to increase in arithmetical progression while, for each change, the quantity of the variant,—the rate of cooling—was observed. Sagacity and experiment, aided by mathematical artifices, then tentatively investigated an algebraic formula of an exponential character, by which the observed relations became congruously expressible. These elements correspond in our case to the Independent Variable of the Age, and the varying numbers of the living or dying. It is true that, in accordance with the established course in Science, Dulong and Petit were guided in their provisional selection of a formula by the physical hypothesis of Prévost respecting the mutual Exchanges of Radiant Heat between contiguous bodies, and hence the mathematical equation they obtained was not merely indicative of a uniformity of Nature, based upon a valid prior induction, but served the ancillary purpose of all true Theories in further establishing and expanding the principle on which it proceeded. In our labours we must be content, so far as we pursue this method, with adopting these mathematical operations upon numbers, since we are devoid of the assistance of any physiological principle of direction which, had it been genuine, would have proved the counterpart of Prévost's Theory of Exchanges.

Without describing the entire series of methods, I select the two which, in Science, have been termed the method of Means and the method of Curves.

The method of Means has been extensively employed in Science, and in the great Report in 1829 of our first President,*—whose honoured name is happily and worthily still continued in our ranks,—we possess an instructive example. In the various modifications which have since been attempted, an important advance consists in the introduction of a completer scheme for utilising the "Weights" of observations.† And certainly, to my mind, no more admirable adaptation of this method has ever been essayed than the system which Woolhouse‡ devised as an instrument of graduation. Notwithstanding its limitation of application to our work, it ranks in our history as a remarkable specimen of mathematical grasp and ingenuity, worthy of being

* J. Finlaison: On the Evidence and Elementary Facts on which the Tables of Life Annuities are founded.

† Journal of the Institute of Actuaries: Vol. xxii, p. 24.

‡ Journal of the Institute: Vol. xv, p. 389.

included in the impressive collection of serviceable processes which the rich accumulations of Science contain.

A further scheme appears likely to prove the most prophetic of success in our approximate efforts. For the relations between the age and the numbers living or dying may be projected into a Curve,* when, by this aid of space-representation, the uniformities seem to spring at once into evident relief, and reveal or suggest, as in a picture, the ideal and general regularity, of which the numbers individually are the fragmentary signs, however interrupted by minor breaks and tortuosities which the errors of observation and collection may have introduced. The eye,—our most intellectual sensual organ, as Bain has remarked,—trained by experience and fashioned by acquired skill into a fineness of perception of symmetry, deciphers, from the flow of the line, the uniformity of which it is in search; and, not infrequently, the disciplined organ of mind, following, in this united venture, the perception of the eye, detects, as the flexibilities of the Curve are pursued, the class of mathematical functions to which the regularity appertains, and thus compresses the pictured scene within the framework of symbolic representation. At all events, if a mathematical formula be not secured, the adjusted flow of the Curve, appropriately modified and adequately tested, will exhibit, with sufficient approximation, the natural progression which the facts embody. This method has been conspicuously productive of result in the investigations of Astronomy, Meteorology, and the Tides.† Admirable examples of the employment of the Method also enrich the literature of our special subject; and I venture to express the general view that our investigations will be most successfully aided and recompensed,—not by any assumed Law of Mortality which we expect the phenomena to disclose; not by the introduction of any physiological principle which possesses no foundation in Biology; not even by the adoption of any method of Means, however elaborate and ingenious, but,—by the judicious and skilful application of the Graphic Method. Some writers‡ have expressed the apprehension, and even the conviction, that the employment of this method by different investigators would produce seriously divergent conclusions on account of its supposed

* Journal of the Institute of Actuaries: Vol. xxvi, p. 77.

† Herschel: Investigation of the Orbits of Double Stars;
Herschel: Meteorology, § 29.
Whewell: Researches on the Tides.

‡ Farren, *e.g.*: The Chances of Premature Death and the Value of Selection.

largely subjective character. This objection appears to me to be futile. We do not entrust the processes of the Differential Calculus to him who has merely mastered the Binomial Theorem; and the use of this method by competent students, disciplined in the practice of our work, may confidently be expected to exhibit a practically uniform result.

The necessity may be urged, in passing, of presenting in every case, the *original* facts, purged only by approved methods from errors incidental to observation and classification, and placing in juxtaposition the graduated figures determined by the mode of adjustment which appears to be appropriate. We should then observe how, in many older graduations, important and capital features, were deliberately (through ignorance or pre-conception) obscured* or lost, especially the significant change in the rate of mortality at early adult male ages. It is obvious that, confirmed as this fact has been by authentic experience, no formula or method which suppresses it can be in conformity with Nature. Just as the mathematical formula for a Curve presents the curious phenomena of Isolated or Conjugate Points and a Continuous Line, each element being equally and essentially involved in the equation itself, so any mathematical expression of uniformities deduced from observations on Mortality should include, as an integral constituent of its construction, the striking phenomenon I have mentioned. This consideration once again pronounces that the expectation of a mathematical statement of a Law of Mortality must remain within the sphere of hope, and directs our anticipations of practical success more conclusively to the resources of the Graphic Method.

VIII. MODES OF COMPUTATION, AND THEIR SIMPLIFICATION AND EXTENSION.

Science inevitably—and as a guarantee of a genuine progress—tends to evolve simpler, and therefore more general, modes of investigation and calculation from scattered, cumbrous, and isolated forms. We can appeal to both the mathematical and physical departments of knowledge for apt and suggestive illustrations of this process.

One of the most admirable simplifications in Pure Mathematics—important in its consequences, and simple in its form—was the

* Dr. Price, *e.g.* under a preconception, arbitrarily altered the decrements of the Northampton table between the ages of 20 and 30. (Journal of the Institute of Actuaries: Vol. v, p. 284.)

suggestion of Harriott* in 1631 of the transposition of the members of an equation to one side and equating to zero.

In the development of Functions, we perceive the Binomial Theorem, which Newton confessedly obtained by induction, comprehended as a particular case in Taylor's Theorem in the Differential Calculus, which again has become merged in the higher mathematical generalisations of Laplace and Lagrange. A wider and successively wider formula in order of simplicity and comprehensiveness emerges as the Science proceeds.

In this region, I need only, again, refer to the foundation and development of the Higher Calculus: the Scheme of Exhaustions of the Greek Geometricians, with its necessary confirmatory supplement of the *Reductio ad Absurdum*; the Method of Indivisibles of Cavalierius, which reduced the ancient process into a compacter compass, and relieved it of the verifying encumbrance; and the absorption, with added rigour of demonstration, of all these symbolic systems in the Fluxions of Newton and the Infinitesimal Calculus of Leibniz.†

In the physical Sciences, the relevant process consists in the introduction of a simplification of the problem presented, and its graduated extension into wider generality of expression by successive approximation. Newton's solution of the problem of planetary motions entirely depended at the outset on a conspicuous simplification by proving that homogeneous material spheres acted as though their masses were condensed into the centres, and thus comprising them within the range of the Law of Gravitation as applicable directly to infinitely small particles.

Dr. Halley pursued, in 1693, the correct method of determining the values of Life Annuities, which had previously been assessed by guess-work, by combining the successive probabilities of survivance with the factor of discount. So masterly was the command of mathematical resources, and so keen and ample the gift of scientific insight, of that remarkable man, that at one felicitous stroke, he laid the permanent fortunes of our Science in the construction equally of the basis and mode of deduction of Annuity computation. But in each series of calculations, the summation of which constituted the value at any age, he was compelled to proceed independently, so that the values of one series failed to be utilised in the construction of the rest. Hence

* Ball: *A History of Mathematics*.

† Carnot: *Reflections on the Metaphysical Principles of the Infinitesimal Analysis*.

the laborious character of the process necessitated the limitation of his results to a fragmentary Table. And he confessed that he perceived no method of generalisation which could diminish the vast extent of toil.

But earlier, in 1671, the illustrious de Witt had pursued a similar, though apparently an equally discontinuous, plan, in the Report presented to the States-General, for though he computed his results by combining the successive values of Annuities-certain with the Annual probabilities of Death, his formula is obviously transformable into the established system devised by Halley.

In 1654, the subtle intellect of Pascal, in his correspondence with the mathematician, Fermat, had securely based the foundation and principles of the Calculus of Probabilities, and, with that instrumental Science thus newly created, gifted minds were alert to prolong its power into practical applications, exhibited in our domain by the absolutely independent investigations of these two distinguished men. To whom should the priority of originality be conceded? The famous Canon, termed Waring's Rule, which has frequently been invoked in deciding controversies of this character, determined priority on the basis of Publication, and the scope of Publication has usually been accepted as defined by public announcement through the press. Judged by this standard, Halley's claim would possess precedence, but seeing that de Witt's Report was actually printed in 1671, though its circulation was limited to the Members of the States-General, it must undoubtedly be conceded that the originator of the principle of the true method is de Witt.* We hold them, however, in equal honour, without invidious appraisement of merit; and the discovery ranks, in its degree, within the category which has been rendered illustrious by the independent discovery of Newton and Leibniz in the Calculus; the suggestion of the Undulatory Theory of Light by Fresnel and Young; the announcement of Natural

* Recent investigations into the early history of Life Contingencies in Holland have been fertile of interesting discoveries. We now learn (i) that when de Witt's Treatise was presented to the Assembly of the States-General of Holland and West Friesland in 1671, it was at once ordered to be officially printed. This was done; and the document was circulated among the Members of the States; probably about 100 in number; (ii) Extracts from the Resolutions of the States of Holland and West Friesland were regularly printed officially shortly after the Resolutions had been passed. In the Volume containing Extracts, dated December 1670, de Witt's Report again appears under date of the 30th of July 1671. This Volume I have had an opportunity of inspecting, and copies are exceedingly rare; and (iii) the manuscript of the Treatise still remains in the Archives of State at the Hague.

Selection as part of the mechanism of Evolution by Darwin and Wallace; and the marvellous intellectual reach of power of Adams and Le Verrier.

In 1725, a striking simplification was effected when these isolated Methods were merged into the wider formula by which De Moivre* proceeded, from the value at any age, to deduce the corresponding value at the next younger age.

De Moivre pursued his investigation in the mode which the historians† of Mathematics describe as Rhetorical Reasoning, where the process is conducted in language without the intervention of Symbols. In 1742, however, Thomas Simpson,‡ though adopting the same procedure, concluded with a mathematical demonstration in which the modern formula is precisely expressed under the more general form of a combination of joint lives. It is not difficult to pronounce, with authoritative certainty, that the old dispute between the claims of De Moivre and Simpson on the question of priority of conception must be decided definitely in favour of the former.

The Summation-formula of Lubbock,§ involving Finite Differences; that of Woolhouse,|| including Differential Coefficients; and the modern methods of approximate Valuation of Annuity and Assurance problems by means of Definite Integrals¶ are interesting examples of wider and more simple methods, in gradual succession of extension.

The history, already furnished, of the Commutation Method is an additional illustration of this development in simplicity and capacity of form, both in respect of its substitution for more cumbrous and isolated modes of computation,** and the progressive improvement of the Method itself.

And regarding the subject generally, Thomas Simpson,†† in 1742, appears to have been the first writer who avowedly

* Annuities on lives: $a_x = vp_x (1 + a_{x+1})$.

The distinguished mathematician, Euler,—as we learn from Todhunter's History of the Mathematical Theory of Probability,—investigated the problem in 1760 (published in 1767), and showed that the value at any age provided a mode of immediately determining the value at the *succeeding* age. This conclusion was apparently deduced quite independently of the enquiries of his predecessors, and the precise form of the modern expression is, of course, at once obtained by a simple algebraical transformation.

† Ball: A History of Mathematics: Cap. v.

‡ Simpson: The Doctrine of Annuities and Reversions.

§ Journal of the Institute of Actuaries: Vol. xviii, p. 305.

|| Journal of the Institute: Vol. xi, p. 301; Vol. xv, p. 95.

¶ Journal of the Institute: Vol. xxiv, p. 95; Vol. xxvi, p. 276 Vol. xxvii, p. 122.

** Journal of the Institute: Vol. i, p. 96*.

†† The Doctrine of Annuities and Reversions.

attempted to devise a universal system of procedure, accompanied by a general Notation, without restriction to the incidents and peculiarities of any specialised class of observations.

IX.—THE APPROXIMATIVE CHARACTER OF SCIENCE.

No teaching is more impressive and valuable in the historical evolution of every physical Science than its frank confession of the approximate character of its processes and results.

The infinite variety of Nature; the complex combination of Causes; the consequent intermixture of Effects; and the bewildering interactions of diverse laws, present, even in comparatively simple phenomena, a tangled maze to the baffled intellect, while, ever existent behind the seeming chaos, shine serene simplicity and symmetric precision.

In almost every mathematical expression in physical research, apparently so complete and definitely inclusive, a vaster number of terms are rejected through mental limitation, than the sparse and larger terms that are retained. In the finest development of Science,—Astronomy,—the problem of the Three Bodies is a purely approximate solution, while the more complicated scheme of attractions existing between the members of a larger system require to be decomposed, approximation upon approximation, into separate sections involving three planets only.* No planet, again, exactly conforms to Kepler's laws, which only hold precisely true of infinitely minute bodies, and the processes applied under the Law of Gravitation assume that each planet is a perfect ellipsoid of homogeneous structure. Professors Thomson and Tait† have furnished a most impressive lesson of pure approximation in the consideration of apparently the simplest problem in Statics,—the employment of a crowbar in raising a heavy body; and, without wearying you with additional illustrations, the stamp of distinct empiricism, limitation, and conditionality is impressed upon practically every physico-mathematical result which the genius of the race has contributed to the opulent treasure-house of Science. It has often occurred to me that an Actuarial student would commence his enquiries into our special subject with a mind more efficiently trained and accustomed to practical considerations, and less liable to be misled by the

* Cheyne: *Treatise on the Planetary Theory*. Godfray: *Treatise on the Lunar Theory*.

† *Treatise on Natural Philosophy*. Vol. ii, cap. 5.

decimal fallacy, as I may term it, of affected and baseless precision, if he rendered himself familiar in some degree with this universal and memorable lesson of Scientific truth.

Sir John Leslie,* in 1804, raised a serious warning to physical investigators against pretended and delusive exactness: I repeat the needed caution to my younger colleagues, and point out to them that this decimal precision is not merely illusory and impracticable, but is absolutely misleading in presenting an ostentatious appearance of exactitude to which no valid pretension can be raised. And to mention a small, perhaps, but significant illustration, I yet hope to survive the time when the idle parade of decimals in our Valuation results will be discarded as an error. But let me not be misunderstood. A sound mathematical equipment is essential to a successful prosecution of our labours in any useful form, but the teaching which I urge with earnestness upon our younger members is that mathematical processes are simply instruments to be employed, over which judgment, experience, and a practical acquaintance with the affairs of life and with the necessary limitations of our work should preside in autoeratic and unquestioned power. Even in the purer realms of Geometry, we are attended by the spirit of approximation, for the ideas to which our reasonings apply possess no reality in Nature, but are simply, so to speak, the intellectual limits of sensible quantities and qualities. The character of rigorous exactness is secured because the elements of calculation are ideal.

X. THE LANGUAGE OF SCIENCE.

The exposition of the evolution of a Science usually includes a reference to the Scheme of Language which embodies and perpetuates its discoveries; and this custom I venture to pursue in relation to our Professional Speech.

The advance of knowledge requires to be expressed in clear and definite Propositions, compacting into manageable masses individual and isolated facts; but, in addition, is demanded a system of Nomenclature and Terminology, which constitutes the current coin of mental gains, and provides a simple and expressive medium of intellectual intercourse.

In the Symbolic Notation which now dignifies the Science of Life Contingencies, we possess, I conceive, a scheme of abbreviated language which is at once simple, lucid, definite, and

* An Experimental Enquiry into the Nature of Heat: Note iv.

connected. The visual relief and mental gain are almost startling when we pass from the bizarre pages of Milne with their bewildering maze of eleven different forms of type to the adequate and congruent system in which we are now competent to present our processes and results. The advantage I have mentioned is not merely the exhibition of symmetry to the eye, but that distinctive aid in investigation which accompanies every wise and organic reform of language, in mental relief, intellectual clearness, and enlarged power of concentration. But when we turn from this admirable lexicon of symbols to our current Nomenclature, we descend from the cultured pages of the philosopher to the cumbrous dialect of the peasant. What consistent or adequate significance is conveyed in our "net", "mathematical", or "risk" premium? Where is the inevitable appropriateness or precision of meaning in our "office" or "gross" premium? And we attain the acme of linguistic barbarity in that abhorrent term, the "loading." Our scientific language should fulfil the requisites of precision and consistency in the work of specialists, and of freedom from alien connotations when our words are uttered by the laymen whom we teach.

The Criteria of a Scientific language have been adequately, and almost finally, expounded by Whewell* and John Stuart Mill,† to whose authority all cultured scientists appeal; and I shall not despair of the advent of the time when the sagacious and constructive skill which has elaborated our refined scheme of analytical speech refurbishes these unrepresentable expressions, or rather substitutes a more consonant triad of technical terms.

XI. THE BENEFIT OF SCIENCE TO THE COMMUNITY.

In no portion of Bacon's *Novum Organum* does his language assume a more sonorous and dignified expression or his predictions a more vivid and impressive tone than in those inspiring passages which expound the social benefits conferred by Science and expatiate upon the widening area over which her future successes should beneficently extend.

And following this custom, which has been observed by historians of Science down to Herschel and Whewell, I conclude this portion of my Address with a few brief reflections in adaptation of such a survey to our special work.

* Aphorisms on the Language of Science: *Nov. Org. Renov.*

† *A System of Logic*: Vol. ii., Lib. iv., cap. 4 to 6.

I need only summarily refer to the vast practical scheme of Assurance which our principles have founded, directed, and expanded, with its countless social, economic, and even political consequences; and addressing myself to a more specialised subject, I point with admiration to the commercial and financial advantages to the community which the introduction of Market Values, by the appropriate adaptation of Actuarial principles, has permanently created. The history of this feature is deserving of a momentary chronicle. In the earlier writers, the purchase and sale of Life Interests were based exclusively upon pure Annuity-Values.* In the Treatises of De Moivre† in 1752 and Dodson‡ in 1753, we discover for the first time a solution of the problem in which Life Assurance was involved as an illustrative element; but the purchaser became his own Assurer. The absence or imperfect conception of a general Assurance system must naturally be remembered when considering these attempts. It was only in 1825 that Griffith Davies§ first practically solved the problem as the commencing stage of a varied and exhaustive application. Davies deduced his result from first principles, and it is interesting to observe, as was subsequently pointed out,|| that, by means of the Conception of Analogy,—so fertile an instrument of discovery in the advance of Science,—he might have detected the process by comparison with the “Sinking-fund” factor in the valuation of an Annuity-Certain.

I recall, too, the memorable monetary advantage to the Public Funds which our first President¶ established, when, in 1819,—guided by the Conception of Appropriateness of data,—he denounced the employment of the Northampton Table in the grant of Government Annuities.

And another illustration may be adduced of the practical efficiency of our methods for commercial purposes, in the admirable and conclusive researches, commenced in 1859, into the Probabilities of Issue deduced from the records of Peerage Families,** which have rendered numberless interests marketable to which no definite value had previously attached.

* Journal of the Institute of Actuaries: Vol. vii, p. 136.

† Treatise on Annuities: 4th edition.

‡ Mathematical Repository.

§ $\frac{1}{d+p} - 1$: Treatise on Annuities: cap. iv., § 5.

|| Journal of the Institute: Vol. xx, p. 435.

¶ Journal of the Institute: Vol. x, p. 147.

** Journal of the Institute: Vol. viii, p. 127; Vol. x, p. 181; Vol. xii, p. 185; and Vol. xxi, p. 406.

It will summarily be observed that the examples and applications of Actuarial principles which I have described throughout this Address, amply and impressively exemplify a general criterion of Science in the progressive advance from the Isolated to the Combined; from the Complex to the Simple: from the Homogeneous to Heterogeneity both of structure and method.

XII.—THE CHARACTER AND AIMS OF THE ACTUARIAL STUDENT.

The advance of Science, and the specialised Education of the enquirer, proceed in mutual interaction. Enhanced resourcefulness of investigation ensues as the accumulated facts and laws become added to the armoury of research. Defective processes assume a keener precision and pliability; each happy interpretation and generalisation of Nature preludes a wider amplitude of survey and more assured confidence of insight; the intellectual obedience of man to the teachings of Nature becomes transformed into a supremacy of power; and thus, besides conferring an exacter and more refined character upon the Science, Man himself, as the *Minister et Interpres Naturæ*, in Bacon's phrase, gradually becomes possessed of a finer sagacity in search, a developed skill in method, and a more vigilant vision of Ends.

And the historians of Science naturally deduce, from their survey of research, the assemblage of attributes, which distinguish the genuine cultivators of this systematised form of knowledge. In pursuance of this customary course, I venture to address a few reflections, specially and earnestly, to my younger colleagues: and though most of us cannot hope to emulate the commanding qualities of the Masters who have contributed to the development of our work, we may yet gain courage from the circumstance that the characteristics they displayed must exist, though only in potential germ, in every student who has rightly discovered his vocation or calling in our sphere. And those higher qualities which win our unstinted admiration, constitute the Ideal form towards which, though we may only dare to cast a wistful regard, we yet sedulously and hopefully direct our way. And despair is banished by the thought that progress in all Sciences presents a corresponding aspect, enlarging in vivid contrast with each advancing stage, and that not simply in Ethics, but also in Intellectual and Practical work, every movement of memorable achievement in History and Life has become the more purified, stimulated, and

enriched as the Ideal End assumed a minuter distinctness and glowed with a steadier and serener light. In the History of our Profession we may, I think, distinguish three definite stages of development of character and equipment: the Mathematical; the Administrative; and the Financial.

(i) In the primitive period, the Actuary was chiefly a Mathematician,* from the necessity of the case and in the absence of that practical exhibition of Actuarial principles which the system of Life Assurance embodies. His pursuit was mainly that of calculation, based upon the application of established mathematical processes to the novel materials of mortality statistics which had then assumed a collected form.

(ii) More practical considerations gradually emerged, as I have shown in the solutions of De Moivre and Dodson, in 1752 and 1753, which introduced within our sphere commercial elements, like market values, and,—with the elaboration of the system of Assurance, in which his principles discovered their fullest sway, involving subtle Actuarial problems of rational premiums,† Valuations, and Distribution of Profits,—the Actuary, while retaining his special character, gradually added the attribute of an Administrator, and thus united to the mathematical aptitude, a practical knowledge of large business affairs and contact with financial and economic problems.

(iii) At the present day, we perceive a further and very definite transformation where general skill in Finance and practical mastery of financial questions have assumed an imperative supremacy. But an acquaintance with problems of finance will be absolutely impotent if confined to theoretic study. The student of our Profession must now, more sedulously than ever, mix with men in business enterprise, and learn from mastered experience in the world itself, the modes of dealing with financial subjects and the methods and meaning of financial work. *Seclusion* in the study implies *exclusion* from the bracing rivalries and the highest usefulness of Professional life; a strict devotion to the region of pure mathematics paralyzes the tact and practical skill which are essential to the estimation of

* As the Science of Numbers is the basis of every mode of Actuarial work, early speculations and applications constituted a rich armoury of weapons with which, as observations became more complete and specialised, succeeding enquirers could successfully attack the increasing complexity of questions which the progress of our science produced. The investigations of Daniel Bernouilli in 1760, of D'Alembert in 1761, and of Laplace in 1812, into the mortality due to smallpox, are instructive examples of mathematical skill applied to limited data.

† Gompertz: Memoir of 1820.

probable evidence: and in obedience to the great Law of Distribution of Force, the specialised cultivation of the Mind, without the concurrent development of Feeling and of Interest in Humanity, produces but a fragmentary and unfinished man.

I counsel the young members, with sincere and earnest hopefulness, to acquire this direct and practical financial training, and I am confident that the resources of the Institute, which should be ever vigilant of the appropriate education of its youthful Alumni, will speedily devise a plan of wisely guiding them in this specialised direction, where their prosperous and honourable usefulness distinctly and permanently lies.

The essential aim of our general and professional life is the quest of a genuine system of Education: the appropriate and laborious educing of the faculties we possess; the provision of an organised method of training enquirers to collect and elaborate knowledge for themselves; so that we may become, to adopt a figurative expression, not mere mental sponges capable of absorbing a vast amount of knowledge, but unhappily simply competent of discharging it minutely as it was acquired, somewhat the muddier perchance from the process: but rather that our minds should assume the functions of a Chemical Laboratory where supplies of information constitute merely the crude materials which the subtle Alchemy of the Intellect and the Practical Power shall convert into fresher and more finished forms.

We possess admirable Text-Books of our Science, worthy of the honoured position which the Institute fulfils, but the mere mental absorption of a Text-Book, however exhaustive be its treatment and extensive its scope, will leave the student a commonplace storehouse of garnered knowledge which remains in the precise condition in which it was imbibed, void of stimulating energy, impotent as an active instrument of mind. Search your Text-Books for hints and directions, but sedulously pursue the real and sovereign mode of education in studying the original works of the great Masters themselves; in following the researches which enrich our *Journal*; and seeking to employ all these acquisitions both as stable elements of learning but essentially as means of sharpening, invigorating, and guiding the activity and vivid perceptiveness of your individual powers. A minute fragment of original thoughtful effort outweighs, in the true scale of educative purpose, the vastest accumulation of unassimilated

and mechanical facts. An Echo, though precisely articulate, is yet but a repeated Voice.

There is, unhappily, no example so common as that of men who fancy that, by introducing into their speech the terms of Philosophy and Science, they are conversing in a philosophic and scientific mode, and the sole corrective of this mischief is the study of original and fruitful works with alert and teachable intelligence. Even the moral qualities of patience and humility which such a study involves constitute an ennobling education of permanent value and hope. We thus learn, again, the impotency of that glib assumption, fatal both in Theology and Life, and too prevalent in this thoughtless age, that the deepest and widest problems are capable of expression within the narrow boundaries of a summary dogmatic statement. We gain the helpful and steadying conviction that social, professional, and scientific questions are usually of too profound an import to become adequately compressible into a portable formula, however specious and ingenious; and I can never hear, in the discussion of comprehensive and complex problems, whether of life or professional work, the frequent assertion, to employ the current phrase, that the solution lies within the compass of a "nutshell", without humbly thinking that the nutshell measures, not the dimensions of the question but, rather the character and capacity of the speaker's head.

I finally urge the importance of the student attending our debates, and modestly taking a part in our discussions. He will never fail of a kindly and cordial reception; even a single question, derived from the speaker's own freshness of enquiry and reflection, is often of material service to us all,—the "*prudens interrogatio*" which is not infrequently the "*dimidium scientiæ*";* and, moreover, an early participation in debate, conducted solely in the pursuit of Truth, affords steadiness of conception, readiness of resource, flexibility of utterance, and that modest self-reliance, which so largely determine the complete command of all our powers.

XIII.—CONCLUSION.

In thus concluding the task which I assigned to myself in this Address, the reflection naturally arises, as I survey the course I have traversed, that, with fragmentary knowledge and imperfect

* Bacon: *De Augmentis Scientiarum*: Lib. v, cap. 3.

enquiry, I have attempted too vast and intricate an enterprise,—a local pilot steering his vessel amid perilous and unnavigated seas,—but a generous interpretation of a feeble effort will, I am confident, ensue in the remembrance that motives of surpassing interest have impelled me in this quest: a sincere affection for our great Institute, in whose honour, devotion and labour can recognise no bounds; a deepening pride in our traditions of memorable public usefulness, and of strenuous and unrelenting toil in fashioning the structure of Science which we now inherit as an honourable Trust; a profound personal concern in the fortunes of the younger Members of our Body, destined to receive, and worthily uplift, the torch of knowledge from our passing hands; and an ambition to kindle within their minds a vivid sympathy with the high vocation of an Institute whose signal services in the past are authentically prophetic of an even ampler history in the future, and a yet loftier station in the hierarchy of Institutions devoted to the Common Weal.

(I). *An Investigation of some of the Methods for deducing the Rates of Mortality, and of Withdrawal, in Years of Duration; with (II) the Application of such Methods to the computation of the Rates experienced, and the Special Benefits granted, by Clerks' Associations.* By THOMAS G. ACKLAND, Fellow of the Institute of Actuaries.

(I). INVESTIGATION OF METHODS FOR DEDUCING THE RATES OF MORTALITY AND OF WITHDRAWAL IN YEARS OF DURATION.

(A). PERIOD OF OBSERVATION LIMITED BY CALENDAR YEARS.*

(Continued from page 96.)

SPECIAL CHARACTERISTICS OF ILLUSTRATIVE EXPERIENCE, AS DISTINGUISHED FROM THOSE OF ASSURED LIVES GENERALLY.

This subject may be conveniently considered under two headings; (1) as to the actual or assumed age at entry (2) as to the distribution of entrants and emergents over the year of duration current at entry and exit.

* From some remarks made in the discussion which followed the reading of this essay, it seems necessary to point out that the limitation of the *period of observation* within complete calendar years is not to be confounded with the *tabulation and scheduling of the facts* according to the "calendar year" method. In both sections (A) and (B) of this paper the *facts* are tabulated according to "policy-years," or years of duration; but in section (A) the *observations* included are comprised within an integral number of calendar years, the period actually selected for illustration being from 1 January 1888, to 31 December 1892.

(1) AS TO THE AGE AT ENTRY.

It will have been seen that the experience here tabulated is set out throughout according to office ages at entry and years of duration. This is a very convenient, and seems to be the most appropriate, course, in a case such as that here dealt with, where the experience of an Association is investigated, and the results applied to the valuation of its own risks; for the rates, as deduced, can be applied to the computation of valuation factors tabulated in the same form; and the cases for valuation being similarly scheduled (according to office age at entry and years of duration as at date of valuation) these factors can be directly applied in valuing the risks, as is more fully shewn in Part II of the present paper. No assumptions are then involved as to the age at entry, nor does any question arise as to the ages at date of valuation. The only assumption made seems to be that the actual entry ages of the members in force at the date of valuation, bear the same relations, on the whole (whether in excess or defect) to their office ages, which obtained in the general body of lives investigated over the past quinquennium; and it does not appear to be material whether the office ages were, in point of fact, taken as at next, or last, or nearest birthdays, so long as the same general relations hold good throughout.

When, however, the case of lives in an assurance office is dealt with, and especially in the case where the aggregate experience of several distinct companies is under investigation, these considerations do not hold good; and the office age at entry appears to be no longer a safe or trustworthy basis. This would appear to be especially the case where the experience is deduced and tabulated in years of duration. Several suggestions have been made as to the most appropriate method of determining the ages at entry in such a case; and a brief consideration of some of the methods suggested may, perhaps, be useful.

There seems to be a general agreement as to the desirability, in the words of Mr. Whittall (*J.I.A.*, xxxi, 166) of adopting "that method of determining the ages at entry which most nearly agrees with the actual ages of the assured at entry." Among the methods that have been suggested are that of (1) "Nearest Age at Entry," called by Dr. Sprague, "commencing age," and adopted by him in an illustrative experience (given in *J.I.A.*, xxxi, 208); that of (2) "Mean Age at Entry," stated by Messrs. G. F. Hardy and Rothery (*J.I.A.*, xxvii, 165); and

that of (3) "Nearest Age at nearest 31 December," given by Mr. G. King (*J.I.A.*, xxvii, 218). These methods may be concisely stated, and their relative advantages briefly considered.

(1) The "*Nearest Entry Age*" is simply arrived at by taking the age as at the birthday nearest to the actual date at entry. The amount of divergence from the true age at entry cannot exceed six months, the greatest difference, of course, arising when the birthday precedes or follows the date of entry by an interval of half a year. This method gives results which compare very favourably with those deduced by other assumptions. It would, however, appear that where a large proportion of the lives have a tendency to enter near their next birthdays, the true ages at entry will on the whole be somewhat overstated by the Nearest Age Method, as all such cases will, by that method, be taken as at entry age next birthday. Judging, however, from the table given by Mr. Whittall (*J.I.A.*, xxxi, 187) of the experience of the Clerical, Medical, and General Office in this respect, as regards entrants in the two years 1853-4 and 1886-7, it may be considered that this tendency of the method in question to overstate the age at entry is not in practice sufficiently marked to affect the accuracy of the results.

(2) The "*Mean Age*" is arrived at by deducting the calendar year of birth from the calendar year of entry. Here the divergence from the true age at entry may be almost a year; the greatest difference arising, in excess, where the date of entry is at the beginning, and the date of birth at the end, of the calendar year (a very unusual case); and, in defect, where the birthday is at the beginning, and the date of entry at the end, of the calendar year (a not unusual case). Some objections to this method have been stated by Mr. Whittall and the late Mr. Sunderland (*J.I.A.*, xxvii, 191, 193); but judging from the table (already referred to) published by Mr. Whittall, it gives in practice very good results. It is, however, clearly inferior in point of accuracy to the method of nearest ages.

(3) The modification of the Nearest Age Method, as suggested by Mr. King, proceeds upon the plan of taking the "Age at Entry" as the age which falls *on the birthday nearest to the nearest 31 December at entry*. This takes practical effect by fixing the age at entry, in the case of entrants in the first half of a calendar year, at the birthday which falls in the year made up of the current and the preceding half-years; and, in the case of entrants in the second half of a calendar year, at the birthday

which falls in the year made up of the current and the following half-years.* The results may differ by almost a year from the true entry ages, the greatest difference arising, in excess, where the date of entry, in second half-year, immediately follows the birthday in the first half-year (a very unusual case); and, in defect, where the date of entry in first half-year immediately precedes the birthday in the second half year (a by no means unusual case). This method has, as I understand, been proposed particularly for the investigation of the experience of an office in an inter-valuation period, and not as specially suited for the aggregate experience of assured lives. It will probably, however, give on the whole good results: but is clearly inferior to the method of Nearest Ages.

In order to show graphically, and in a tabular form, the effect of adopting these three methods, I append a comparative statement of the results arrived at, on different assumptions as to the date of entry and the birthdays of the entrants. In column (1) I have graphically delineated the calendar year with quarterly or half-yearly divisions; and the letters (*e*) and (*b*) represent respectively the dates of entry and of the birthday, the incidence of each being supposed to occupy a mean position in the divisions in which they are respectively located. In column (2) I have set out the office age at entry next birthday, which I have taken as (*x*) throughout; and in column (3) is given the true average age at entry, as deduced from the mean positions of the date of entry and of the birthday in column (1). In columns (4), (6) and (8) I have given the "Age at Entry" as deduced by the "Mean Age" method, Mr. King's method, and the "Nearest Age" method; and in columns (5), (7) and (9) I have stated the divergence between the ages so estimated and the true ages as given in column (3). For convenience, the cases are separately tabulated and summarized where the date of entry respectively precedes and follows the birthday in the same calendar year.

* I cannot entirely agree with the conclusion of Mr. Whittall (*J.I.A.*, xxxi, 184), that this method may be classed as one of "mean ages"; and Mr. King has himself expressed his dissent from this view (*J.I.A.*, xxxi, 201). The method, as stated by Mr. King, gives, not true, but *modified* "mean ages", which coincide with those arrived at by deducting the year of birth from the year of entry, *in the case only where both years are reckoned from 1 July to 30 June*. If, however, the method be so applied as to deduce the ages which fall on the birthdays nearest to the nearest 30 June to the dates of entry (a modification contemplated by Mr. King, *J.I.A.*, xxvii, 218), the results will in that case precisely coincide with *true* "mean ages," computed by deducting the *calendar* years of birth from the *calendar* years of entry. I am also unable to agree with Mr. King that the error of age can never exceed six months. This seems to me to be demonstrably inaccurate, at least, as regards the age at entry, with which alone I am now concerned; and I can only suppose that Mr. King is referring to the average error, and not to individual cases of deviation.

Comparative Statement showing results of different methods of estimating Ages at Entry, according to the distribution of the Date of Entry, and the birthday, in the calendar year of entry.

Calendar Year: Showing assumed location of Dates of Entry and Birthdays in Quarters and Half-Years.	Office Age next Birthday	True Average Entry Age	"MEAN AGE" METHOD		MR. KING'S METHOD		"NEAREST AGE" METHOD	
(1)	(2)	(3)	Entry Age	Average Error	Entry Age	Average Error	Entry Age	Average Error
(1) Date of Entry preceding Birthday in Calendar Year.								
(1)	x	$x - \frac{1}{2}$	x	$+\frac{1}{2}$	x	$+\frac{1}{2}$	x	$+\frac{1}{2}$
(2)	x	$x - \frac{1}{2}$	x	$+\frac{1}{2}$	x	$+\frac{1}{2}$	x	$+\frac{1}{2}$
(3)	x	$x - \frac{1}{2}$	x	$+\frac{1}{2}$	$x-1$	$-\frac{1}{2}$	$\left\{ \begin{matrix} x \\ x-1 \end{matrix} \right\}$	0
(4)	x	$x - \frac{3}{4}$	x	$+\frac{3}{4}$	$x-1$	$-\frac{1}{4}$	$x-1$	$-\frac{1}{4}$
(5)	x	$x - \frac{1}{4}$	x	$+\frac{1}{4}$	$x-1$	$-\frac{3}{4}$	x	$+\frac{1}{4}$
(6)	x	$x - \frac{1}{2}$	x	$+\frac{1}{2}$	$x-1$	$-\frac{1}{2}$	$\left\{ \begin{matrix} x \\ x-1 \end{matrix} \right\}$	0
Total	$= 6x$	$6x - 2\frac{1}{2}$	$6x$	$+ 2\frac{1}{2}$	$6x - 4$	$- 1\frac{1}{2}$	$6x - 2$	$+ \frac{1}{2}$
(2) Date of Entry following Birthday in Calendar Year.								
(7)	x	$x - \frac{1}{2}$	$x-1$	$-\frac{1}{2}$	$x-1$	$-\frac{1}{2}$	$x-1$	$-\frac{1}{2}$
(8)	x	$x - \frac{1}{2}$	$x-1$	$-\frac{1}{2}$	$x-1$	$-\frac{1}{2}$	$x-1$	$-\frac{1}{2}$
(9)	x	$x - \frac{1}{2}$	$x-1$	$-\frac{1}{2}$		$+\frac{1}{2}$	$\left\{ \begin{matrix} x-1 \\ x \end{matrix} \right\}$	0
(10)	x	$x - \frac{1}{4}$	$x-1$	$-\frac{3}{4}$	x	$+\frac{1}{4}$	x	$+\frac{1}{4}$
(11)	x	$x - \frac{3}{4}$	$x-1$	$-\frac{1}{4}$	x	$+\frac{3}{4}$	$x-1$	$-\frac{1}{4}$
(12)	x	$x - \frac{1}{2}$	$x-1$	$-\frac{1}{2}$	x	$+\frac{1}{2}$	$\left\{ \begin{matrix} x-1 \\ x \end{matrix} \right\}$	0
Total	$= 6x$	$6x - 3\frac{3}{4}$	$6x - 6$	$- 2\frac{1}{2}$	$6x - 2$	$+ 1\frac{1}{2}$	$6x - 4$	$- \frac{1}{2}$
Grand Total	$= 12x$	$12x - 6$	$12x - 6$	0	$12x - 6$	0	$12x - 6$	0

The aggregate results over the 12 cases cited, are identical by the three methods, giving an average entry age of $(x - \frac{1}{2})$, which agrees also with the true average age at entry of the 12 cases. But, looking at the cases in detail, and bearing in mind that the 12 typical examples given will not be equally numerous in actual experience, the great superiority of the "Nearest Age" method will be readily seen by comparison of the column (9), with columns (5) and (7). It may, however, be added that the cases which

will be likely to be relatively most numerous (at least, in the ordinary case where premiums are quoted for integral ages only) will be those numbered (1), (2), (5), and (10); and it will be noted that in these four typical cases, the "Nearest Age" method always overstates the age at entry (by taking it at next birthday); while both the Mean Method and that of Mr. King overstate the age in three of the typical examples, and understate it in the fourth case.

I cannot here consider this question further; but these brief remarks and the appended statement may, perhaps, be of some service in elucidating the points involved.

(2) DISTRIBUTION OF ENTRANTS AND EMERGENTS OVER YEARS OF DURATION CURRENT AT ENTRY AND AT EXIT.

Reverting now to the comparative statement of aggregate numbers exposed to risk, as deduced by the three methods exemplified in Schedules (B), (C), and (D) or (E), it is to be remarked that, in the particular experience here investigated, the members' contributions are payable at short periodical intervals; and there is no apparent reason why cases should not enter upon observation, emerge during the period, or survive to its close, so as to be equally distributed over the years of duration. In point of fact, it will be seen, upon reference to Schedule (A) (page 75), that the fractional exposures, whether in individual years of duration, or in the aggregate, do not materially differ from one-half of the corresponding number of cases. The ratios of the aggregate fractional exposures to the aggregate number of cases are as follow:—

"Survivors" (at entry)	= 566
"Existing" (at close of period)	= 570
Withdrawals	= 529
Deaths	= 506

If it be borne in mind that all the cases enter, and all (excepting the deaths) emerge, on the first day of a calendar month, and that (assuming an equal distribution of entrants, and of emergents, throughout the year) the average exposure during the year would thus be equal to

$$\frac{12 + 11 + 10 + 9 + 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1}{12}$$

= 6.5 months = .5417 of a year; it will be seen that the cases, whether of entry or of emergence, are practically distributed equally over the year of duration.

The particular experience here investigated, differs, in this

respect, materially from that usually obtaining among assured lives, and it may be of interest to give some consideration to this point. In the case of a period of observation limited by calendar years, the survivors at the commencement will (among assured lives generally) tend to come under observation, and the cases existing will tend to terminate their experience, in the first half of their then current years of duration; while the withdrawals will tend to congregate towards the end of the year of duration current at exit.

The average fractional duration of the withdrawals will depend to some extent upon the proportions of cases effected at yearly, half-yearly, and quarterly premiums. If, however, the reasonable assumption be made that cases effected at half-yearly premiums are equally likely to withdraw (by lapse) at the date of the first or second half-yearly payment, and that cases effected at quarterly premiums are equally likely to withdraw (by lapse) at the date of the first, second, third or fourth quarterly payment, the average fractional duration of the withdrawals will be, for half-yearly cases

$$\frac{1}{2}(\frac{1}{2} + 1) = \frac{3}{4}$$

and for quarterly cases,

$$\frac{1}{4}(\frac{1}{4} + \frac{1}{2} + \frac{3}{4} + 1) = \frac{5}{8}$$

so that there will be a marked tendency (upon the above assumptions) apart from the actual proportions of yearly, half-yearly and quarterly cases, for the withdrawals by lapse to fall, on the average, in the latter half of the year of duration.

Mr. Chatham, in his recent Messenger Prize Essay, assumes (*J.I.A.*, xxxii, 412) that the proportions of the yearly, half-yearly, and quarterly cases will be respectively 75 per-cent, 20 per-cent, and 5 per-cent. After some consideration and enquiry, I have preferred, for present purposes, to assume that the proportions are as $62\frac{1}{2}$ per-cent, $32\frac{1}{2}$ per-cent, and 5 per-cent; that is, that of every 400 cases, 250 are effected at yearly payments, 130 at half-yearly payments, and 20 at quarterly payments, of premium. Assuming that 400 withdrawals take place, in a given year of duration, in the above proportions, and that the withdrawals of half-yearly and quarterly cases are equally likely to fall at either of the two half-yearly, or at any of the four quarterly, epochs for payment, the aggregate exposure of the 400 cases in their year of withdrawal would be equal to

$$250 + (.75 \times 130) + (.625 \times 20) = 360,$$

thus giving an average exposure of $\frac{9}{10}$ ths of a year, for each

case. If the cases were in the proportions assumed by Mr. Chatham, the aggregate exposure of the 400 withdrawals would be, upon the same assumptions, 372.5 years, with an average exposure of .93125 of a year for each case.

In order to ascertain the effect of employing the methods of Mean Duration, and of Nearest Duration, in the case of an experience where the average exposure of the entrants and the emergents in the years of duration current at entry or at exit, materially varied from half a year, I have applied the Nearest Duration Method to the same illustrative experience which has been already here employed; but upon the assumption that the surviving entrants, at the commencement of the period of observation, and the cases existing at its close, had, on the whole, completed *one-fourth* only of the year of duration current at the commencement, and at the end, of the period of observation, respectively; also that the withdrawals during the period had, on the whole, an average duration of *nine-tenths* of a year in the year of duration current at exit.

These proportions are not intended to represent the actual, or most probable, experience of any company or group of companies; but are adopted merely for purposes of illustration. These assumptions do not, of course, in any case affect the total number of cases surviving, existing or withdrawing, as observed in successive years of duration, but solely affect their distribution over the years of duration current at entry and at exit.

The results are set out in tabular form in Schedule (F); and it will be seen that the above assumptions are given effect to, upon the Nearest Duration Method, by so dividing the total cases in any year of duration that $(as) = 3(bs)$; $(ae) = 3(be)$; and $(bw) = 9(aw)$; these values being computed to the nearest integer. The death cases were tabulated throughout in half-years, as actually experienced.

To illustrate more clearly the methods followed, and the effects of the assumptions made as to the distribution of the cases, I have tabulated the cases surviving, existing, withdrawing and dying, in half-years of duration throughout. It will be understood, however, that as regards the "surviving" and "existing," in columns (2) and (3), it is not necessary to sort or tabulate the cases in half-years, for the purpose of computing the numbers exposed to risk, or the rates of mortality and withdrawal.

The number exposed to risk in each year of duration ($\bar{E}_{[x]+t}$) is computed by formula (11), and the values of $E_{[x]+t}$ and of

(wE)_{[x]+t}, and of q _{[x]+t} and (wq)_{[x]+t}, are deduced as in Schedule (E), pp. 92, 93.

If now the numbers exposed to risk of death and of withdrawals, as arrived at in Schedule (F), are compared with those deduced in Schedule (E) upon the basis of a (practically) uniform distribution of the entrants and emergents throughout the year, it will be seen that the aggregate results are as follows:—

NEAREST DURATION METHOD.

	$\Sigma(E)$	$\Sigma(wE)$
Uniform Distribution (Schedule E)	6,140	6,667
Assumed Distribution (Schedule F)	6,664	6,683
Difference	+224	+16

Thus the numbers exposed to the risk of death are, in this particular experience, considerably increased when the special assumptions as to distribution are given effect to; while the numbers exposed to the risk of withdrawal, are (in this particular case) not substantially varied.

It will be readily seen that the difference between the numbers exposed to the risk of death, as deduced (by the Nearest Duration Method) from the assumed distribution of entrants and withdrawals, and as deduced (by the same method) from their uniform distribution, may be expressed as equal to

$$\left\{ \frac{1}{4}(e-s) - [(be) - (bs)] \right\} + \left\{ \frac{9w}{10} - (bw) \right\}$$

which expression holds good in any given year of duration, or in the aggregate. Taking aggregate values, we have,

$$\begin{aligned} & \left\{ \frac{1}{4}(1225 - 1333) - (668 - 711)* \right\} + \left\{ \frac{9}{10} \times 481 - 225 \right\} \\ & = (16 + 207.9) = 223.9, \end{aligned}$$

which agrees closely with the actual difference of the aggregate numbers.

Similarly, the difference in the numbers exposed to the risk of withdrawal is equal to

$$\begin{aligned} & \frac{1}{4}(e-s) - [(be) - (bs)] \\ & = \frac{1}{4}(1225 - 1333) - (668 - 711)* = 16. \end{aligned}$$

Here the distribution of the death-cases, in the year of death, does not affect the numbers exposed to the risk of death; and similarly the distribution of the withdrawals, in the year of

* The values of (be) and (bs) are not separately stated in Schedule (E); but I have taken them from working sheets, which include the values throughout in half-years of duration.

SCHEDULE (F).—OBSERVATION EXTENDING OVER
ASSUMED DISTRIBUTION OF

*Table showing methods of deducing the Numbers Exposed to Risk, and the
Fractional Exposures being taken to the*

Dura- tion	Survivors	Existing	Withdrawals	Deaths	Total Decrement	Net Movement	$n_{[x]} + \Sigma d_{[x]}^{(g)}$
t	$\left. \begin{array}{l} 25s_{[x]+t-1} \\ 75s_{[x]+t} \end{array} \right\} = s_{[x]+t}$	$\left. \begin{array}{l} 25e_{[x]+t-1} \\ 75e_{[x]+t} \end{array} \right\} = e_{[x]+t}$	$\left. \begin{array}{l} 9w_{[x]+t-1} \\ 1w_{[x]+t} \end{array} \right\} = w_{[x]+t}$	$\left. \begin{array}{l} (bd)_{[x]+t-1} \\ (ad)_{[x]+t} \end{array} \right\} = d_{[x]+t}$	$\left. \begin{array}{l} (e \\ + w + d) \\ = f_{[x]+t} \end{array} \right\}$	$\left. \begin{array}{l} (s-f) \\ = g_{[x]+t} \end{array} \right\}$	$= E_{[x]+t}$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
0	103	61	5	1	67	+ 36	$n_{[x]} = 421$
1	34	21	46	0	135	- 25	457
2	76	62	5	1	79	+ 45	432
3	25	21	42	4	102	- 21	477
4	99	5	5	2	123	+ 4	498
5	33	2	39	1	126	- 18	502
6	90	54	5	1	107	- 24	484
7	30	18	40	0	117	- 38	460
8	97	56	6	3	84	- 61	442
9	32	19	49	0	81	- 63	419
10	76	52	4	2	117	- 38	381
11	25	17	40	1	91	- 24	357
12	58	46	3	..	78	- 26	331
13	19	16	30	..	74	- 55	102
14	67	55	2	1	62	- 50	74
15	22	19	21	3	21	- 7	67
16	65	61	3	3	16	- 5	62
17	22	21	24	2	18	- 12	50
18	57	67	2	1	10	- 2	48
19	19	22	14	0	14	- 4	44
20	48	52	2	1	12	+ 1	45
21	16	17	17	2	9	+ 7	52
22	36	36	2	4	6	+ 6	58
23	12	12	18	2	8	- 1	57
24	11	49	1	2	13	- 11	46
25	4	16	10	0	12	- 12	34
26	14	52	1	2	15	- 15	19
27	5	17	12	0	12	- 12	7
28	14	44	1	0	6	- 6	1
29	5	15	10	1	1	- 1	..
30	7	35	0	1
31	3	11	2	1
32	10	26	1
33	8	9	4
34	11	8	0
35	4	3	3
36	7	10	0
37	3	3	3
38	3	12	0
39	1	4	1
40	7	5	0
41	2	2	3
42	8	8	0	1
43	2	3	1	0
44	11	8
45	4	2
46	12	7	0
47	4	2	1
48	8	3
49	2	1
50	5	7	0	0
51	2	2	1	1
52	..	8	0	1
53	..	2	1	0
54	..	9	..	0
55	..	3	..	1
56	..	11	0
57	..	3	1
58	..	6	..	2
59	..	2	..	0
60	..	4
61	..	1
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FIVE CALENDAR YEARS.—NEAREST DURATION METHOD.—SCHEDULE (F).
ENTRANTS AND WITHDRAWALS.*Rates, of Mortality and of Withdrawal, in true years of duration; the nearest integer.—Central Age at Entry (20).*

MORTALITY			WITHDRAWAL			Duration
Exposed	Deaths	Rate	Exposed	Withdrawals	Rate	
$\frac{E_{[x]+t}}{= \bar{E} + (ad)}$	$d_{[x]+t}$	$\frac{q_{[x]+t}}{d} = \frac{q}{\bar{E}}$	$\frac{(wE)_{[x]+t}}{= \bar{E} + (aw)}$	$w_{[x]+t}$	$\frac{(wq)_{[x]+t}}{w} = \frac{(wq)}{(wE)}$	t
(9)	(10)	(11)	(12)	(13)	(14)	(15)
458	1	·00218	462	51	·1104	0
433	5	·01155	437	47	·1075	1
479	3	·00626	482	44	·0913	2
499	1	·00200	503	45	·0895	3
505	3	·00594	508	55	·1083	4
486	3	·00617	488	44	·0902	5
460	463	33	·0713	6
443	4	·00903	444	23	·0518	7
422	5	·01185	422	27	·0640	8
382	1	·00262	383	16	·0418	9
358	3	·00837	359	19	·0529	10
335	6	·01719	333	20	·0600	11
272	2	·00735	271	11	·0406	12
209	2	·00957	208	13	·0625	13
152	1	·00658	153	11	·0719	14
103	2	·01942	102	2	·0196	15
74	75	5	·0667	16
67	67	3	·0448	17
62	62	3	·0484	18
50	50	1	·0200	19
48	48	3	·0625	20
45	1	·02222	44	1	·0227	21
45	45	22
52	52	1	·0192	23
58	58	24
57	1	·01754	57	1	·0175	25
47	1	·02128	46	1	·0217	26
34	1	·02941	34	27
19	19	1	·0526	28
9	2	·22222	7	29
1	1	30
..	31
6,664	48	..	6,683	481	..	

withdrawal, does not affect the numbers exposed to the risk of withdrawal. This would of course be anticipated, and evidently arises from the fact that the deaths and withdrawals respectively are given a full year's exposure in their year of exit.

It will also be remarked that the difference in the aggregate numbers exposed to the risk of death and of withdrawal, is considerably affected by the fact that the numbers existing (e) and surviving (s) are approximately equal, and nearly balance one another. If the experience were such that the value of ($s - e$) was relatively large (as, for example, where the cases are under observation from original entry up to a fixed date, and the "survivors" wholly disappear), the results above shown would be materially varied.

A comparison of the values of q and of (wq), in columns (10) and (12) of Schedule (F), with those given in columns (11) and (14) of Schedule (E) will show the effect, upon the rates of mortality and withdrawal, of the assumed variation in the distributions of the entrants and emergents. It will be seen that the rate of mortality is, under the assumed distribution, lower in the first 12 years of duration, and throughout higher, from the 13th year of duration to the end of the table; while the rates of withdrawal, although somewhat lower in the first six years of duration, do not, upon the whole, show any very marked deviations.

Let us now consider the effect of the distribution of entrants and emergents, in the case where the Mean Duration Method is applied. It is evident that this method, being based upon the assumption of an equal distribution of the entrants and emergents, will give precisely the same results, whatever the actual or assumed distribution may be; and that those results will, in the case of this particular experience, be those shown in Schedule (C), pp. 82, 83. Comparing, then, the numbers exposed to risk, as given in Schedules (C) and (F), we find that the aggregate values are as follow:—

ASSUMED DISTRIBUTION OF ENTRANTS AND EMERGENTS.

	$\Sigma(E)$	$\Sigma(wE)$
Mean Duration Method (Schedule C) .	6,444.5	6,661.0
Nearest Duration Method (Schedule F)	6,664.	6,683.
Difference	<u>+ 219.5</u>	<u>+ 22.0</u>

Here, the difference in the numbers exposed to the risk of death is evidently equal to

$$\begin{aligned} & \frac{1}{4}(s-e) + \left(\frac{9w}{10} - \frac{w}{2} \right) \\ & = 27 + 192.4 = 219.4 \end{aligned}$$

and the difference in the numbers exposed to the risk of withdrawal is equal to

$$\begin{aligned} & \frac{1}{4}(s-e) + \left[bd - \frac{d}{2} \right] \\ & = 27 + [19 - 24] = 22 \end{aligned}$$

The remarks made above as to the counter-balancing effects upon these results of the nearly equal values of (*e*) and (*s*) will also apply here.

It is thus evident that the Mean Duration Method gives, as might have been anticipated, erroneous results as regards the numbers exposed to risk, and the resulting rates of mortality and of withdrawal, in the case supposed where the average fractional exposures of the entrants and emergents materially differs from half a year.

SPECIAL SUITABILITY OF NEAREST DURATION METHOD FOR THE EXPERIENCE OF ASSURED LIVES GENERALLY.

The Nearest Duration Method, as here applied (in Schedule F) to the case of an assumed distribution of entrants and emergents, cannot be compared with the Exact Duration Method, as in this illustrative case the exact durations have not been ascertained, or rather have been assumed to be faithfully reproduced by the Nearest Duration Method. There can, however, be no doubt of the special suitability of this method to the experience of assured lives; for this method gives effect, to a very great extent, to the actual average durations of the assurances in their several years of duration, and that by a sort of automatic reference to the beginning or end of those years. As compared with the Mean Duration Method, which gives an average of half-a-year's exposure to all entrants and emergents (and thus deduces a result which can hardly fail to be erroneous in the case of assured lives) the Nearest Duration Method has manifest advantages.

This method also seems to me to be greatly superior to the method [suggested and illustrated by Mr. Ryan (*J.I.A.*, xxvi, 259-264)], of taking a *constant ratio* of the entrants and emergents, to represent their assumed fractional exposure in all years of duration. The Nearest Duration Method, by giving approximate

effect to the true fractional exposures in each year of duration, will allow for any marked deviation from the general average ratio in individual years; and thus possesses an elasticity which compares very favourably with the rigidity of the "Constant Ratio" method.

Let us now consider briefly the probable deviations of the Nearest Duration Method from the true exposures, as regards the several cases of withdrawals (1) by lapse, (2) by surrender, (3) by miscellaneous causes; also of cases "surviving" and "existing"; and cases of death.

Taking first the cases of withdrawal (1) by lapse, it will be borne in mind that cases of non-renewal of premium, occurring at the end of the year of duration (whether in respect of yearly, half-yearly, or quarterly premiums), will, in all cases, be given their true integral exposures; also, that cases of non-renewal occurring in the middle of the year of duration (whether in respect of half-yearly or quarterly premiums) will, by the system of alternate reference to the beginning and end of the year of exit, be also given their true exposures, taken one with another. There only remain the quarterly cases of non-renewal at the first and third quarters of the year of duration; and those occurring at the first quarter (and referred by the Nearest Duration Method to the beginning of the year) may fairly be considered to be balanced by those occurring in the third quarter (and referred to the end of the year).

As regards (2) surrenders, there will be a certain error in the estimated exposures in respect of the period intervening between the date of surrender and the next following renewal date. This period is, in practice, usually found to be very short; and any error will be much reduced by the fact that the intervening period is understated, if the surrender take place in the first half of the year of exit, and overstated if it occur in the second half. If there be (*a*) yearly cases, (*b*) half-yearly cases, and (*c*) quarterly cases, surrendered in any year of duration; if $\left(\frac{1}{m}\right)$ represent the period intervening between surrender and the next renewal date; and if it be assumed that a surrender is equally likely to take place before any of the four quarterly renewal dates, or before either of the half-yearly renewal dates; it can readily be shown that the aggregate exposures of the yearly cases surrendering are overstated by $\frac{a}{m}$ years; and that the aggregate exposures of the

half-yearly and quarterly cases are respectively understated by $b\left(\frac{1}{4} - \frac{1}{m}\right)$ years and by $c\left(\frac{1}{8} - \frac{1}{m}\right)$ years. The aggregate error would thus amount to

$$\frac{a}{m} - b\left(\frac{1}{4} - \frac{1}{m}\right) - c\left(\frac{1}{8} - \frac{1}{m}\right)$$

and the average error would be found by dividing this expression by $(a + b + c)$. If we assume (as before) that the cases are in the proportions of 250 yearly, 130 half-yearly, and 20 quarterly, and that the average interval between surrender and the next renewal date is one month, the above expression becomes

$$\frac{250}{12} - \frac{130}{6} - \frac{20}{24} = -\frac{10}{6}$$

showing an *aggregate* error (for 400 cases) of $1\frac{2}{3}$ years' exposures; and an *average* error of $\frac{1}{240}$ th of a year, or say $1\frac{1}{2}$ days, in each case. The remarkable power of the Nearest Duration Method, in adapting itself to the conditions of actual practice, could, perhaps, hardly be better exemplified.

The miscellaneous cases of withdrawal (3) include maturity of endowment assurances; expiration of term assurances; cancellation by forfeiture, &c. These cases would not be relatively numerous; and while the majority would usually fall at the end of the year current at exit, the remainder may fairly be considered as likely to be equally distributed over that year.

As regards cases of surviving entrants, they will, as already stated, tend to enter upon observation in the first half of the year of duration current at entry; at least, in the usual case, where the commencement of the period of observation is also the commencement of a financial year of the office. These cases will, by the Nearest Duration Method, be given an exposure for the whole of the year of duration current at entry, which will be in excess of the true exposures.

Upon the other hand, the cases existing at the close of the period of observation will, on the whole, tend to pass out of observation in the first half of the year of duration then current; and such cases will, by the Nearest Duration Method, be given no exposure in that year of duration, thus involving an understatement of the true exposures. There is thus a compensating influence at work, in reduction of the balance of error; and where the number of cases of surviving entrants does not greatly differ from that of the cases "existing", the ultimate error may be

expected to be relatively small. In the case, however, of an experience where there are no "survivors", there would be no such compensating influence; and the cases "existing" would, as a whole, have their exposures understated by the Nearest Duration Method.

Finally, as regards the cases of death. Upon the usual assumption of an equal distribution of the deaths over the year of duration current at death, the average exposure would be half a year; and this would be correctly tabulated, taking one case with another, both by the Mean Duration Method, and the Nearest Duration Method. (See, however, Appendix D).

(B) PERIOD OF OBSERVATION LIMITED BY YEARS OF DURATION.
ILLUSTRATIVE EXPERIENCE.

We have hitherto investigated the case where the period of observation extends over an integral number of calendar years, so that survivors enter at fractional durations at the commencement, and cases exist, also at fractional durations, at the close. If now the experience is so investigated that the period of observation runs concurrently with years of duration, that is, so that the "survivors" come under observation from the commencement, and the cases "existing" are traced up to the close, of a year of duration, the formulæ and the tabular operations are throughout very much simplified. As these are precisely the conditions under which the New Institute Experience is being investigated, the cases being traced from their policy anniversary in the calendar year 1863 (or from subsequent entry) to their policy anniversary in 1893 (or previous exit), I have deemed it useful to investigate this condition of things somewhat in detail.

For this purpose I eliminated, from the illustrative experience previously employed (1) all cases emerging (by death or withdrawal) prior to their policy anniversaries in 1888; (2) all cases entering in the year 1892; while cases emerging (by death or withdrawal) during the year 1892, and subsequently to their policy anniversaries in that year, were treated as "existing." There were thus eliminated 65 "survivors" and 89 original entrants, leaving 1,268 and 332 respectively; of which 1,184 existed at their policy anniversaries in 1892, 385 withdrew during the period of observation, and 31 died.

The period of observation thus extended over four years of duration, from the policy anniversary in 1888 (in the case of "survivors") to the policy anniversary in 1892 (in the case of

“existing”); and included all cases of entry, withdrawal, and death falling between those anniversaries.

I have, in Appendix (B), investigated the modifications introduced, in the formulæ already given, to meet the particular case of an experience limited by years of duration, in the several cases where the Exact Duration Method, the Mean Duration Method, and the Nearest Duration Method, are adopted.

EXACT DURATION METHOD.

I have computed, in the appended Schedule (G), the numbers exposed to risk, and deduced the rates of mortality and withdrawal, by the formulæ

$$E_{[x]+t} = \sum_0^t (G) + (w' + d)_{[x]+t} \quad . \quad . \quad . \quad (18)$$

$$\text{and} \quad (wE)_{[x]+t} = \sum_0^t (G) + (w + d')_{[x]+t} \quad . \quad . \quad . \quad (21)$$

$$\text{where} \quad G_{[x]+t} = [s^1 - (e^1 + w + d)]_{[x]+t}$$

and s^1 and e^1 indicate the numbers respectively surviving and existing at the integral duration of t years; the withdrawals (w) and the deaths (d) being tabulated as before, according to their curtate durations at exit.

Upon comparing Schedule (G) with Schedule (B)* it will be seen that the tabular operations are much simplified, in the case here investigated, where the period of observation is limited by years of duration.—In Schedule (G), fractional expressions enter only into column (4) of withdrawals, and column (5) of deaths; and by the formulæ given above, the values of E and of (wE) are deduced by a direct process, and one not involving the calculation of the function \bar{E} . It will also be noted that the “new entrants” (332 in number) during the period of observation here conveniently figure as “surviving entrants” at precise age (x); and that the table, as a whole, is thus rendered more symmetrical and convenient.

This Schedule (G) appears to me to present, in a very simple form, the whole of the tabular work involved in deducing the numbers exposed to risk of death and of withdrawal, and the resulting rates, in true years of duration, and with exact fractional exposures. It need hardly be added that the values of q and of (wq) deduced in columns (11) and (14) cannot properly be compared with those set out in columns (15) and (17) of Schedule (B), as the period of observation, and the data involved, are not identical in the two cases.

* See pp. 78, 79.

SCHEDULE (G).—OBSERVATION EXTENDING OVER
 Table showing methods of deducing the Numbers Exposed to Risk, and the
 with exact Fractional Exposures.

Curtate Duration	Surviving	Existing	Withdrawals	Deaths	Total Decrement	Net Movement	$\Sigma_0^t(G)$
t	$s^1_{[x]+t}$	$e^1_{[x]+t}$	$\frac{w_{[x]+t}}{w'_{[x]+t}}$	$\frac{d_{[x]+t}}{d'_{[x]+t}}$	$(e^1 + w + d)$ $= F_{[x]+t}$	$(s^1 - F)$ $= G_{[x]+t}$	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
0	332	..	35 16'6	..	35	+ 297	297
1	128	85	40 21'9	3 2'5	128	0	297
2	93	7	36 19'1	2 0'4	45	+ 48	345
3	124	75	40 18'5	1 0'5	116	+ 8	353
4	116	80	42 18'2	2 0'8	124	- 8	345
5	121	77	32 18'5	2 0'5	111	+ 10	355
6	96	62	29 15'9	..	91	+ 5	360
7	74	78	17 10'3	2 0'8	97	- 23	337
8	84	84	24 14'5	2 1'3	110	- 26	311
9	82	91	11 5'5	1 0'3	103	- 21	290
10	74	74	15 7'9	2 1'1	91	- 17	273
11	63	50	17 8'9	6 3'3	73	- 10	263
12	48	65	10 5'6	1 0'5	76	- 28	235
13	13	70	12 5'6	1 0'1	83	- 70	165
14	18	61	9 5'1	1 0'6	71	- 53	112
15	19	47	1 0'3	1 0'1	49	- 30	82
16	9	36	4 2'6	..	40	- 31	51
17	13	11	3 2'3	..	14	- 1	50
18	14	14	14	0	50
19	9	16	1 0'8	..	17	- 8	42
20	4	7	8 1'8	..	10	- 6	36
21	9	12	1 0'6	..	13	- 4	32
22	10	10	10	0	32
23	15	9	9	+ 6	38
24	15	4	4	+ 11	49
25	10	9	1 0'1	1 0'6	11	- 1	48
26	7	10	1 0'8	1 0'5	12	- 5	43
27	..	12	..	1 0'9	13	- 13	30
28	..	14	1 0'9	..	15	- 16	15
29	..	9	..	1 0'2	10	- 10	5
30	..	5	5	- 5	..
	1,600	1,184	385 202'3	31 15'0	1,600	0	4,941

FOUR YEARS OF DURATION.—EXACT DURATION METHOD.—SCHEDULE (G).
Rates, of Mortality and of Withdrawal, in true years of duration; and
—Central Age at Entry (20).

$(w'_{[x]-t} + d'_{[x]-t})$	MORTALITY		$(w_{[x]-t} + d'_{[x]-t})$	WITHDRAWAL		Curtate Duration t
	Exposed	Rate		Exposed	Rate	
	$E_{[x]-t}$ $= \sum_j t(G)$ $+ (w' + d)_{[x]-t}$	$q_{[x]-t}$ $= \frac{d}{E}$		$(wE)_{[x]-t}$ $= \sum_j t(G)$ $+ (w - d')_{[x]-t}$	$wq_{[x]-t}$ $= \frac{w}{(wE)}$	
(9)	(10)	(11)	(12)	(13)	(14)	(15)
16.6	313.6	..	35.0	332.0	.1054	0
24.9	321.9	.00082	42.5	339.5	.1175	1
21.1	366.1	.00546	36.4	351.4	.0944	2
19.5	372.5	.00269	40.5	363.5	.1017	3
20.2	365.2	.00543	42.8	357.8	.1088	4
20.5	375.5	.00583	32.5	357.5	.0826	5
15.9	375.9	..	29.0	359.0	.0746	6
12.3	349.3	.00573	17.8	354.8	.0489	7
16.5	327.5	.00611	25.8	336.8	.0714	8
6.5	296.5	.00887	11.3	301.3	.0365	9
9.9	282.9	.00707	16.1	289.1	.0519	10
14.9	277.9	.02159	20.3	283.3	.0609	11
6.6	241.6	.00414	10.5	245.5	.0407	12
6.6	171.6	.00583	12.1	177.1	.0377	13
6.1	115.1	.00847	9.6	121.6	.0740	14
1.3	88.3	.01209	1.1	88.1	.0120	15
2.6	53.6	..	4.0	55.0	.0727	16
2.3	52.3	..	3.0	53.0	.0566	17
..	50.0	50.0	..	18
0.8	42.8	..	1.0	43.0	.0233	19
1.5	37.5	..	3.0	39.0	.0769	20
0.6	32.6	..	1.0	33.0	.0303	21
..	32.0	32.0	..	22
..	35.0	35.0	..	23
..	49.0	49.0	..	24
1.1	49.1	.02087	1.6	49.6	.0262	25
1.8	44.8	.02232	1.5	44.5	.0225	26
1.0	31.0	.03226	0.9	30.9	..	27
0.9	15.9	..	1.0	16.0	.0625	28
1.0	6.0	.16667	0.2	5.2	..	29
..	30
233.3	5,174.3	..	400.0	5341.0	..	

MEAN DURATION METHOD.

In this case, the formulæ for the numbers exposed to risk become respectively

$$E_{[x]+t} = \sum_0^t (G) + \left(\frac{w}{2} + d \right)_{[x]+t} \cdot \cdot \cdot \quad (24)$$

and

$$(wE)_{[x]+t} = \sum_0^t (G) + \left(w + \frac{d}{2} \right)_{[x]+t} \cdot \cdot \cdot \quad (27)$$

and the tabular results are set out in Schedule (H).

SCHEDULE (H).—OBSERVATION EXTENDING OVER

Table showing methods of deducing the Numbers Exposed to Risk, and the mean or average Fractional Exposures.

Curta-te Dura- tion	Surviving	Existing	With- drawals	Deaths	Total Decrement	Net Movement	$\sum_0^t (G)$
t	$s^1_{[x]+t}$	$e^1_{[x]+t}$	$w_{[x]+t}$	$d_{[x]+t}$	$(e^1 + w + d)$ $= F_{[x]+t}$	$(s^1 - F)$ $= G_{[x]+t}$	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
0	332	...	35	...	35	+ 297	297
1	128	85	40	3	128	0	297
2	93	7	36	2	45	+ 48	345
3	124	75	40	1	116	+ 8	353
4	116	80	42	2	124	- 8	345
5	121	77	32	2	111	+ 10	355
6	96	62	29	...	91	+ 5	360
7	74	78	17	2	97	- 23	337
8	84	84	24	2	110	- 26	311
9	82	91	11	1	103	- 21	290
10	74	74	15	2	91	- 17	273
11	63	50	17	6	73	- 10	263
12	48	65	10	1	76	- 28	235
13	13	70	12	1	83	- 70	165
14	18	61	9	1	71	- 53	112
15	19	47	1	1	49	- 30	82
16	9	36	4	...	40	- 31	51
17	13	11	3	...	14	- 1	50
18	14	14	14	0	50
19	9	16	1	...	17	- 8	42
20	4	7	3	...	10	- 6	36
21	9	12	1	...	13	- 4	32
22	10	10	10	0	32
23	15	9	9	+ 6	38
24	15	4	4	+ 11	49
25	10	9	1	1	11	- 1	48
26	7	10	1	1	12	- 5	43
27	...	12	...	1	13	- 13	30
28	...	14	1	...	15	- 15	15
29	...	9	...	1	10	- 10	5
30	...	5	5	- 5	...
	1,600	1,184	385	31	1,600	0	4,941

Here again the operations are simplified as compared with those shown in Schedule (C),* and the values of E and of (wE) are deduced by direct processes. This method is especially rapid in its operations, and is very suitable for the investigation of the experience of a body of lives, where the average fractional exposure at exit does not greatly differ from half a year.

* See pp. 82, 83.

FOUR YEARS OF DURATION.—MEAN DURATION METHOD.—SCHEDULE (H).

Rates of Mortality and of Withdrawal, in true years of duration; and with—Central Age at Entry (20).

$\left(\frac{w[x]-t}{2} + d_{[x]-t}\right)$	MORTALITY		$\left(\frac{w[x]-t}{2} - \frac{d_{[x]-t}}{2}\right)$	WITHDRAWAL		Currate ¹ Duration t
	Exposed	Rate		Exposed	Rate	
	$E_{[x]-t}$ $= \sum_i (G)$ $+ \left(\frac{w}{2} + d\right)$	$q_{[x]-t}$ $= \frac{d}{E}$		$(wE)_{[x]-t}$ $= \sum_i (G)$ $+ \left(w + \frac{d}{2}\right)$	$(wq)_{[x]-t}$ $= \frac{w}{(wE)}$	
(9)	(10)	(11)	(12)	(13)	(14)	(15)
17.5	314.5	...	35.0	332.0	.1054	0
23.0	320.0	.00938	41.5	338.5	.1182	1
20.0	365.0	.00548	37.0	382.0	.0942	2
21.0	374.0	.00267	40.5	393.5	.1017	3
23.0	368.0	.00544	43.0	388.0	.1082	4
18.0	373.0	.00536	33.0	388.0	.0825	5
14.5	374.5	...	29.0	389.0	.0746	6
10.5	347.5	.00576	18.0	355.0	.0479	7
14.0	325.0	.00615	25.0	336.0	.0714	8
6.5	296.5	.00337	11.5	301.5	.0365	9
9.5	282.5	.00708	16.0	289.0	.0519	10
14.5	277.5	.02163	20.0	283.0	.0601	11
6.0	241.0	.00415	10.5	245.5	.0407	12
7.0	172.0	.00551	12.5	177.5	.0676	13
5.5	117.5	.00851	9.5	121.5	.0741	14
1.5	83.5	.01198	1.5	83.5	.0120	15
2.0	53.0	...	4.0	55.0	.0727	16
1.5	51.5	...	3.0	53.0	.0566	17
...	50.0	50.0	...	18
0.5	42.5	...	1.0	43.0	.0233	19
1.5	37.5	...	3.0	39.0	.0769	20
0.5	32.5	...	1.0	33.0	.0303	21
...	32.0	32.0	...	22
...	38.0	38.0	...	23
...	49.0	49.0	...	24
1.5	49.5	.02020	1.5	49.5	.0202	25
1.5	44.5	.02247	1.5	44.5	.0225	26
1.0	31.0	.03226	0.5	30.5	...	27
0.5	15.5	...	1.0	16.0	.0625	28
1.0	6.0	.16667	0.5	5.5	...	29
...	30
223.5	5,164.5	...	400.5	5,341.5	...	

NEAREST DURATION METHOD.

In this case the formulæ become respectively

$$E_{[x]+t} = \Sigma_0^t (G) + (ad)_{[x]+t} \quad . \quad . \quad . \quad (29)$$

and

$$(wE)_{[x]+t} = \Sigma_0^t (G) + (aw)_{[x]+t} \quad . \quad . \quad . \quad (30)$$

where

$$G_{[x]+t} = [s^1 - (e^1 + w + d)]_{[x]+t}$$

w and d representing, as before, the numbers of withdrawals and deaths as tabulated by the Nearest Duration Method.

The values are tabulated in Schedule (J).

The cases "surviving" and "existing" are respectively set out, in columns (2) and (3), at their true integral durations, the new entrants $n_{[x]}$ figuring as $s_{[x]}$, and in columns (4) and (5) the withdrawals and deaths are respectively tabulated, according to their half-years of duration at exit. The numbers existing, withdrawing, and dying, as set out in columns (3), (4) and (5), are then summed (as indicated by the brackets) in column (6); their sum being deducted from the numbers "surviving" in column (2), and the difference entered in column (7). The values thus arrived at, continuously summed in column (8), give the values of $\Sigma_0^t (G) = \bar{E}_{[x]+t}$ the numbers exposed to risk up to the dates of death or withdrawal; from which the values of E and of (wE) are deduced, by the above formulæ, in columns (9) and (12) respectively. In columns (10) and (13) the total number of deaths and of withdrawals are tabulated for convenience, these numbers being simply the sums of those set out, in adjacent half-years, in columns (5) and (4) respectively. Finally, in columns (11) and (14), the rates of mortality and of withdrawal are computed, for each year of duration.

I have explained these operations in detail, because this method appears to me to be, upon the whole, that best suited for practically dealing with a large body of assured lives, in such a way as to deduce, by a very simple and rapid series of operations, the rates of mortality and of withdrawal, in their true years of duration.

Comparing Schedule (J) with Schedule (E),* where the observation was limited by calendar years, it will be seen that the tabular operations are practically identical throughout, and as

* See pp. 92, 93.

SCHEDULE (J).—OBSERVATION EXTENDING OVER

Table showing methods of deducing the Numbers Exposed to Risk, and the Fractional Exposures being taken to the

Duration	Surviving	Existing	Withdrawals	Deaths	Total Decrement	Net Movement	$\bar{E}_{[x]+t}$ $= \sum_0^t (G)$
t	$s^1_{[x]+t}$	$e^1_{[x]+t}$	$\left. \begin{array}{l} (bw)_{[x]+t-1} \\ (aw)_{[x]+t} \end{array} \right\}$ $= w_{[x]+t}$	$\left. \begin{array}{l} (bd)_{[x]+t-1} \\ (ad)_{[x]+t} \end{array} \right\}$ $= d_{[x]+t}$	$(e^1 + w + d)$ $= F_{[x]+t}$	$(s^1 - F)$ $= G_{[x]+t}$	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
0	332	..	21 14	..	21	+311	311
1	128	85	23 17	0 3	122	+ 6	317
2	93	7	19 17	2 0	48	+ 45	362
3	124	75	24 16	1 0	117	+ 7	369
4	116	80	28 14	2 0	126	- 10	359
5	121	77	14 18	2 0	107	+ 14	373
6	96	62	16 13	..	96	0	373
7	74	78	5 12	1 1	97	- 23	350
8	84	84	11 13	1 1	109	- 25	325
9	82	91	7 4	1 0	113	- 31	294
10	74	74	8 7	1 1	87	- 13	281
11	63	50	11 6	4 2	73	- 10	271
12	48	65	6 4	1 0	80	- 32	239
13	13	70	7 5	1 0	82	- 69	170
14	18	61	5 4	0 1	71	- 53	117
15	19	47	1 0	1 0	54	- 35	82
16	9	36	1 3	..	37	- 28	54
17	13	11	0 3	..	14	- 1	53
18	14	14	17	- 3	50
19	9	16	0 1	..	16	- 7	43
20	4	7	1 2	..	9	- 5	38
21	9	12	0 1	..	14	- 5	33
22	10	10	11	- 1	32
23	15	9	9	+ 6	38
24	15	4	4	+ 11	49
25	10	9	1 0	0 1	10	0	49
26	7	10	0 1	1 0	12	- 5	44
27	..	12	..	0 1	13	- 13	31
28	..	14	0 1	..	15	- 15	16
29	..	9	..	1 0	11	- 11	5
30	..	5	5	- 5	..
	1,600	1,184	209 176	20 11	1,600	0	5,128

FOUR YEARS OF DURATION.—NEAREST DURATION METHOD.—SCHEDULE (J).

Rates of Mortality and of Withdrawal, in true years of duration; the nearest integer.—Central Age at Entry (20).

MORTALITY			WITHDRAWAL			Duration
Exposed	Deaths	Rate	Exposed	Withdrawals	Rate	
$E_{[x]+t}$ $= \sum_j^t (G) + (ad)$	$d_{[x]+t}$	$\frac{q_{[x]+t}}{d}$ $= \frac{E}{E}$	$(wE)_{[x]+t}$ $= \sum_j^t (G) + (aw)$	$w_{[x]+t}$	$\frac{(wq)_{[x]+t}}{w}$ $= \frac{w}{(wE)}$	t
(9)	(10)	(11)	(12)	(13)	(14)	(15)
311	332	35	1054	0
317	3	00046	340	49	1177	1
364	2	00549	381	36	0045	2
370	1	00270	393	40	1018	3
361	2	00554	387	42	1085	4
375	2	00533	387	32	0827	5
373	389	29	0746	6
351	2	00570	355	17	0479	7
326	2	00614	336	24	0714	8
295	1	00339	301	11	0366	9
282	2	00709	289	15	0519	10
275	6	02182	282	17	0603	11
240	1	00417	245	10	0408	12
171	1	00585	177	12	0678	13
117	1	00855	122	9	0738	14
83	1	01205	88	1	0120	15
54	55	4	0727	16
53	53	3	0566	17
50	50	18
43	43	1	0233	19
38	39	3	0779	20
33	33	1	0393	21
32	32	22
38	38	23
49	49	24
49	1	02041	50	1	0200	25
45	1	02222	44	1	0227	26
31	1	03226	31	27
16	16	1	0625	28
6	1	16667	5	29
..	30
5,148	31	..	5,337	385	..	
..	

SCHEDULE (K).—OBSERVATION EXTENDING OVER
ASSUMED DISTRIBUTION

*Table showing methods of deducing the Numbers Exposed to Risk, and the
Fractional Exposures being taken to the nearest*

Duration	Survivors	Existing	Withdrawals	Deaths	Total Decrement	Net Movement	$\bar{E}_{[x]+t}$ $= \Sigma ({}^uG)$
t	$s^1_{[x]+t}$	$e^1_{[x]+t}$	$\left. \begin{array}{l} {}^9w_{[x]+t-1} \\ {}^1w_{[x]+t} \end{array} \right\}$ $= w_{[x]+t}$	$\left. \begin{array}{l} (bd)_{[x]+t-1} \\ (ad)_{[x]+t} \end{array} \right\}$ $= d_{[x]+t}$	$e^1 + w + d$ $= F_{[x]+t}$	$(s^1 - F)$ $= G_{[x]+t}$	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
0	332	..	4 31	..	4	+ 328	328
1	128	85	4 36 3	0 3	120	+ 8	336
2	93	7	4 32	2 0	52	+ 41	377
3	124	75	4 36 1	1 0	112	+ 12	389
4	116	80	4 38 2	2 0	122	- 6	383
5	121	77	4 28 3	2 0	121	0	383
6	96	62	3 26	93	+ 3	386
7	74	78	2 15 1	1 1	107	- 33	353
8	84	84	2 22 1	1 1	103	- 19	334
9	82	91	1 10 1	1 0	116	- 34	300
10	74	74	1 14 1	1 1	86	- 12	288
11	63	50	2 15 1	4 2	71	- 8	280
12	48	65	1 9 0	1 0	84	- 36	244
13	13	70	1 11 0	1 0	81	- 68	176
14	18	61	1 8 1	0 1	73	- 55	121
15	19	47	0 1 0	1 0	57	- 38	83
16	9	36	0 4	37	- 28	55
17	13	11	0 3	15	- 2	53
18	14	14	17	- 3	50
19	9	16	0 1	16	- 7	43
20	4	7	0 3	8	- 4	39
21	9	12	0 1	15	- 6	33
22	10	10	11	- 1	32
23	15	9	9	+ 6	38
24	15	4	4	+ 11	49
25	10	9	0 1 ..	0 1	9	+ 1	50
26	7	10	0 1 0	1 0	13	- 6	44
27	..	12	0 1	13	- 13	31
28	..	14	0 1	15	- 15	16
29	..	9	1 0	11	- 11	5
30	..	5	5	- 5	..
	1,600	1,184	38 347	20 11	1,600	0	5,299

FOUR YEARS OF DURATION.—NEAREST DURATION METHOD.—SCHEDULE (K).
OF WITHDRAWALS.

Rates of Mortality and of Withdrawal, in true years of duration; the integer.—Central Age at Entry (20).

MORTALITY			WITHDRAWAL			Duration
Exposed	Deaths	Rate	Exposed	Withdrawals	Rate	
$E_{[x]+t}$ $= E + (ad)$	$d_{[x]+t}$	$\frac{q_{[x]+t}}{d}$ $= \frac{q}{E}$	$(wE)_{[x]+t}$ $= \bar{E} + (aw)$	$w_{[x]+t}$	$\frac{(wq)_{[x]+t}}{w}$ $= \frac{(wE)}{(wE)}$	t
(9)	(10)	(11)	(12)	(13)	(14)	(15)
328	332	35	·1054	0
336	3	·00893	340	40	·1177	1
379	2	·00528	381	36	·0945	2
390	1	·00256	393	40	·1018	3
385	2	·00519	387	42	·1085	4
385	2	·00519	387	32	·0827	5
386	389	29	·0746	6
354	2	·00565	355	17	·0479	7
335	2	·00597	336	24	·0714	8
301	1	·00332	301	11	·0366	9
289	2	·00692	289	15	·0519	10
284	6	·02113	282	17	·0603	11
245	1	·00408	245	10	·0408	12
177	1	·00565	177	12	·0678	13
121	1	·00826	122	9	·0738	14
84	1	·01190	83	1	·0120	15
55	55	4	·0727	16
53	53	3	·0566	17
50	50	18
43	43	1	·0233	19
39	39	3	·0770	20
33	33	1	·0303	21
32	32	22
38	38	23
49	49	24
50	1	·02000	50	1	·0200	25
45	1	·02222	44	1	·0227	26
31	1	·03226	31	27
16	16	1	·0625	28
6	1	·16667	5	29
..	30
5,319	31	..	5,337	335	..	

NEAREST DURATION METHOD.

	$\Sigma(E)$	$\Sigma(wE)$
Uniform Distribution (Schedule J)	5,148	5,337
Assumed Distribution (Schedule K)	5,319	5,337
Difference	<u>+171</u>	<u>0</u>

Here the difference in the numbers exposed to risk of death is evidently equal to

$$\frac{9w}{10} - (bw) = 346.5 - 176 = 170.5.$$

and it will be seen that the rate of mortality q as set out in column (11) of Schedule (K), is throughout lower than (or equal to) that tabulated in the corresponding column of Schedule (J).

As regards the rate of withdrawal, the numbers exposed to risk, and the resulting rates, columns (12) and (14), are throughout identical in Schedules (J) and (K), and are thus unaffected by the distribution of the withdrawals over the year of duration current at exit. This is, of course, self-evident, the withdrawals being credited with a full year's exposure in their year of exit; but it has, I think, been sometimes overlooked in discussions as to the (assumed) effect upon the rate of withdrawal of the distribution of the withdrawals over the year of duration current at exit.

Comparing now the results of Schedule (K) with those of Schedule (H), we shall see the results of adopting the Mean Duration Method, where the fractional exposure of the withdrawals in their year of exit differs materially from half a year. The following are the aggregate numbers exposed to risk.

ASSUMED DISTRIBUTION OF WITHDRAWALS.

	$\Sigma(E)$	$\Sigma(wE)$
Mean Duration Method (Schedule H)	5,164.5	5,341.5
Nearest Duration Method (Schedule K)	5,319	5,337
Difference	<u>+154.5</u>	<u>-4.5</u>

Here the difference in the numbers exposed to the risk of death is evidently equal to

$$\frac{9w}{10} - \frac{w}{2} = 346.5 - 192.5 = 154.0$$

and the rates of mortality by the Nearest Duration Method (Schedule K) are throughout lower than (or equal to) those deduced by the Mean Duration Method.

As regards the number exposed to the risk of withdrawal, the difference is limited to the estimated fractional exposures of the death cases by the two methods, and is equal to

$$(bd) - \frac{d}{2} = 11 - 15.5 = -4.5.$$

The numbers exposed to risk, and the rates, of withdrawal, are thus practically identical throughout.

It is thus again shown that the actual distribution of the withdrawals over their year of exit has a somewhat material effect upon the rates of mortality experienced, and in cases where the average fractional exposure of the withdrawals differs materially from half a year, the Nearest Duration Method may be expected to give a more faithful representation of the rates of death, than that which would be indicated by the Mean Duration Method.

COMPARATIVE SUMMARY OF OPERATIONS UNDER THE EXACT DURATION, THE MEAN DURATION, AND THE NEAREST DURATION, METHODS. SCHEME OF NOTATION.

In the appended Comparative Statement I have tried to set out a concise summary of the successive operations, under each of the three methods here discussed, for deducing the numbers exposed to risk, and the rates, of mortality and withdrawal; also the number of columns involved; the extent to which + and - signs, and fractions, enter into the computations; and the assumptions made as to the incidence of entrants and emergents in the years of duration current at entry and exit.

I also append a complete "Scheme of Notation", in which I have, for convenience of reference, brought together in a general view the several distinctive symbols used throughout Part I of this essay, according to the limitation of the period of observation and the different methods employed, as illustrated in the tabular Schedules (B) to (K).

In selecting the several symbols for the functions or quantities involved, I have tried (1) to depart as little as possible from those adopted and sanctioned by previous writers; (2) to employ symbols which should (as far as practicable) graphically suggest the functions or quantities which they represent; and (3) to distinguish by a variation of type those quantities, which, while functionally similar, differ in detail or in value when applied or computed according to a particular method.

COMPARATIVE

Of the successive Operations required for the computation of the Numbers of Duration:—by the application of the Exact Duration Method,

Under all three methods, the cards are first sorted according to original age at entry; (1) the separation into (a) new entrants, (b) surviving entrants; (2) the counting and operations in respect of each age (or group of ages) at entry:—

Method	"SURVIVING ENTRANTS" AND CASES "EXISTING"		WITHDRAWALS AND DEATHS	
	Duration Recorded on Cards	Cards Sorted and Tabulated	Duration Recorded on Cards	Cards Sorted and Tabulated
(1)	(2)	(3)	(4)	(5)
(A) PERIOD OF OBSERVATION LIMITED				
EXACT DURATION	Exact fractional duration at entry, or at exit	According to curtate* duration at entry, or at exit (1) Number of cases (2) Aggregate fractional exposures	Exact fractional duration at exit	According to curtate* duration at exit (1) Number of cases (2) Aggregate fractional exposures
MEAN DURATION	Curtate* duration at entry, or at exit	According to curtate* duration at entry, or at exit (1) Number of cases	Curtate* duration at entry, or at exit	According to curtate* duration at exit (1) Number of cases
NEAREST DURATION	Nearest integral duration, at entry or at exit†	According to nearest integral duration† (1) Number of cases	Half-year of duration at exit†	According to half-year of duration at exit† (1) Number of cases in each half-year
(B) PERIOD OF OBSERVATION LIMITED				
EXACT DURATION	Integral duration at entry, or at exit	According to integral duration recorded on cards (1) Number of cases	Exact fractional duration at exit	According to curtate* duration (1) Number of cases (2) Aggregate fractional exposures
MEAN DURATION	Integral duration at entry, or at exit	According to integral duration recorded on cards (1) Number of cases	Curtate* duration at exit	According to curtate* duration (1) Number of cases
NEAREST DURATION	Integral duration at entry, or at exit	According to integral duration recorded on cards (1) Number of cases	Half-year of duration at exit†	According to half-year of duration at exit† (1) Number of cases in each half-year

* Integral durations of $(t+1)$ years being treated

† Durations of $(t+\frac{1}{2})$ being treated as of durations

‡ Columns in which figures are duplicated being

STATEMENT

Exposed to Risk, and the Rates of Mortality and Withdrawal, in Years the Mean Duration Method, and the Nearest Duration Method.

and the preliminary operations at each age at entry (or group of entry ages), are recording of the total number of new entrants. The following are the successive

No. of Columns: involved in tabulation of No. Exposed to Risk		No. of Cols. involving attention to — and — signs		Fractions intro- duced		Assumptions made as to incidence of Entrants or Emergents in years of Duration current at Entrance or Exit		Type of Operation in Schedule		Method	
Of Death	Of Death and With- drawal	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)
BY CALENDAR YEARS.											
15	16	4	Yes		None.		(B)	EXACT DURATION			
10	11	4	.5 only		That the average fractional ex- posure of entrants and emer- gents = .5		(C)	MEAN DURATION			
10	11	2	None		(1) As regards cases entering and emerging exactly in middle or at end of year:—None (2) As regards cases entering and emerging at other points:— that the errors involved by reference to the nearest end of year counterbalance one another		(E), (F)	NEAREST DURATION			
BY YEARS OF DURATION.											
11	13	2	Yes		None.		(G)	EXACT DURATION			
9	11	2	.5 only		That the average exposure of the emergents at exit = .5		(H)	MEAN DURATION			
10	11	2	None		(1) As regards cases emerging exactly in middle or at end of year:—None (2) As regards cases emerging at other points:—that the errors involved by reference to the nearest end of year counter- balance one another.		(J), (K)	NEAREST DURATION			

as of "curtate" duration (t), and "fractional exposure", ($1\cdot0$).
(t), and ($t+1$), alternately.
each counted as two.

SCHEME OF NOTATION

CLASS OF OBSERVATION, AND METHODS EMPLOYED	Schedule	Survivors	Existing
		Cases as tabulated,	
(A) PERIOD OF OBSERVATION LIMITED BY CALENDAR YEARS.			
(1) EXACT DURATION METHOD—	(B)		
Duration (t) as tabulated	curtate*	curtate*
Cases as tabulated	$s[x]+t$	$e[x]+t$
Aggregate Fractional Exposures	$s'[x]+t$	$e'[x]+t$
(2) MEAN DURATION METHOD—	(C)		
Duration (t) as tabulated	curtate*	curtate*
Cases as tabulated	$s[x]+t$	$e[x]+t$
(3) NEAREST DURATION METHOD—	(D) (a)		
(a) For Mortality only—			
Duration (t) as tabulated	nearest integral	nearest integral
Cases as tabulated	$s[x]+t =$ $(bs)_{[x]+t-1} + (as)_{[x]+t}$	$e[x]+t =$ $(be)_{[x]+t-1} + (ae)_{[x]+t}$
(b) For Withdrawal only—	(D) (b)		
Duration (t) as tabulated	nearest integral	nearest integral
Cases as tabulated	$s[x]+t =$ $(bs)_{[x]+t-1} + (as)_{[x]+t}$	$e[x]+t =$ $(be)_{[x]+t-1} + (ae)_{[x]+t}$
(c) For Mortality and With- drawal—	(E), (F)		
Reference to beginning or end of Year of Duration	$(as)_{[x]+t}$ $(bs)_{[x]+t}$	$(ae)_{[x]+t}$ $(be)_{[x]+t}$
Duration (t) as tabulated	nearest integral	nearest integral
Cases as tabulated	$s[x]+t =$ $(bs)_{[x]+t-1} + (as)_{[x]+t}$	$e[x]+t =$ $(be)_{[x]+t-1} + (ae)_{[x]+t}$
(B) PERIOD OF OBSERVATION LIMITED BY YEARS OF DURATION.			
(1) EXACT DURATION METHOD—	(G)		
Duration (t) as tabulated	true integral	true integral
Cases as tabulated	$s^1[x]+t$	$e^1[x]+t$
Aggregate Fractional Exposures
(2) MEAN DURATION METHOD—	(H)		
Duration (t) as tabulated	true integral	true integral
Cases as tabulated	$s^1[x]+t$	$e^1[x]+t$
(3) NEAREST DURATION METHOD—	(J), (K)		
Reference to beginning or end of Year of Duration
Duration (t) as tabulated	true integral	true integral
Cases as tabulated	$s^1[x]+t$	$e^1[x]+t$

* Integral durations of $(t+1)$ years being considered as of

SYMBOLS common to

Numbers exposed to Risk—

Of Death	$E_{[x]+t}$
Of Withdrawal	$(wE)_{[x]+t}$
Of Death or Withdrawal	$\bar{E}_{[x]+t}$
New Entrants at age $[x]$	$n_{[x]}$

Withdrawals	Deaths	Total Decrement	Net Movement
-------------	--------	-----------------	--------------

at Age at Entry [x], and Duration (t).

curtate*	curtate*
$w_{[x]-t}$	$d_{[x]+t}$	$f_{[x]-t}$	$g'_{[x]-t}$
		$= (e + w + d)_{[x]-t}$	$= (s - f)_{[x]-t}$
$w'_{[x]-t}$	$d'_{[x]-t}$	$f'_{[x]-t}$	$g'_{[x]-t}$
		$= (e' + w' + d')_{[x]-t}$	$= (s' - f')_{[x]-t}$
curtate*	curtate*
$w_{[x]-t}$	$d_{[x]-t}$	$f_{[x]-t}$	$g'_{[x]-t}$
		$= (e + w + d)_{[x]-t}$	$= (s - f)_{[x]-t}$
nearest integral	current year
$w_{[x]+t} =$	$d_{[x]-t-1}$	$f_{[x]-t}$	$g'_{[x]-t}$
$(bw)_{[x]-t-1} + (aw)_{[x]-t}$		$= (e + w + d)$	$= (s - f)_{[x]-t}$
current year	nearest integral
$w_{[x]-t-1}$	$d_{[x]+t} =$	$f'_{[x]-t}$	$g'_{[x]-t}$
	$(bd)_{[x]+t-1} + (ad)_{[x]-t}$	$= (e + w + d)$	$= (s - f')_{[x]-t}$
$(aw)_{[x]-t}$	$(ad)_{[x]-t}$	$(af)_{[x]-t}$	$(ag)_{[x]-t}$
$(bw)_{[x]-t}$	$(bd)_{[x]-t}$	$(bf)_{[x]-t}$	$(bg)_{[x]-t}$
nearest integral	nearest integral
$w_{[x]-t} =$	$d_{[x]+t} =$	$f_{[x]-t}$	$g'_{[x]-t}$
$(bw)_{[x]-t-1} + (aw)_{[x]-t}$	$(bd)_{[x]-t-1} + (ad)_{[x]-t}$	$= (e + w + d)_{[x]-t}$	$= (s - f)_{[x]-t}$
curtate*	curtate*
$w_{[x]-t}$	$d_{[x]-t}$	$F_{[x]-t}$	$G_{[x]-t}$
		$= (e + w + d)_{[x]-t}$	$= (s + F)_{[x]-t}$
$w'_{[x]-t}$	$d'_{[x]-t}$	$F'_{[x]-t}$	$G'_{[x]-t}$
		$= (e' + w' + d')_{[x]-t}$	$= (s' + F')_{[x]-t}$
curtate*	curtate*
$w_{[x]-t}$	$d_{[x]-t}$	$F_{[x]-t}$	$G_{[x]-t}$
		$= (e + w + d)_{[x]-t}$	$= (s + F)_{[x]-t}$
$(aw)_{[x]-t}$	$(ad)_{[x]-t}$	$(af)_{[x]-t}$	$(ag)_{[x]-t}$
$(bw)_{[x]-t}$	$(bd)_{[x]-t}$	$(bf)_{[x]-t}$	$(bg)_{[x]-t}$
nearest integral	nearest integral
$w_{[x]-t} =$	$d_{[x]-t} =$	$F_{[x]-t}$	$G_{[x]-t}$
$(bw)_{[x]-t-1} + (aw)_{[x]-t}$	$(bd)_{[x]-t-1} + (ad)_{[x]-t}$	$= (e + w + d)_{[x]-t}$	$= (s + F)_{[x]-t}$

"curtate" duration (t), and "fractional exposure" (1.0).

all above Methods.

Rates—

Of Mortality	$Q_{[x]+t}$
Of Withdrawal	$(wQ)_{[x]-t}$
Central Death-rate	$m_{[x]-t}$
Central Withdrawal Rate	$(wm)_{[x]-t}$

(II). APPLICATION OF METHODS TO THE COMPUTATION OF THE
RATES EXPERIENCED, AND THE SPECIAL BENEFITS
GRANTED, BY CLERKS' ASSOCIATIONS.

GENERAL CHARACTERISTICS OF CLERKS' ASSOCIATIONS, AND
NATURE OF SPECIAL BENEFITS GRANTED.

It will be convenient, at the outset, to set out as concisely as possible the leading objects and general characteristics of these Clerks' Associations, which are established in several of the cities and larger towns of England and Scotland.

The membership is restricted to *bonâ fide* clerks employed within a definite local area: and the main object of the Associations is to render assistance to the members when out of employment, or when payment of their salaries is temporarily suspended by reason of sickness.

The following is a condensed summary of the main provisions of the rules of one of these Associations established in an English city. These particulars are given by way of illustration merely, and while the main principles will not vary, the actual provisions as to age, amount, incidence of allowances, &c., may materially differ in individual Associations.

Members are admitted between the ages of 18 and 45. The rates of monthly subscriptions vary from 2s. to 2s. 9d. per month, under the "Low" Scale: but members can, at their option, subscribe at an increase of 50 per-cent upon the above rates (on the "Middle" Scale), or at double the minimum rates (on the "High" Scale). The benefits secured under the three scales are strictly proportionate to the increased subscriptions.

The benefits granted under the Low Scale of Subscriptions are as follow:—

- (1) A weekly allowance during non-employment, or during sickness involving temporary suspension of pay, granted to members of six months' standing and upwards, of 24s. weekly during the first four weeks; 15s. weekly during the following nine weeks; and 8s. weekly during the remaining 13 weeks, at the end of which the allowance ceases, but is subject to renewal, for similar amounts and durations, after the member shall have held a permanent situation for at least six months. This restrictive term is reduced to three months in respect of members who have been three years upon the

books without receiving benefit: and is altogether abrogated in the case of members who have been five years upon the books without receiving benefit. There are also limitations as to the maximum aggregate amounts which may be received during the whole of membership by way of allowance, which need not be here specified in detail.

- (2) An increasing death-benefit on the following scale:—To members of between six months' and five years' standing, £7. 10s.; to members of between five and ten years' standing, £11. 5s.; to members of 10 years' standing and upwards, £15.
- (3) Allowances to members of not less than five years' standing, who shall be permanently unfitted for employment; and to members of not less than 15 years' standing, who shall, from old age or permanent ill-health, have no prospect of obtaining remunerative employment; also to necessitous widows and orphans of members. These allowances may be either by way of benevolent grants, or annuities, and they are at the entire discretion of the executive as to grant, amount, and duration, and are usually made from a "Special Annuity Fund" or "Reserve Fund", created and augmented by voluntary contributions, and appropriations of surplus moneys. They are not, therefore, a charge upon the members' monthly contributions.
- (4) Medical attendance and medicine during sickness, the charges under this head being provided by the allocation of a fixed portion of the members' subscriptions, averaging about 4s. per member per annum.
- (5) An "Employment Bureau", or classified register of candidates and situations, by which the members are assisted in obtaining situations when out of employment; this being equally a benefit to the individual member, and to the Association, which is thus liberated from a charge upon its funds in respect of the allowance during non-employment.
- (6) A restricted allowance of death-benefit upon the Minimum Scale, or of death-benefit and medical attendance only, to members who, by removal outside the local district, or by entry into business as employers (and thus ceasing to be "clerks") have forfeited the right to the allowance

during non-employment; reduced subscriptions of 15s. and of 21s. per annum respectively being paid by such members.

The management expenses of the Association are usually defrayed (1) by an allocation of a fixed proportion (say 35 per cent) of the members' subscriptions; (2) by incidental receipts from entry fees, donations, and miscellaneous sources of income.

FORM OF CARD ADOPTED, AND METHOD OF RECORDING THE EXPERIENCE.

The following is the form of card, which was, on the whole, found to be that most appropriate for recording the data:—

FRONT.

Number	4614	Scale	L
Age at Entry	24		
Subscription	£1 . 40		
Date of Entry	1 . 11 . 82		
Date of Exit	1 . 4 . 91		
Mode of Exit	L		
Duration (Entry)	5 . 2		
Duration (Exit)	8 . 4		
Remarks			

A few explanatory notes are needed to make clear the mode of filling up the several particulars upon the cards:—

- (1) *Number*.—This was filled in with the consecutive number of membership, corresponding to the policy number in an insurance office.
- (2) *Scale*.—This was entered as “H” (High), “M” (Middle), or “L” (Low).
- (3) *Age at Entry*.—This was, in all cases, the office age at entry, as charged, and stated in the office books. As, however, the scale of subscriptions was constant over groups of entry ages (18–30, 30–35, 35–40, and 40–45) it is probable that there was no great exactitude in stating the precise age at entry; nor does it appear

BACK.

[illegible]

- that any evidence of the true age was required or furnished. Upon the whole, and after enquiry, it seemed probable that the office age at entry was that at nearest birthday, which would be identical with Dr. Sprague's "commencing age." (*J.I.A.*, xxxi, 208.)
- (4) *Subscription*.—This was filled in from the office books, the amount entered on the card being the year's subscription. In practice all subscriptions were paid at intervals of calendar months, excepting the relatively few cases of restricted benefit, where the reduced subscription of 15s. or 21s. was paid annually.
- (5) *Date of Entry*.—This was entered up with the month and year in which the first subscription was paid. The subscriptions being payable in all cases on the first day of a calendar month, the date of entry always coincides with the commencement of the month recorded upon the cards.
- (6) *Date of Exit*.—This was recorded, in cases of withdrawal, as the month and year in which the subscription ceased; and in cases of death, as the month and year in which death actually took place.
- (7) *Mode of Exit*.—This was either by *withdrawal*, including (a) non-payment of subscription (b) removal outside the local district (c) entering upon business as an employer—all of which were marked "L"—or by *death*, which was marked "D."
- (8) *Duration (Entry)*.) These particulars were not entered
- (9) *Duration (Exit)*.) upon the cards from the office books, but were computed and recorded later on, according to the methods specified in the first part of this paper.* It may be added that, where it is desired to deduce and tabulate the experience according to ages attained, these headings would be replaced by "Age at Entry on Observation" and "Age at Exit" respectively. The additional lines afford opportunity for similar entries at subsequent investigations.
- (10) *Remarks*.—Here particulars were entered of any special incidents of the case, including changes of scale during membership; revival after lapse; transfer to restricted scale of benefit, by removal or entry upon business; transfers to Annuity or Benevolent Funds; &c.

* See pp. 71, 72.

- (11) *Allowances during Sickness or Non-Employment.*—The entries under this head being sometimes numerous, and increasing with the duration of membership, it was found preferable to preserve the back of the cards for these particulars. The first column “duration” (representing the duration of actual membership at the date when allowance commenced) was not supplied from the office books, but computed and recorded later. The “date” was that of the month and year when benefit commenced; the “weeks” the number of weeks (and days) during which the allowance was continuously paid; and the “amount” the aggregate sum received in respect of each such series of continuous payments.

METHODS ADOPTED IN DEDUCING RATES OF MORTALITY, WITHDRAWAL, AND ALLOWANCE DURING NON-EMPLOYMENT.

The method actually selected for investigating the particular experience here under review was that of Mean Durations, as this method offered great facilities in the actual computations, and undoubtedly gave very accurate results in this particular case.

The processes of deducing the numbers exposed to risk, and the rates of mortality and withdrawal, in central ages at entry and individual years of duration, have been fully explained in the first part of this paper, the tabulation being in the form set out in Schedule (C), on pages 82 and 83.

The *rate of mortality* was experimentally computed in this form; but it was seen from the outset that the aggregate number of deaths (128) was quite too few to give trustworthy, or indeed, workable, results; and it was ultimately decided to treat the rate of mortality as a function of the age only, irrespective of duration of membership. The age attained was assumed to be equal to the sum of the office age at entry (x) and the curtate duration (t); and all cases having an (assumed) age of ($x+t$) were brought together so that the numbers exposed to risk (E_{x+t}) and the deaths (d_{x+t}) could be compared, and the rate of mortality computed. The rate thus deduced was equal to the central death-rate (m_{x+t}), and the values of (q_{x+t}) were deduced by the formula of relation

$$q_{x+t} = \frac{2m_{x+t}}{2 + m_{x+t}}$$

It will be seen that, as the cases had been grouped in central ages at entry, and years of duration, the effect of this was that the values of m_{x+t} (and consequently those of q_{x+t}) were deduced from the numbers exposed to risk, and the deaths at five grouped ages $(x+t-2)$, $(x+t-1)$, $(x+t)$, $(x+t+1)$, and $(x+t+2)$. This was in effect a first graduation by summation in fives. The results thus deduced were combined with the experience of the previous seven years 1880-7, and a graduated rate was ultimately adopted, which represented approximately the experience of the 12 years 1880-1892.

The rate of mortality thus adopted, as a basis for money-values, is set out in Table I.* The actual experience did not extend beyond age 60, and at this age a junction was conveniently effected with a table of mortality representing the experience of males in city districts employed in similar occupations to those of the members whose experience was under investigation.

As regards the *rate of withdrawal*, the experience was much more extensive, there having been 1,222 cases of withdrawal during the period of observation. The actual rate of withdrawal was deduced for each central age at entry and each year of duration by the formula

$$(wm)_{[x]+t} = \frac{w_{[x]+t}}{E_{[x]+t}}$$

the values of $(wq)_{[x]+t}$ being then computed by the formula of relation

$$(wq)_{[x]+t} = \frac{2(wm)_{[x]+t}}{2 + (wm)_{[x]+t}}$$

The resulting values are set out in Table II for each central age at entry and each year of duration. It will be seen that the rate of withdrawal (at all entry ages) averages about 16 per-cent in the first year of duration, about 9 per-cent in the fifth year, 5 per-cent in the tenth year, 4 per-cent in the fifteenth year, and thence diminishes steadily to practical extinction at about the thirtieth year. It is also to be noted that the rates are somewhat materially lower at central age at entry 20 (especially in the earlier years of duration) than the rates at higher entry ages in corresponding years of duration. As there was a considerable preponderance of cases at the earlier entry ages and durations, this diminution in the rate of withdrawal was somewhat noteworthy, and could hardly be disregarded in the computation of money-values.

* For Tables I to VIII, see pages 180-6.

A rate of withdrawal was ultimately adopted, which, while having regard to the results set out in Table II, was well within that actually experienced, and thus left a fair margin for contingencies in this respect.

As regards the *rate of non-employment*, the method adopted was as follows: The dates of the several allowances, the number of weeks and days during which they were continuously paid, and the aggregate amounts paid in respect of each continuous allowance, having been recorded on the backs of the cards, as already explained, the duration of membership as at the date of each such allowance was computed to one decimal place, and entered in the column headed "Duration." The cards were then sorted (1) according to central ages at entry; (2) according to year of duration when entering on benefit; and the cases counted and tabulated, so as to show, for each central age at entry and each successive year of duration, the number of cases of benefit, the total number of weeks during which benefit was actually paid, and the aggregate amount paid in each case.

The allowance was, as has been said, a variable one, diminishing after 4 weeks', and again after 13 weeks', continuous pay, and ceasing after 26 weeks of total pay. In order to give effect to this, in a form convenient for computation, a further column was added, giving the number of weeks over which the allowance would have extended, if the amount of the weekly pay had been, throughout, the maximum amount which was in fact payable during the first four weeks only. This was readily arrived at, by dividing the aggregate amount received, by the maximum weekly allowance, which, of course, varied according to the scale of membership and subscription (High, Middle, or Low); and could be computed in each individual case, or in each group of cases where the central entry age, the year of duration, and the scale of membership, were all identical.

The number of weeks thus deduced, which I have termed "equivalent weeks", numbered 2,776 in the aggregate, as compared with 4,206 weeks during which the reducible allowance was actually paid; and the effect of this was to reduce the tabulated number of weeks in the average ratio of about 3 to 2.

By the adoption of this device, a good deal of labour was saved in the following stages. It proceeds, as will have been seen, upon the reasonable assumption that the value of an allowance extending, for example, over fifteen weeks, of £1. 4s. during the first four weeks, of 15s. during the following nine weeks,

and of 8s. for the remaining two weeks—or £12. 7s. in all—cannot differ materially from the value of an allowance at the constant rate of £1. 4s. weekly extending over $10\cdot3 \left(= \frac{12\cdot35}{1\cdot2} \right)$ weeks, and also amounting, in the aggregate to £12. 7s.

Upon the basis of the number of “equivalent weeks” thus deduced, the average number of weeks’ allowance per member was deduced (1) as a function of the year of duration only; (2) as a function of the age attained only; the formulæ being respectively

$$u_t = \frac{v_t}{\bar{E}_t}$$

and

$$u_{x+t} = \frac{v_{x+t}}{\bar{E}_{x+t}}$$

where v_t and v_{x+t} represent the aggregate number of “equivalent weeks” arising in the $(t+1)$ th year of duration, and in the year following age $(x+t)$, respectively; \bar{E}_t and \bar{E}_{x+t} represent the numbers exposed to risk in the $(t+1)$ th year of duration, and in the year following age $(x+t)$ respectively; and u_t and u_{x+t} represent the resulting rates of non-employment. In deducing these rates, as at ages attained, a special correction was made, in respect of the non-payment of benefit during the first six months of membership.

In Table III, I have set out the rate of non-employment computed upon these two bases. As regards the rate experienced in years of duration, it might be anticipated that there would be some indications of a selection against the Association in the early years of duration, arising from members entering who were in expectation of benefitting by the allowances in this respect at an early date. If the rates, as tabulated in years of duration, are closely examined, as set out in column (4) of Table III, regard being given to the fact that the rate during the first year should practically be doubled for purposes of comparison, it will be seen that there are indications, in the early years of duration, of the effects of some such causes: but these indications cannot be said to be strongly marked.

As regards the tabulation of the rates according to ages attained, it will be seen from column (2) that the rate tends to diminish from age 20 to about age 31, after which it tends on the whole to increase with the age, up to age 64, the highest age under observation.

It was, upon the whole, thought to be preferable to deduce the money values upon the basis of the rate of non-employment as computed at ages attained. As this function is one of some interest, I have appended in Table IV the rate of non-employment as experienced in the several periods 1862-79, column (2); 1880-87, column (3); and 1888-92, column (4), at ages attained; and also, in column (5), the rates adopted for the computation of money values, after consideration of the whole experience thus available.

The rates above age 64 not being obtainable, an assumed rate of allowance was tentatively adopted; and, having regard to the restrictions under the rules as to the duration of allowance, and the conditions under which the allowance could be resumed, as well as those limiting the aggregate amount receivable by a member, a rate increasing up to a maximum of two weeks' allowance per annum per member, was deemed to be fully sufficient to meet the case.

METHODS OF COMPUTING VALUES OF BENEFITS, AND VALUATION TABLES AND RESULTS.

Upon the bases specified above, as to the rates of mortality, of withdrawal, and of allowance during non-employment, there were computed, for each central age at entry, and for each successive year of duration, the value (1) of annuities payable throughout membership (2) of the varying assurance at death and (3) of the benefit payable during non-employment; interest being taken throughout at a rate of 3 per-cent per annum.

(1) The values of the annuity payable throughout membership, that is, until cessation by death or withdrawal, were computed upon the basis of the approximate formula (See Appendix E)

$$q''_{[x]+t} = q_{[x]+t} + (wq)_{[x]+t} - [q \times (wq)]_{[x]+t}$$

where q'' represents the probability of either death or withdrawal during the year of duration; whence we have

$$p''_{[x]+t} = 1 - q''_{[x]+t}$$

and

$$\log rp''_{[x]+t} = \Delta \log D''_{[x]+t}$$

whence the values of $N''_{[x]+t}$ and of $a''_{[x]+t}$ can be deduced by the ordinary processes.

(2) The values of the assurance, payable at death, with allowance for mortality and withdrawal during life, were based upon the approximate formula (See Appendix E)

$$q'_{[x]+t} = q_{[x]+t} - \frac{1}{2} [q \times (wq)]_{[x]+t}$$

where q' represents the probability of death, allowing for withdrawals: and we have

$$q'_{[x]+t} \times D''_{[x]+t} \times v = d'_{[x]+t} v^{x+t+1} = C'_{[x]+t}$$

or

$$\log q'_{[x]+t} + \log D''_{[x]+t} + \log v = \log C'_{[x]+t}$$

whence the values of $M'_{[x]+t}$, of $A'_{[x]+t}$, and of the varying assurance, can readily be deduced in the usual way.

(3) The values of the allowance during non-employment were obtained by the formula

$$U'_{[x]+t} = \frac{\sum (D''_{[x]+t+\frac{1}{2}} \cdot u_{[x]+t})}{D''_{[x]+t}}$$

where $U'_{[x]+t}$ represents the value of a constant benefit of one per week during non-employment, with allowance for mortality and withdrawals, and

$$\sum (D''_{[x]+t+\frac{1}{2}} \cdot u_{[x]+t}) = D''_{[x]+t+\frac{1}{2}} \cdot u_{[x]+t} + D''_{[x]+t+1\frac{1}{2}} \cdot u_{[x]+t+1} + \dots$$

The tabular values, computed as above, represent the values of the several benefits to members entering at office age $[x]$, who were in existence (at the date of valuation) at the precise durations 0, 1, 2, 3, . . . t years. By taking the means of these values throughout, factors were obtained appropriate for the valuation of the benefits or contributions of members whose curtate durations were, at the date of valuation 0, 1, 2, 3, . . . t years, and whose fractional exposures were, on the average, half-a-year.

The cards representing the cases "existing" at the date of the valuation having been already sorted according to central ages at entry and curtate durations, the mean valuation factors, arrived at as above, were then applied to the valuation of the cases in each year of duration, regard being also given to the scale under which the members had severally subscribed. By this means the liability in respect of the death-benefit, and of the allowance during non-employment, was ascertained; and the values of the members' annual subscriptions were similarly computed by the employment of mean annuity-values.

In computing the values of the subscriptions, allowance was made for the non-payment of the monthly subscriptions by

members whilst in receipt of benefit. This was arrived at by increasing the amount of the maximum weekly allowance during non-employment, by the amount of the average *weekly* subscription of the members (in each of the three scales) as a whole, increased in the proportion of actual to "equivalent" weeks of benefit.

The value of the estimated net premiums was arrived at by deducting from the value of the office subscriptions payable (1) the percentage available under the rules for management expenses, (2) the value of the constant deduction in respect of medical attendance, &c. Finally, by deducting the aggregate value of the estimated net premium from the aggregate values of the death benefit and the allowance during non-employment, and carefully eliminating negative values, the amount of the estimated liability was arrived at.

AS TO THE EFFECT OF AN ESTIMATED ALLOWANCE FOR WITH-
DRAWALS UPON THE NET PREMIUMS AND VALUATION
FACTORS COMPUTED, AND UPON THE RESERVES OR NET
LIABILITY.

The effect of making an allowance for secessions or withdrawals, in the valuation of Benefit Societies, is a subject that does not appear to have received much attention in our published transactions; although it must, as I imagine, come frequently under the attention of the actuary. It is quite impossible for me, within the limits of the present paper, to discuss this interesting question at any length, or at all adequately; but I have thought that it would be of interest to append some comparative results, as to the effect of making an allowance for withdrawals, in the case of the particular benefits here dealt with (1) on the amounts of the net premium and valuation factors, (2) on the amount of the reserves, or net liability.

I have assumed, for the sake of simplicity, that the benefit at death is a constant amount of £10: that the allowance during non-employment is at the constant rate of £1 per week, without reduction, but limited as to incidence and duration according to the conditions specified on pages 164 and 165; and I have adopted the rate of mortality as specified in Table I; a graduated rate of withdrawal based upon the rates shown in Table II, and ceasing at the expiration of 30 years' duration; and a rate of non-employment as shown in Table IV, column (5): with interest throughout at 3 per-cent.

I have preferred, in these illustrative examples, to assume a rate of withdrawal which, upon the whole, is fully equal to that actually experienced; it will, however, be understood that in the practical valuation of such an Association, the rate of withdrawal assumed as likely to operate in the future should be materially below that actually obtaining in the immediate past.

I have computed, and give in Table V, the valuation factors $\bar{A}'_{[x]+t}$, $U'_{[x]+t}$, and $\bar{a}''_{[x]+t}$, with allowance for withdrawals; also the factors $A_{[x]+t}$, $U_{[x]+t}$, and $a_{[x]+t}$, without allowance for withdrawals; taking values of $[x]=20, 30$, and 40 ; and of $t=0, 1, 3, 5, 10, 15, 20, 25$, and 30 . I have also computed the value of $\pi'_{[x]}$, the net premium, with allowance for withdrawals, required to provide £10 at death, and £1 weekly during non-employment; and of $\pi_{[x]}$ the net premium, without allowance for withdrawals, computed to provide the same benefits. These valuation factors and net premiums are, as might have been expected, materially reduced where the element of withdrawal is introduced, and especially in the early years of duration.

In Table VI are given what may be termed the true net premium reserve values at the ages at entry, and after the several durations, above mentioned. Here the reserve values in columns (2), (3), and (4) are computed on the basis of a net premium and valuation factors allowing for withdrawals; while in columns (5), (6), and (7) the reserves are upon the basis of a net premium and of valuation factors without allowance for withdrawals. These reserve values will be built up respectively by the accumulations of the net premiums as computed, assuming, of course, that the rates of mortality, of withdrawal, and of non-employment do not differ from those assumed.

It will be seen that in these examples the reserve values are, in the early years of duration, less when the element of withdrawal is introduced; but that after between 10 and 15 years' duration, the introduction of the withdrawals increases the reserve values throughout. It does not of course follow, that these relations would always hold good, and the comparative results would probably be modified, according to the rate of withdrawal assumed in successive years of duration, and the progression of that rate. It is, however, clear, where (as in the present examples) the rate of withdrawal is assumed to operate over a fixed term of years only, that the reserve values *after the expiration of that term* must throughout be greater where the element of withdrawal enters into the net premium: for the other valuation factors (the effect

of withdrawal having ceased to operate) are now identical, and the lower net premium will necessarily produce a greater reserve value.

In the above examples it is assumed that the Actuary has a free hand in the selection and computation of his valuation factors, and that the office scale of subscriptions will, after a reasonable allowance for expenses and other charges, be sufficient to provide the larger net premium required where no allowance is made for withdrawals. As, however, the expenses of management and other charges will usually absorb a stated proportion of the office subscriptions, the more usual case will be that where the *available net premium* applicable to benefits is represented by the fixed proportion of the subscriptions remaining.

Let us first take the case where this available benefit premium is found to be just sufficient to provide the risks without any allowance for withdrawals. I have computed in Table VII the comparative reserve values upon this basis, with and without allowance for withdrawals; but always upon the assumption that the net premium valued is that available as above, and which will provide for the benefits, assuming that there are no withdrawals.

Taking the columns (2), (3), and (4), and comparing the results with those set out in columns (5), (6), (7), it will be seen that the element of withdrawal introduces (upon this basis) negative values in the early years of duration, and that the reserve values are throughout diminished, where allowance is made for withdrawals. After 30 years' duration, however, when the effect of withdrawals ceases, the reserve values will be identical under both assumptions; as the net premium employed in this case is throughout the same in the two cases.

It may be added that the reserve values set out in columns (5), (6), and (7) will be throughout built up (without allowance for withdrawals) precisely by the net premiums assumed; and that the reserve values set out in columns (2), (3), and (4) will consequently be materially less than those that would have been built up (with allowance for withdrawals) by the assumed net premiums; the difference being represented by the anticipated value of the future profit arising from withdrawals.

Taking now the case where the available net premium, after providing for expenses and charges, is only just sufficient to provide the benefits granted, with full allowance for withdrawals, we have the results set out in Table VIII. Here the element

of withdrawal materially reduces the reserve values in columns (2), (3), and (4), which are throughout less than those in columns (5), (6), and (7), until the effects of withdrawals ceases, when they are identical with those deduced without allowance for withdrawals.

The values in columns (2), (3), and (4) are here precisely built up (with allowance for withdrawals) by the net premiums assumed; but these premiums would be quite insufficient to build up the greater reserve values (without allowance for withdrawals) set out in columns (5), (6), and (7); the difference representing the present value of the loss which would arise if there should in the future be no withdrawals.

In the case of the Association whose experience has been under review in the earlier portion of this paper, it was found that the available office subscriptions, after allowing for expenses and medical charges, was somewhat more than sufficient to provide the benefits granted, upon the assumption that the future rate of withdrawal would be reasonably below that actually experienced. It will probably be found, in many cases, that there is a very narrow margin in this respect; and that, therefore, the premium available for benefits would not be sufficient to provide reserve values computed without allowance for withdrawals. The rate of withdrawal likely to obtain in the future history of the Association thus becomes a most important element in the case; as, if from any cause the rate experienced falls materially below that assumed in the valuation, the net premiums available will no longer be sufficient to provide the necessary reserves.

It therefore behoves the Actuary to exercise great caution in his assumptions as to the rate of withdrawal. If, on the one hand, he altogether ignore this element, or adopt a rate materially below that indicated by the experience, he will probably bring out an immediate and large (but at the same time somewhat illusory) deficiency in the funds; while if, on the other hand, he assume a rate of withdrawal practically identical with that actually experienced in the past, he runs the risk that the rate obtaining in the future may be materially below his estimates, and that a grave deficiency may thus arise at future valuations. In practice, a careful judgment must be exercised, as to the sufficiency, upon reasonable assumptions as to the rate of withdrawal, of the available net premium to provide for the risks; and the basis upon which to proceed in the computations of the reserves, must be largely determined by a careful analysis of the

circumstances and experience of the particular Association, and its general financial position.

As an illustration of the important financial considerations involved in these questions, I have roughly computed the aggregate amount of reserves which would have been required, upon the different assumptions made in Tables VI, VII, and VIII, in respect of the 2,881 members actually existing, as at 31 December 1892, in the Association whose experience I have dealt with in this paper. I have taken, throughout, supposititious benefits of £10 at death, and of £1 weekly during non-employment. The reserve values were computed, at quinquennial ages and durations, by means of the values given in Tables VI, VII, and VIII, so as to give the aggregate reserves (1) with allowance for withdrawals, (a) in both valuation factors and net premium, (b) in valuation factors only; (2) without allowance for withdrawals, (a) in both valuation factors and net premium, (b) in valuation factors only. The application of these four cases to the "model office" necessarily involves the assumption that the available benefit premiums are, throughout, sufficient to cover the net premiums computed without allowance for withdrawals. The resulting Aggregate Reserve Values are as follow:—

AGGREGATE RESERVE VALUES.

Basis of Reserves	Basis of Net Premiums	Aggregate Reserves	Ratios per-cent	Table
With Allowance for Withdrawals	With	£8,856	100.0	VI (a)
Without " "	Without	8,901	100.5	VI (b)
With " "	Without	5,327	60.2	VII (a)
Without " "	With	14,156	159.9	VIII (b)

TABLE I.—CLERKS' ASSOCIATION.

Graduated Rate of Mortality, as employed in computation of money values, based upon actual experience, 1880-1892.

Age ($x+t$)	Rate of Mortality q_{x+t}
(1)	(2)
20	·0046
21	·0047
22	·0048
23	·0049
24	·0052
25	·0054
26	·0057
27	·0059
28	·0062
29	·0065
30	·0068
31	·0071
32	·0073
33	·0076
34	·0079
35	·0082
36	·0085
37	·0088
38	·0092
39	·0096
40	·0101
41	·0106
42	·0112
43	·0119
44	·0126
45	·0133
46	·0142
47	·0150
48	·0159
49	·0167
50	·0176
51	·0186
52	·0197
53	·0211
54	·0230
55	·0255
56	·0285
57	·0321
58	·0366
59	·0402

TABLE II.—CLERKS' ASSOCIATION.

Rates of Withdrawal, as actually experienced (1888-92) scheduled according to Central Ages at Entry and Years of Duration.

Year of Duration	CENTRAL AGE AT ENTRY \bar{x}						All Entry Ages	Year of Duration
	20	25	30	35	40	45		
t	Rate of Withdrawal $100(w_{t,x;-})$							t
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
0	11.4	19.7	17.2	18.4	17.8	22.1	16.7	0
1	10.9	13.1	10.2	18.4	23.4	...	12.0	1
2	9.8	11.9	13.1	14.5	9.0	...	10.9	2
3	9.2	14.9	13.9	5.9	14.5	18.9	12.2	3
4	11.1	9.6	7.2	1.3	10.5	...	9.4	4
5	9.2	9.2	9.7	9.8	9.8	...	9.7	5
6	7.2	8.6	6.8	9.2	6.1	10.5	8.0	6
7	5.2	6.2	3.7	7.3	7.1	...	5.5	7
8	6.4	6.3	4.7	4.7	12.1	8.7	6.1	8
9	4.1	3.7	4.3	3.2	4.3	...	3.8	9
10	5.3	5.1	4.7	3.6	4.8	10
11	6.0	5.6	5.3	6.3	5.8	11
12	3.9	7.9	2.6	7.3	5.2	12
13	5.9	2.2	6.4	3.0	4.3	13
14	6.8	3.9	4.0	3.9	20.0	...	5.2	14
15	1.8	5.6	2.8	5.2	3.7	15
16	6.2	3.5	7.2	4.7	16
17	4.6	...	4.1	2.5	17
18	4.7	2.5	4.8	3.7	18
19	1.9	3.4	...	13.3	2.8	19
20	6.4	4.4	4.6	20
21	2.3	4.6	3.8	21
22	22
23	2.0	1.0	23
24	24
25	1.7	2.6	...	7.4	2.2	25
26	2.0	5.0	3.5	3.0	26
27	4.3	0.9	27
28	4.4	1.4	28
29	...	6.7	2.3	29

TABLE III.--CLERKS' ASSOCIATION.

Rates of Non-Employment, as actually experienced (1888-92), scheduled (1) according to ages attained (2) according to years of duration.

Age attained $x+t$	Rate of Non- Employment u_{x+t}	Year of Duration t	Rate of Non- Employment u_t
(1)	(2)	(3)	(4)
20	·106	0	·087
21	·262	1	·245
22	·174	2	·122
23	·236	3	·204
24	·283	4	·311
25	·139	5	·159
26	·219	6	·167
27	·175	7	·248
28	·143	8	·165
29	·151	9	·100
30	·216	10	·185
31	·127	11	·222
32	·167	12	·247
33	·141	13	·113
34	·165	14	·152
35	·151	15	·139
36	·278	16	·189
37	·217	17	·304
38	·195	18	·349
39	·283	19	·177
40	·125	20	·012
41	·109	21	·290
42	·143	22	·061
43	·196	23	...
44	·171	24	·013
45	·139	25	·206
46	·341	26	·158
47	·509	27	·508
48	·360	28	·383
49	·117	29	·864
50	·269
51	·066
52	·709
53
54	·255
55	·141
56	·660
57	·535
58
59
60	·680
61
62
63	·300
64	2·787

TABLE IV.—CLERKS' ASSOCIATION.
Graduated Rates of Non-Employment, based upon the Experience of successive periods 1862-1879, 1880-87, and 1888-92; also the rate adopted for computation of Money Values.

Age attained	PERIOD OF EXPERIENCE			Rate employed in computation of Money Values	Age attained
	1862-1879	1880-1887	1888-1892		
(t)	Rate of Non-Employment (u_{x-t})				(t)
(1)	(2)	(3)	(4)	(5)	(6)
20	·266	·259	·214	·252	20
21	·268	·256	·213	·250	21
22	·268	·258	·212	·249	22
23	·263	·260	·211	·249	23
24	·250	·264	·209	·249	24
25	·243	·269	·199	·248	25
26	·232	·270	·192	·246	26
27	·226	·269	·182	·242	27
28	·228	·266	·172	·237	28
29	·232	·259	·166	·231	29
30	·238	·248	·165	·226	30
31	·247	·241	·160	·219	31
32	·256	·231	·163	·214	32
33	·260	·224	·169	·211	33
34	·260	·216	·177	·210	34
35	·256	·217	·189	·213	35
36	·261	·223	·203	·220	36
37	·265	·239	·204	·229	37
38	·269	·253	·200	·238	38
39	·270	·277	·194	·244	39
40	·281	·288	·179	·249	40
41	·291	·292	·166	·250	41
42	·308	·289	·168	·252	42
43	·341	·284	·188	·256	43
44	·355	·280	·215	·261	44
45	·421	·277	·244	·268	45
46	·435	·275	·277	·276	46
47	·446	·274	·290	·284	47
48	·433	·269	·297	·293	48
49	·448	·252	·283	·303	49
50	·408	·249	·276	·316	50
51	...	·274	·259	·335	51
52	...	·344	·277	·358	52
53	...	·440	·280	·384	53
54	...	·585	·297	·416	54
55	...	·719	·298	·454	55
56	...	·820	·322	·499	56
57	...	·841	·305	·538	57
58	...	·845	·292	·577	58
59	...	·774	·271	·616	59
60	...	·733	·303	·655	60
61	·295	·710	61
62	·352	·780	62
63	·496	·870	63
64	·682	·980	64

TABLE V.—CLERKS' ASSOCIATION.

Values of Assurances at Death, of Allowance during Non-Employment, of Annuities, and of Net Premiums, based upon the Rate of Mortality as shown in Table I; a Graduated Rate of Withdrawal (where assumed), based upon Table II; and the Rate of Non-Employment as shown in Table IV column (5), with Interest at 3 per-cent. The Net Premiums are computed (with and without Withdrawals) to provide Benefits of £10 at Death, and of £1 weekly during Non-Employment.

Duration	WITH ALLOWANCE FOR WITHDRAWALS			WITHOUT ALLOWANCE FOR WITHDRAWALS			Duration
	Value of Assurance of 1 at Death	Value of Allowance of 1 Weekly during Non-Em- ployment	Value of Annuity of 1 per annum	Value of Assurance of 1 at Death	Value of Allowance of 1 Weekly during Non-Em- ployment	Value of Annuity of 1 per annum	
(t)	$\bar{A}_{[x]+t}$	$U'_{[x]+t}$	$\bar{a}''_{[x]+t}$	$\bar{A}_{[x]+t}$	$U_{[x]+t}$	$\bar{a}_{[x]+t}$	(t)
	$\pi'_{[20]} = \cdot 397$ Age at Entry $[x] = 20$			$\pi_{[20]} = \cdot 476$			
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
0	·105	2·65	9·33	·337	7·31	22·45	0
1	·123	3·08	10·25	·343	7·44	22·22	1
3	·156	3·64	11·68	·358	7·45	21·72	3
5	·189	4·09	12·68	·373	7·47	21·21	5
10	·267	5·06	14·12	·413	7·60	19·86	10
15	·355	6·26	15·01	·457	7·95	18·39	15
20	·442	7·36	14·98	·505	8·38	16·77	20
25	·528	8·41	14·31	·557	8·87	15·01	25
30	·611	9·50	13·16	·611	9·50	13·16	30
	$\pi'_{[30]} = \cdot 456$ Age at Entry $[x] = 30$			$\pi_{[30]} = \cdot 585$			
0	·117	2·30	7·61	·413	7·60	19·86	0
1	·144	2·85	8·72	·422	7·65	19·57	1
3	·192	3·62	10·27	·439	7·78	18·99	3
5	·238	4·33	11·36	·457	7·95	18·39	5
10	·332	5·63	12·21	·505	8·38	16·77	10
15	·437	7·03	12·44	·557	8·87	15·01	15
20	·538	8·38	11·85	·611	9·50	13·16	20
25	·636	9·65	10·69	·669	10·13	11·19	25
30	·723	10·72	9·39	·723	10·72	9·39	30
	$\pi'_{[40]} = \cdot 606$ Age at Entry $[x] = 40$			$\pi_{[40]} = \cdot 794$			
0	·154	2·62	6·86	·505	8·38	16·77	0
1	·189	3·26	7·78	·515	8·46	16·42	1
3	·250	4·17	8·97	·535	8·65	15·73	3
5	·307	5·00	9·73	·557	8·87	15·01	5
10	·420	6·60	9·92	·611	9·50	13·16	10
15	·545	8·26	9·48	·669	10·13	11·19	15
20	·651	9·62	8·55	·723	10·72	9·39	20
25	25
30	30

TABLE VI.—CLERKS' ASSOCIATION.

Comparison of Reserve Values for Benefits of £10 at death, and of £1 weekly during Non-Employment; computed upon the basis of the factors given in Table V, respectively with and without allowance for Withdrawals.

Duration	(a) WITH ALLOWANCE FOR WITHDRAWALS			(b) WITHOUT ALLOWANCE FOR WITHDRAWALS			Duration
	Age at Entry			Age at Entry			
	(t)	20	30	40	20	30	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
1	·24	·31	·43	·29	·42	·57	1
3	·56	·86	1·23	·69	1·06	1·51	3
5	·95	1·53	2·17	1·11	1·76	2·52	5
10	2·12	3·38	4·79	2·28	3·62	5·16	10
15	3·85	5·73	7·96	3·77	5·66	7·94	15
20	5·83	8·36	10·95	5·45	7·91	10·49	20
25	8·01	11·13	...	7·29	10·27	...	25
30	10·38	13·67	...	9·34	12·46	...	30
$\pi_{[x]}$ (as valued)	·397	·456	·606	·476	·585	·794	$\pi_{[x]}$ (as valued)

TABLE VII.—CLERKS' ASSOCIATION.

Comparison of Reserve Values for Benefits of £10 at death, and of £1 weekly during Non-Employment; computed upon the basis of the factors given in Table V; the net premiums employed being in all cases those computed WITHOUT allowance for Withdrawals.

Duration	(a) WITH ALLOWANCE FOR WITHDRAWALS			(b) WITHOUT ALLOWANCE FOR WITHDRAWALS			Duration
	Age at Entry			Age at Entry			
	20	30	40	20	30	40	
(<i>t</i>)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
1	(- ·57)	(- ·81)	(- 1·03)	·29	·42	·57	1
3	(- ·36)	(- ·48)	(- ·45)	·69	1·06	1·51	3
5	(- ·06)	·05	·34	1·11	1·76	2·52	5
10	1·01	1·81	2·92	2·28	3·62	5·16	10
15	2·66	4·15	6·18	3·77	5·66	7·94	15
20	4·65	6·80	9·34	5·45	7·91	10·49	20
25	6·88	9·75	...	7·29	10·27	...	25
30	9·34	12·46	...	9·34	12·46	...	30
$\pi_{[x]}$ (as valued)	·476	·585	·794	·476	·585	·794	$\pi_{[x]}$ (as valued)

MEAN DURATION FORMULÆ. (SCHEDULE C.)

Here $s' = \frac{s}{2}$, $e' = \frac{e}{2}$, $w' = \frac{w}{2}$, $d' = \frac{d}{2}$;

also, $f' = e' + w' + d' = \frac{e + w + d}{2} = \frac{f}{2}$

and $g' = s' - f' = \frac{s - f}{2} = \frac{g}{2}$.

Inserting these mean values in formulæ (1) (3) and (4) respectively, we have:

$$\bar{E}_{[x]} = n_{[x]} + \frac{g_{[x]}}{2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

$$\bar{E}_{[x]+t} = \bar{E}_{[x]+t-1} + \frac{g_{[x]+t} + g_{[x]+t-1}}{2} \quad . \quad . \quad . \quad (6)$$

$$\bar{E}_{[x]+t} = n_{[x]} + \Sigma_0^t (g) - \frac{g_{[x]+t}}{2} \quad . \quad . \quad . \quad . \quad . \quad (7)$$

NEAREST DURATION FORMULÆ, AS APPLIED (IN SCHEDULE D)
FOR DEDUCING EITHER THE RATE OF MORTALITY OR
THAT OF WITHDRAWAL.

(a) RATE OF MORTALITY.

Let s , e and w , represent the numbers surviving, existing and withdrawing, as tabulated by this method; so that we have, for initial values:

$$s_{[x]} = (as)_{[x]} \quad e_{[x]} = (ae)_{[x]} \quad w_{[x]} = (aw)_{[x]}$$

and for subsequent values,

$$s_{[x]+t} = [(bs)_{[x]+t-1} + (as)_{[x]+t}]$$

$$e_{[x]+t} = [(be)_{[x]+t-1} + (ae)_{[x]+t}]$$

$$w_{[x]+t} = [(bw)_{[x]+t-1} + (aw)_{[x]+t}]$$

the symbols a and b , representing throughout the reference to the beginning and the end respectively of the year of duration.

Also let $f_{[x]+t} = (e + w)_{[x]+t} + d_{[x]+t-1}$

where $d_{[x]+t-1}$ represents, as usual, the deaths actually occurring in the t th year of duration,

and let $g_{[x]+t} = (s - f)_{[x]+t}$.

Then we have for the numbers exposed to the risk of death,

$$E_{[x]} = n_{[x]} + g_{[x]} \quad . \quad . \quad . \quad . \quad . \quad (a)$$

$$E_{[x]+1} = E_{[x]} + g_{[x]+1} \quad . \quad . \quad . \quad . \quad . \quad (b)$$

and, generally, $E_{[x]+t} = E_{[x]+t-1} + g_{[x]+t} \quad . \quad . \quad . \quad . \quad . \quad (c)$

Inserting in formula (b) the value of $E_{[x]}$ from formula (a), we have,

$$\begin{aligned} E_{[x]+1} &= n_{[x]} + g_{[x]} + g_{[x]+1} \\ &= n_{[x]} + \Sigma_0^1(g) \end{aligned}$$

and, generally,

$$E_{[x]+t} = n_{[x]} + \Sigma_0^t(g) \quad . \quad . \quad . \quad . \quad . \quad (d)$$

the formula employed in Schedule (D) for deducing the rate of mortality.

It can be shown that

$$\Sigma_0^t(g) = \Sigma_0^t(g) - (bg)_{[x]+t} + (ad)_{[x]+t}$$

where g = the net movement of cases according to the Exact Duration Method; and we have, therefore, by the Nearest Duration Method (Schedule D)

$$E_{[x]+t} = n_{[x]} + \Sigma_0^t(g) - (bg)_{[x]+t} + (ad)_{[x]+t},$$

and by the Exact Duration Method (Schedule B)

$$E_{[x]+t} = n_{[x]} + \Sigma_0^t(g) - g'_{[x]+t} + (d-d')_{[x]+t},$$

(bg) and (ad) in the former formula being replaced by g' and $(d-d')$ in the latter respectively.

(b) RATE OF WITHDRAWAL.

Let s , e , represent, as before, the numbers tabulated as surviving and existing; and let

$$d_{[x]} = (ad)_{[x]} \quad d_{[x]+t} = [(bd)_{[x]+t-1} + (ad)_{[x]+t}]$$

also let

$$f'_{[x]+t} = (e + d)_{[x]+t} + w_{[x]+t-1}$$

where $w_{[x]+t-1}$ represents the actual withdrawals in the t th year of duration; and let $g'_{x+t} = (s - f')_{[x]+t}$.

Then we have, for the numbers exposed to the risk of withdrawal

$$(wE)_{[x]} = n_{[x]} + g'_{[x]} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (e)$$

$$(wE)_{[x]+t} = (wE)_{[x]+t-1} + g'_{[x]+t} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (f)$$

$$= n_{[x]} + \Sigma_0^t(g') \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (g)$$

the formulæ employed in Schedule (D) for deducing the rate of withdrawal.

It can similarly be shown here that

$$\Sigma_0^t(g') = \Sigma_0^t(g) - (bg)_{[x] \div t} + (aw)_{[x] \div t}$$

and that, therefore,

$$(wE)_{[x] \div t} = n_{[x]} + \Sigma_0^t(g) - (bg)_{[x] \div t} + (aw)_{[x] \div t}$$

which may be compared with the Exact Duration formula,

$$(wE)_{[x]+t} = n_{[x]} + \Sigma_0^t(g) - g'_{[x]+t} + (w - w')_{[x]+t}$$

(bg) in the former formula being again replaced by (g') in the latter; and (aw) by ($w - w'$).

NEAREST DURATION FORMULE, AS APPLIED (IN SCHEDULES
E AND F) FOR DEDUCING THE RATES OF MORTALITY AND OF
WITHDRAWAL.

As the cases are sorted and tabulated by this method in a distinctive way, it will be well to investigate the formulæ independently. Let s, e, w and d, represent the numbers surviving, existing, withdrawing and dying, as tabulated by this method, so that the initial values

$$s_{[x]} = (as)_{[x]} \quad w_{[x]} = (aw)_{[x]} \quad d_{[x]} = (ad)_{[x]} \quad e_{[x]} = (ae)_{[x]}$$

also

$$s_{[x]_{\vdash} t} = [(bs)_{[x]_{\vdash} t-1} + (as)_{[x]_{\vdash} t}] \quad e_{[x]_{\vdash} t} = [(be)_{[x]_{\vdash} t-1} + (ae)_{[x]_{\vdash} t}]$$

$$w_{[x]+t} = [(bu)_{[x]+t-1} + (av)_{[x]+t}] \quad d_{[x]+t} = [(bd)_{[x]+t-1} + (ad)_{[x]+t}]$$

$$\text{and} \quad f_{[x]+t} = (e + w + d)_{[x]+t} \quad g_{[x]+t} = (s - f)_{[x]+t}.$$

Then we have

$$\bar{\mathbf{E}}_{[x]} = n_{[x]} + (as)_{[x]} - (ae)_{[x]} - (aw)_{[x]} - (ad)_{[x]} = n_{[x]} + g_{[x]} \quad (8)$$

similarly,

$$\begin{aligned}\bar{\mathbf{E}}_{[x]+1} &= \bar{\mathbf{E}}_{[x]} + (bs)_{[x]} - [(be) + (bw) + (bd)]_{[x]} \\ &\quad + (as)_{[x]+1} - [(ae) + (aw) + (ad)]_{[x]+1} \\ &= \bar{\mathbf{E}}_{[x]} + (\mathbf{s}-\mathbf{f})_{[x]+1} \\ &= \bar{\mathbf{E}}_{[x]} + \mathbf{g}_{[x]+1} \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \quad (9)\end{aligned}$$

and generally $\bar{E}_{[x]+t} = \bar{E}_{[x]+t-1} + g_{[x]+t} \quad . \quad . \quad . \quad (10)$

a formula for deducing the numbers exposed to risk by a continuous method.

Inserting now in formula (9) the value of $\bar{E}_{[x]}$ from formula (8), we have:

$$\bar{E}_{[x]+1} = n_{[x]} + g_{[x]} + g_{[x]+1}$$

and generally

$$\bar{E}_{[x]+t} = n_{[x]} + \sum_0^t (g) \quad . \quad . \quad . \quad . \quad (11)$$

a formula for deducing the numbers exposed to risk by a process of summation, as applied in Schedule (E).

If now we analyze this formula, we have:

$$\begin{aligned} \bar{E}_{[x]+t} &= n_{[x]} + (g_{[x]} + g_{[x]+1} + \dots + g_{[x]+t}) \\ &= n_{[x]} + [(as)_{[x]} + (bs)_{[x]} + (as)_{[x]+1} + \dots + (as)_{[x]+t}] \\ &\quad - [(ae)_{[x]} + (be)_{[x]} + (ae)_{[x]+1} + \dots + (ae)_{[x]+t}] \\ &\quad - [(aw)_{[x]} + (bw)_{[x]} + (aw)_{[x]+1} + \dots + (aw)_{[x]+t}] \\ &\quad - [(ad)_{[x]} + (bd)_{[x]} + (ad)_{[x]+1} + \dots + (ad)_{[x]+t}] \\ &= n_{[x]} + \sum_0^t (s) - (bs)_{[x]+t} - \sum_0^t (e) + (be)_{[x]+t} \\ &\quad - \sum_0^t (w) + (bw)_{[x]+t} - \sum_0^t (d) + (bd)_{[x]+t} \\ &= n_{[x]} + \sum_0^t (g) - (bg)_{[x]+t} \quad . \quad . \quad . \quad . \quad (12) \end{aligned}$$

Comparing this now with formula (4) of Exact Durations,

$$\bar{E}_{[x]+t} = n_{[x]} + \sum_0^t (g) - g'_{[x]+t} \quad . \quad . \quad . \quad . \quad (4)$$

we see that the two formulæ differ only in their last terms, and that the difference between the numbers exposed to risk, as deduced by the Nearest and by the Exact Duration Methods, solely arises from the error introduced in the year of duration at entry (for survivors), or at exit (for cases emerging), by the assumption that $(bg)_{[x]+t}$, the balance of nearest integral durations in the year, is approximately equal to $g'_{[x]+t}$ (see pages 95, 96), the balance of true fractional exposures in the year.

APPENDIX (B).

PERIOD OF OBSERVATION LIMITED BY YEARS OF DURATION.

EXACT DURATION FORMULÆ. (SCHEDULE G).

Here $s_{[x]+t}$, $e_{[x]+t}$, become $s^1_{[x]+t}$, $e^1_{[x]+t}$ (the numbers surviving and existing at precise duration t), and, as the new entrants, during the period $(n_{[x]})$ now fall into rank as "survivors" at precise age $[x]$, ($s^1_{[x]}$), and, as there are now no cases existing at precise age $[x]$, we have $n_{[x]} = s^1_{[x]}$; $e^1_{[x]} = 0$.

Representing by G and G' respectively, the modified values of g and g' under these conditions, we have

$$G'_{[x]} = s'_{[x]} - (w + d)_{[x]}, \quad G_{[x]-t} = s'_{[x]-t} - (e' + w + d)_{[x]-t},$$

also, as $s'_{[x]-t} = 0$, $e'_{[x]-t} = 0$, we have

$$G'_{[x]-t} = - (w' + d')_{[x]-t}$$

for all values of t .

Then formula (1) becomes

$$\bar{E}_{[x]} = G_{[x]} - G'_{[x]} = G_{[x]} + (w' + d')_{[x]} \quad (13)$$

Similarly, formula (3) becomes

$$\begin{aligned} \bar{E}_{[x]-t} &= \bar{E}_{[x]-t-1} - G_{[x]+t} + \Delta G'_{[x]-t-1} \\ &= E_{[x]-t-1} + G_{[x]+t} + \Delta (w' + d')_{[x]-t-1} \end{aligned} \quad (14)$$

also formula (4) becomes

$$\begin{aligned} \bar{E}_{[x]-t} &= \sum_0^t G - G'_{[x]-t} \\ &= \sum_0^t (G + (w' + d')_{[x]-t}) \end{aligned} \quad (15)$$

These formulæ give the numbers exposed to risk (\bar{E}), from which the usual functions (E) and wE , can be deduced. It is however, in this case equally convenient to deduce these latter functions by a direct process. For we have

$$\bar{E} = E + (d' - d)$$

and $\bar{E} = (wE) - (w' - w)$,

and inserting these values successively in formulæ (13), (14) and (15) we have for the numbers exposed to the risk of death,

$$E_{[x]} = G_{[x]} + (w' + d)_{[x]} \quad (16)$$

$$E_{[x]-t} = E_{[x]-t-1} + G_{[x]-t} + \Delta (w' + d)_{[x]-t-1} \quad (17)$$

and $E_{[x]-t} = \sum_0^t (G) + (w' + d)_{[x]-t} \quad (18)$

Similarly, for the numbers exposed to the risk of withdrawal, we have

$$wE_{[x]} = G_{[x]} + (w + d')_{[x]} \quad (19)$$

$$(wE)_{[x]-t} = (wE)_{[x]-t-1} + G_{[x]-t} + \Delta (w + d')_{[x]-t-1} \quad (20)$$

$$(wE)_{[x]-t} = \sum_0^t (G) + (w + d')_{[x]-t} \quad (21)$$

MEAN DURATION FORMULÆ. SCHEDULE H.)

Here $w' = \frac{w}{2}$, $d' = \frac{d}{2}$, and formulæ (16), (17) and (18) become respectively

$$E_{[x]} = G_{[x]} + \left(\frac{w}{2} + d\right)_{[x]} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (22)$$

$$E_{[x]+t} = E_{[x]+t-1} + G_{[x]+t} + \Delta\left(\frac{w}{2} + d\right)_{[x]+t-1} \quad . \quad . \quad (23)$$

$$E_{[x]+t} = \Sigma_0^t(G) + \left(\frac{w}{2} + d\right)_{[x]+t} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (24)$$

Similarly, for the numbers exposed to the risk of withdrawal, formulæ (19), (20) and (21) become respectively

$$(wE)_{[x]} = G_{[x]} + \left(w + \frac{d}{2}\right)_{[x]} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (25)$$

$$(wE)_{[x]+t} = (wE)_{[x]+t-1} + G_{[x]+t} + \Delta\left(w + \frac{d}{2}\right)_{[x]+t-1} \quad (26)$$

$$(wE)_{[x]+t} = \Sigma_0^t(G) + \left(w + \frac{d}{2}\right)_{[x]+t} \quad . \quad . \quad . \quad . \quad . \quad . \quad (27)$$

NEAREST DURATION FORMULÆ. (SCHEDULES J, K.)

Reverting to, and analyzing formula (11) we have

$$\begin{aligned} \bar{E}_{[x]+t} &= n_{[x]} + \Sigma_0^t(g) \\ &= n_{[x]} + \Sigma_0^t[s - (e + w + d)] \end{aligned}$$

which becomes, when the observation is limited by years of duration (since $s = s^1$, $e = e^1$, and $n_{[x]} = s^1_{[x]}$),

$$\bar{E}_{[x]+t} = \Sigma_0^t[s^1 - (e^1 + w + d)],$$

which we may call $\Sigma_0^t(G)$; therefore

$$\bar{E}_{[x]+t} = \Sigma_0^t(G) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (28)$$

bnt, as $\bar{E}_{[x]+t} = E_{[x]+t} - (ad)_{[x]+t}$

$$= (wE)_{[x]+t} - (aw)_{[x]+t}$$

we obtain the following direct formulæ for the numbers exposed to the risk of death and of withdrawal respectively

$$E_{[x]+t} = \Sigma_0^t(G) + (ad)_{[x]+t} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (29)$$

$$\text{and} \quad (wE)_{[x]+t} = \Sigma_0^t(G) + (aw)_{[x]+t} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (30)$$

Here it may be shown that

$$\Sigma_0^t(G) = \Sigma_0^t(G) + [(bw) + (bd)]_{[x]+t},$$

and the above formulæ thus become

$$E_{[x]+t} = \Sigma_0^t(G) + [(bw) + d]_{[x]+t} \quad . \quad . \quad . \quad . \quad . \quad . \quad (31)$$

$$\text{and} \quad (wE)_{[x]+t} = \Sigma_0^t(G) + [w + (bd)]_{[x]+t} \quad . \quad . \quad . \quad . \quad . \quad . \quad (32)$$

which may be compared with the Exact Duration formulæ (18) and (21), the quantities w' and d' being replaced, in formulæ (31) and (32), by (bw) and (bd) , respectively.

APPENDIX (C).

The values of $E_{[x]+t}$ and of $(wE)_{[x]+t}$, as deduced from that of $\bar{E}_{[x]+t}$ by the several formulæ (see pages 77, 81, and 94)

$$E_{[x]+t} = \bar{E}_{[x]+t} + (d-d')_{[x]+t} \quad (wE)_{[x]+t} = \bar{E}_{[x]+t} + (w-w')_{[x]+t}$$

$$E_{[x]+t} = \bar{E}_{[x]+t} + \frac{d'_{[x]+t}}{2} \quad (wE)_{[x]+t} = \bar{E}_{[x]+t} + \frac{w'_{[x]+t}}{2}$$

$$E_{[x]+t} = \bar{E}_{[x]+t} + (ad)_{[x]+t} \quad (wE)_{[x]+t} = \bar{E}_{[x]+t} + (aw)_{[x]+t}$$

according to the Exact, Mean, and Nearest Duration Methods respectively, do not appear, in the particular case here investigated, to represent quite correctly the true relations. As the period of observation terminates at the close of a calendar year, the cases "existing" are necessarily under observation for a portion only of the year of duration then current, and some of the cases of death (and of withdrawal) during the last calendar year, would, if treated as "existing", have in like manner completed only a portion of the year of duration current at exit. In strictness, therefore, such cases should contribute to the number exposed to risk, not the full year of duration current at exit, but only that portion of the year which actually fell within the period of observation.

The effect of the assumptions made, and the nature and extent of the correction required, will be seen if we suppose that 1,000 members complete their year of duration in the last calendar year of the period of observation, on 30 June; and that of these 1,000 cases none withdraw, but five die during the following six months. The number exposed to risk in respect of the 995 survivors would clearly be

$$= \frac{995}{2} = 497.5; \text{ and to this would be added, by the above formulæ, a}$$

full year's exposure in respect of each of the five deaths, making the total number exposed to risk 502.5. The annual rate of mortality

$$\text{deduced in respect of these cases only would thus be } \frac{5}{502.5} = .00995;$$

but it would seem to be clear that the true annual rate of mortality = .01, and that the number exposed to risk should, in strictness, be $497.5 + 2.5 = 500$; that is, that the death cases should be considered as exposed to risk for six months each only.

The cases affected are those of death (or withdrawal) occurring in the last calendar year of the period of observation, where the anniversary of entry in that year precedes the date of death (or withdrawal). Assuming that this would include about one-half of the number of cases emerging in the year; that the number of emergents in each year of a quinquennial period do not greatly vary; and that, upon the average, two-thirds of the year of duration current at exit falls, in these cases, within the period of observation; the amount by which the exposures of the cases of death (or of withdrawal) would be overstated, may be roughly estimated at about one-thirtieth part (or $3\frac{1}{3}$ per-cent) of the total number of deaths (or withdrawals) occurring in the quinquennium.

I find that, of the 481 withdrawals included in the illustrative experience employed in this paper, 34 occurred in the calendar year 1892, subsequently to their anniversaries of entry in that year; and that, of these 34 cases, the actual portion of the year of duration current at exit which fell within the period of observation, was, in the aggregate 25.5 years. Of the 48 deaths, seven occurred in the calendar year 1892, subsequently to their anniversaries of entry in that year; in respect of which the actual portion of the current year of duration which fell within the period was 5.7 years. The number exposed to the risk of withdrawals was thus overstated by $(34 - 25.5 =)$ 8.5 years; and the number exposed to the risk of death, by $(7 - 5.7 =)$ 1.3 years. The aggregate years of risk being 6,660 and 6,458 respectively, the amount of the error was (in this particular case) about 13 per 10,000 in the numbers exposed to the risk of withdrawal, and about two per 10,000 in the numbers exposed to the risk of death. The correction is, therefore, extremely small, and may, in practice, be disregarded.

APPENDIX (D) (see page 146).

In the case of an experience where the value of d_x tends to increase steadily with the age, the probability will be for the death cases to congregate towards the end of the year of duration, if we assume that the progression of the decrements in *half-years* follows generally the same law as their annual progression. In the H^M Table, for example, the value of d increases between ages 14 and 22, and between ages 25 and 74, after which it steadily diminishes until the end of the table; and with such an experience, it would appear to be more probable that death will happen (on the whole) in the later half of the year of duration. Under the H^F Table the same tendency is observable, but in a less strongly marked form; the value of d increasing between ages 10 to 16, 19 to 28, 39 to 45, 49 to 76, after which it steadily diminishes.

APPENDIX (E).

The formulæ given on pp. 173, 174, for the values of q' and of q'' , are closely approximate, the true formulæ being, upon the assumption of a uniform distribution of deaths throughout the year,

$$q' = \frac{q - \frac{q \cdot (wq)}{2}}{1 - \frac{q \cdot (wq)}{4}} \qquad q'' = \frac{q + (wq) - q \cdot (wq)}{1 - \frac{q \cdot (wq)}{4}}$$

These formulæ are deduced directly from those given by Dr. Sprague (*J.I.A.*, xxi, 416-7) for computing the elements of the mortality table in the case of a double decrement, which, expressed in the notation here employed, become

$$d = l \cdot \frac{q \left(1 - \frac{(wq)}{2} \right)}{1 - \frac{q \cdot (wq)}{4}} = l \cdot \frac{m}{1 + \frac{m + (wm)}{2}}$$

$$w=l \frac{(wq)\left(1-\frac{q}{2}\right)}{1-\frac{q \cdot (wq)}{4}} = l \cdot \frac{(wm)}{1+\frac{m+(wm)}{2}}$$

$$l_{+1}=l \left[1-\frac{q+(wq)-q(wq)}{1-\frac{q \cdot (wq)}{4}} \right] = l \cdot \frac{1-\frac{m+(wm)}{2}}{1+\frac{m+(wm)}{2}}$$

The alternative expressions given on the right-hand side are convenient for deducing the mortality elements in terms of the *central rates* of death and of withdrawal; and, in the case of an experience such as that here investigated, it is, on the whole, more convenient to compute the values of $m_{[x]+t}$ and of $(wm)_{[x]+t}$ direct from that of $\bar{E}_{[x]+t}$, and thence to deduce the values of $l_{[x]+t}$, $d_{[x]+t}$, and $w_{[x]+t}$. The values of $E_{[x]+t}$ and of $(wE)_{[x]+t}$, as well as those of $q_{[x]+t}$, and of $(wq)_{[x]+t}$, can then be dispensed with.

It will be convenient to call

$$m_{[x]+t} = \frac{m}{1+\frac{m+(wm)}{2}} \quad (wm)_{[x]+t} = \frac{(wm)}{1+\frac{m+(wm)}{2}}$$

$$\text{and} \quad 1 - [m_{[x]+t} + (wm)_{[x]+t}] = \frac{1 - \frac{m+(wm)}{2}}{1 + \frac{m+(wm)}{2}}$$

then we have,

$$d_{[x]+t} = l_{[x]+t} \cdot m_{[x]+t} \quad w_{[x]+t} = l_{[x]+t} (wm)_{[x]+t}$$

$$\text{and} \quad l_{[x]+t+1} = l_{[x]+t} [1 - (m_{[x]+t} + (wm)_{[x]+t})]$$

and by these formulæ the values of d , w , and l , can be readily computed.

The appended Table IX shows, for age at entry (20), the central rates of mortality and of withdrawal, the logarithms of the factors m , (wm) , and $\{1 - [m + (wm)]\}$, respectively, and the numbers living, dying, and withdrawing, as deduced in each successive year of duration by the above formulæ. The rate of withdrawal here employed is a graduated rate, based upon that experienced in years of duration, as set out in column (8) of Table (II).

In Table X the Commutation columns are deduced, at a rate of interest of 3 per-cent per annum, from the mortality elements given in Table IX; also the values of assurances and annuities, and of the benefit during non-employment; with allowance, throughout, for mortality and withdrawal.

TABLE IX.—MORTALITY TABLE.

Table showing the Central Rates of Mortality and of Withdrawal, and the Mortality Table (with decrements by Death and Withdrawal) as deduced therefrom.—Central Age at Entry $[x]=20$.

Duration	CENTRAL RATE OF		LOGARITHMS OF FACTORS FOR DEDUCING MORTALITY TABLE			MORTALITY TABLE			
	Mortality	Withdrawal				Numbers Living	Deaths	Withdrawals	Deaths and Withdrawals
(t)	$m_{[x]+t}$	$(wm)_{[x]+t}$	$\log m_{[x]+t}$	$\log (wm)_{[x]+t}$	$\log [1 - (m+wm)_{[x]+t}]$	$l_{[x]+t}$	$d_{[x]+t}$	$w_{[x]+t}$	$(d+w)_{[x]+t}$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
0	·00461	·16220	7·62891	9·17526	9·92739	100,000	426	14,970	15,396
1	·00471	·13330	·64404	·09585	·93997	84,604	373	10,549	10,922
2	·00481	·11080	·65775	·02014	·94974	73,682	335	7,717	8,052
3	·00491	·09124	·67007	8·95323	·95691	65,630	307	5,892	6,199
4	·00521	·08333	·69803	·90199	·96153	59,431	297	4,741	5,038
5	·00541	·07792	·71547	·87392	·96379	54,393	282	4,070	4,352
6	·00572	·06718	·74185	·81169	·96832	50,041	276	3,244	3,520
7	·00592	·05656	·75896	·73915	·97286	46,521	267	2,551	2,818
8	·00622	·05128	·78148	·69764	·97502	43,703	264	2,179	2,443
9	·00652	·04604	·80298	·65187	·97716	41,260	262	1,852	2,114
10	·00682	·04604	·82245	·65181	·97704	39,146	260	1,756	2,016
11	·00713	·04604	·84170	·65175	·97691	37,130	258	1,664	1,922
12	·00733	·04604	·85366	·65170	·97681	35,208	251	1,580	1,831
13	·00763	·04082	·87214	·60048	·97896	33,377	249	1,329	1,578
14	·00793	·04082	·88881	·60041	·97882	31,799	246	1,268	1,514
15	·00823	·03562	·90598	·54227	·98095	30,285	244	1,056	1,300
16	·00854	·03046	·92307	·47534	·98306	28,985	243	865	1,108
17	·00884	·02532	·93909	·39610	·98516	27,877	242	695	937
18	·00924	·02532	·95823	·39602	·98499	26,940	245	670	915
19	·00965	·02020	·97809	·29891	·98703	26,025	248	518	766
20	·01015	·02020	·99993	·29881	·98682	25,259	253	502	755
21	·01066	·02020	8·02111	·29870	·98660	24,504	257	487	744
22	·01126	·02020	·04476	·29857	·98633	23,760	263	474	737
23	·01197	·01511	·07225	·17342	·98824	23,023	272	343	615
24	·01268	·01511	·09712	·17326	·98792	22,408	280	334	614
25	·01339	·01511	·12064	·17312	·98763	21,794	288	324	612
26	·01430	·01511	·14900	·17292	·98723	21,182	298	316	614
27	·01511	·01005	·17383	7·99674	·98907	20,568	307	204	511
28	·01603	·01005	·19930	·99654	·98867	20,057	317	200	517
29	·01684	·00501	·22162	·69512	·99051	19,540	325	97	422
30	·01776	...	·24560	...	·99229	19,118	337
31	·01877	...	·26940	...	·99185	18,781	349
32	·01990	...	·29455	...	·99136	18,432	363
33	·02132	...	·32419	...	·99075	18,069	381
34	·02327	...	·36177	...	·98989	17,688	407
35	·02583	...	·40655	...	·98879	17,281	440
36	·02891	...	·45482	...	·98744	16,841	480
37	·03262	...	·50645	...	·98583	16,361	525
38	·03728	...	·56346	...	·98381	15,836	580
39	·04102	...	·60418	...	·98218	15,256	613

TABLE X.—VALUES OF BENEFITS.

Table showing the Commutation Columns, allowing for Mortality and Withdrawal, and the values of Annuities, Assurances and Allowance during Non-Employment, as deduced therefrom.—Central Age at Entry $[x]=20$.—Interest at 3 per-cent per annum.

Duration	COMMUTATION COLUMNS, ALLOWING FOR MORTALITY AND WITHDRAWAL				VALUES OF ANNUITY AND ASSURANCE		RATE OF NON-EMPLOYMENT	COMMUTATION COLUMNS, ALLOWING FOR MORTALITY AND WITHDRAWAL		VALUES OF ALLOWANCE
(<i>t</i>)	$D''_{[x]+t}$	$N''_{[x]+t}$	$C'_{[x]+t}$	$M'_{[x]+t}$	$a''_{[x]+t}$	$A'_{[x]+t}$	$u_{[x]+t}$	$D''_{[x]+t} \cdot u_{[x]+t}$	$\Sigma(D''u)$	$U_{[x]+t}$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
0	55,368	488,939	228.7	5,721.0	8.831	.104	.252	6,353.0	146,545	2.652
1	45,479	443,460	194.5	5,492.3	9.751	.121	.250	10,492.0	140,442	3.088
2	38,454	405,006	169.8	5,297.8	10.532	.138	.249	8,927.7	129,949	3.379
3	33,254	371,752	151.0	5,128.0	11.170	.155	.249	7,780.0	121,022	3.640
4	29,236	342,516	141.6	4,977.0	11.716	.170	.249	6,874.2	113,242	3.873
5	25,978	316,538	131.0	4,835.4	12.185	.187	.248	6,098.6	106,368	4.095
6	23,204	293,334	124.3	4,704.4	12.612	.203	.246	5,430.2	100,269	4.321
7	20,944	272,390	116.7	4,580.1	13.006	.219	.242	4,845.6	94,839	4.528
8	19,102	253,288	112.1	4,463.4	13.260	.234	.237	4,338.5	89,993	4.711
9	17,509	235,779	108.0	4,351.3	13.467	.249	.231	3,885.2	85,655	4.892
10	16,128	219,651	104.0	4,243.3	13.620	.263	.226	3,500.7	81,770	5.070
11	14,852	204,799	100.1	4,139.3	13.790	.279	.219	3,123.6	78,269	5.270
12	13,673	191,126	94.8	4,039.2	13.979	.295	.214	2,809.6	75,145	5.496
13	12,584	178,542	91.0	3,944.4	14.188	.313	.211	2,555.7	72,336	5.748
14	11,640	166,902	87.5	3,853.4	14.339	.331	.210	2,352.4	69,780	5.995
15	10,763	156,139	84.1	3,765.9	14.507	.350	.213	2,211.6	67,428	6.265
16	10,001	146,138	81.3	3,681.8	14.613	.368	.220	2,127.3	65,216	6.521
17	9,338.1	136,800	78.8	3,600.5	14.650	.386	.229	2,072.4	63,089	6.756
18	8,761.7	128,938	77.3	3,521.7	14.614	.402	.238	2,020.6	61,016	6.964
19	8,217.5	119,821	75.8	3,444.4	14.581	.419	.244	1,947.2	58,996	7.179
20	7,743.4	112,078	75.2	3,368.6	14.474	.435	.249	1,872.0	57,048	7.369
21	7,293.1	104,785	74.3	3,293.4	14.367	.452	.250	1,769.9	55,176	7.566
22	6,865.6	97,919	73.9	3,219.1	14.262	.469	.252	1,666.3	53,406	7.743
23	6,459.1	91,460	74.1	3,145.2	14.160	.487	.256	1,608.0	51,740	8.010
24	6,103.4	85,356	74.1	3,071.1	13.985	.503	.261	1,548.6	50,132	8.214
25	5,763.0	79,593	73.9	2,997.0	13.811	.520	.268	1,501.0	48,584	8.430
26	5,438.1	74,155	74.4	2,923.1	13.635	.537	.276	1,457.9	47,083	8.658
27	5,126.7	69,029	74.3	2,848.7	13.465	.556	.284	1,417.2	45,625	8.899
28	4,833.7	64,175	74.6	2,774.4	13.222	.572	.293	1,384.5	44,207	9.108
29	4,591.0	59,584	74.2	2,699.8	12.979	.588	.303	1,356.2	42,823	9.328
30	4,360.9	55,223	74.5	2,625.6	12.663	.602	.316	1,346.2	41,467	9.509
31	4,159.4	51,064	75.1	2,551.1	12.277	.613	.335	1,360.6	40,121	9.646
32	3,963.2	47,100	75.8	2,476.0	11.884	.625	.358	1,384.6	38,760	9.780
33	3,772.0	43,328	77.3	2,400.2	11.487	.636	.381	1,412.5	37,375	9.909
34	3,584.9	39,743	80.1	2,322.9	11.086	.648	.416	1,453.0	35,963	10.032
35	3,400.4	36,343	84.2	2,242.8	10.688	.660	.454	1,502.2	34,510	10.150
36	3,217.3	33,126	89.0	2,158.6	10.296	.671	.499	1,559.8	33,008	10.260
37	3,034.5	30,061	94.6	2,069.6	9.916	.682	.538	1,583.3	31,441	10.363
38	2,851.5	27,240	101.3	1,975.0	9.553	.693	.577	1,592.2	29,865	10.473
39	2,667.2	24,573	104.1	1,873.7	9.213	.702	.616	1,587.0	28,272	10.600

DISCUSSION.

The PRESIDENT (Mr. A. J. FINLAISON, C.B.) said that the paper the members had just listened to with so much interest was of great value at the present time. Mr. Ackland, in the first part of his paper, had opportunely summarized, in a complete and lucid manner, the various known methods of deducing rates of mortality and withdrawals from observed facts. Various arrangements had been traced in the most elaborate manner, and the results had been impartially shown. The author had been good enough not to leave the members in doubt as to the arrangement he preferred, for he stated that the "Nearest Duration Method" appeared to him that best suited for dealing with a large observation of assured lives. He (the President) was inclined to agree entirely with Mr. Ackland, to the extent that it was advisable to limit the period of observation by years of duration of policies on the lives of the assured persons included in an inquiry. He would go further and say, that the Nearest Exact Age in years was a desirable assumption to make for the exact age at entry of each person into the observation. He thought discrimination should be observed between the various modes of withdrawal, as those from some causes would group themselves towards the end of policy years, while those from other causes would be distributed over the whole duration of the policy years in which the risk ceased. He did not think it was important to record in what period of each policy year deaths happened to occur; the object should be to determine the probability of living over each year or of dying in each year, and whatever other functions might be wanted should be deduced from those, when they had been discovered. The second part of Mr. Ackland's paper dealt with a very interesting class of societies, some of which had been in successful operation for a considerable number of years. In dealing with the rates of secession from such societies, and the amount of claims that might be made upon them for non-employment, he should like to emphasize what Mr. Ackland had said, namely, that the most careful judgment should be exercised as to whether the experience of the past in those respects was in any degree likely to be repeated in future. To those who had studied the effect of withdrawals, the Abstract Table, given by the author, of "Aggregate Reserve Values", either with or without allowance for withdrawals, would prove of great interest.

Mr. H. W. MANLY said that Mr. Ackland's paper contained a great amount of matter for thought, and that thought, he was sorry to say, he had been unable to give to it. The paper had come at a most opportune moment, when the Council was preparing to deal with that vast mass of material which had been collected for the purposes of the "New Experience." The author had examined all the various methods which had ever been suggested for the collation of material for obtaining an experience in years of duration, and that alone was a valuable work. To his mind the most important part of the paper was the author's remark that, in the particular experience investigated, it was immaterial whether they used the Exact Duration Method, the Mean Duration Method, or the Nearest Duration Method; as, in that particular case, "the values in individual years of duration, both of the numbers exposed to risk and of the rates of

mortality and withdrawal are practically identical throughout." When, however, the author applied the several methods to the general experience amongst assured lives, he understood him to say that from his very extensive inquiry he had formed a preference for the Nearest Duration Method. He thought the paper was an extremely valuable one, and would repay a most careful study.

Mr. G. H. RYAN said that the first thing that struck him, in reading the first of the two great divisions into which the author's paper might be cast, was the following remark:—" Bearing in mind that the Exact Duration Method gives effect to the precise exposures of the cases, and deduces the rates, both of mortality and of withdrawal, strictly as experienced in each successive year of duration, it may, I think, be considered as not unduly laborious for the valuable results obtained." When he read those few lines in Mr. Ackland's paper, he involuntarily recalled the fact that, about ten years ago, the late Mr. Makeham had shown him a manuscript table of Mortality Experience, in which the exposures were calculated exactly to one place of decimals. He remembered at the time being greatly struck with the simplicity and compactness of that method, but the details of the system had passed away from his memory. The fact that Mr. Makeham considered the Exact Duration Method the best for the purposes he had in hand, was, he thought, at any rate some confirmation of the opinions to which Mr. Ackland had placed his name. Now, he would venture to submit that, if it could be shown that, by calculating these exposures exactly, they were obtaining the raw material in the form best suited for the objects in view, no considerations of time, labour, or expense ought to stand in the way of their adopting the Exact Duration system. It must be remembered that it was more than thirty years since an investigation of that sort had been conducted by the Institute of Actuaries, and it might be thirty years more before the Institute again took up a similar work. He maintained that, for the chief task of the Institute for a whole generation, it was desirable to take a broad view of their responsibilities, and to endeavour to obtain the facts in the most perfect form available, so that in their deductions they might feel certain that any defects that existed were inherent in the observations themselves, and were not due to their method of manipulating them. He should like to express his warm congratulations to Mr. Ackland for the very ingenious symbolical formulas which he had prepared to give full expression to an intricate subject. In the past, the papers which had been produced on the subject had, he thought, lost something of interest and value, because they had been produced by different authors and at different times, and the notation had not always been consistent. But Mr. Ackland, having the advantage of bringing the whole subject into focus, had succeeded in producing a scheme of notation which enabled it to be studied very conveniently. In the second part of the paper, the author gave new formulas, methods, and tables relating to some unusual forms of benefit, illustrating the principle of collective assurance in some of its varied phases. It had occurred to him as very singular that, bearing in mind the large number of societies, having a somewhat similar constitution, which must have employed the services of actuaries for

a great number of years, they should have had so seldom brought before them the inner workings of those bodies. He had in mind the large railway superannuation funds, the many banking schemes, and orphans' and widows' funds, all of which must have employed actuarial advice, but with regard to which their literature was very deficient at the present moment. In addition to the new material produced by Mr. Ackland, he had touched upon a subject which one day or another might come to be of very great importance, namely, the effect of the rate of discontinuance upon valuation reserves, annuities, net premiums, and so on. The rate of discontinuance was a very volatile element, being affected, in the case of proprietary companies, sometimes even by Stock Exchange fluctuations; and in the case of mutual societies it might rise or fall according to some temporary or trivial cause. Now, when they came to consider whether it was wise to employ a mercurial element of that sort in the financial estimates of the liabilities of a life assurance company, they might hesitate before adopting such a course, as involving too much uncertainty, if not danger. But for all that, there was no doubt that, whether they thought it expedient to employ that element in the valuation of a single company or not, it was none the less a factor which had had its due effect on the results brought out by the valuation. It had occurred to him that a most interesting investigation might one day be made, when they had the "New Experience" before them, into the effect of the rate of discontinuance upon the financial results of life assurance offices, or life assurance itself in general. He remembered reading, in an address by Mr. Sorley, President of the Actuarial Society of Edinburgh,* that, notwithstanding a greater fall in the rate of interest, or the total disappearance of the rate of interest, life assurance companies would still be able to carry on their functions. Mr. Sorley also gave a very interesting table of the net premiums which would be required to be charged upon the assumption that capital would earn no interest whatever. He (Mr. Ryan) thought that a useful corollary to that would be to consider how the rate of discontinuance would act in modifying Mr. Sorley's net premiums. It was clear it would operate in aid of life companies—he meant companies in general—as a partial set-off against the fall in the rate of interest. It was a force which clearly acted in one direction, while the reduction in the rate of interest operated in another.

Mr. R. TODHUNTER said that the case for the Nearest Duration Method would be strengthened if they could see their way to regard the question of discontinuance from a somewhat different point of view from that usually adopted. The author, looking at the matter from the customary point of view, regarded withdrawals as due to a force similar in its operation to mortality. It seemed that that view might be questioned. The force of withdrawal was really a periodic, and not a continuous force; it operated at certain particular epochs in the history of the policy. Of course, if they made up their experience by calendar years, the withdrawals would be spread over the year; there would then be something in the experience similar to a continuous

*"The Evolution of Premium Rates: a chapter in the History of Life Assurance." (*Transactions of the Actuarial Society of Edinburgh*, vol. iii, No. 14.)

force of withdrawal, but if it was made up by policy-years there was a discontinuous force of withdrawal. The withdrawals arose from lapses and from surrenders. Lapses took place at one of four epochs—at the end of either the first quarter, the half-year, the third quarter, or the year. It might be objected that they did not take place then, but at the expiration of the days of grace; but that simply deferred the epoch one month in each case. It did not, therefore, seem satisfactory, so far as the lapses were concerned, to treat the rate of withdrawal as though it were strictly analogous to the rate of mortality. It was a rate that came into operation at one of four specific intervals in the history of the policy. Four times a year the holder of a policy at a quarterly premium had the option offered to him to pay or not to pay, and he exercised that option; it was not an option which he exercised continuously throughout the year. Then there was the question of surrenders. Surrenders might be scattered over a year, but he would submit that a very large proportion of them were located at the same point of time at which the entire body of lapses was located, namely, at the time when the premium was renewable. How did the Nearest Duration Method apply to the calculation of these periodic rates of withdrawal? In discussing that question he would take the Nearest Duration Method as exhibited in Mr. Ackland's Section (B), the section in which the experience was limited by policy years, so that the only class of cases in which there would be fractional exposures (leaving the deaths out of account) would be the withdrawals. The author had taken a typical case of 400 policies, of which 250 were renewable yearly, 130 half-yearly, and 20 quarterly. Of the half-yearly lapses, one-half would occur at the end of the year; and in the same way one quarter of the quarterly lapses would occur at the end of the year, and they would have altogether four out of every five lapses actually occurring at the end of the policy year. The proportion would be still larger, assuming Mr. Chatham's proportions of yearly, half-yearly, and quarterly cases to be more nearly correct than Mr. Ackland's. The Nearest Duration Method consequently left four-fifths of the withdrawals exactly where they occurred. It collected the remaining one-fifth at the end of the year, taking one-tenth from one side and one-tenth from the other. If the withdrawals were regarded as occurring at renewal dates, instead of as spread over the year, then they would have a much simpler task in applying the Nearest Duration Method, for the "Exposed to risk of withdrawal" could be derived from the "Exposed to risk of death" by simply adding on the withdrawals. He did not know whether he had made his point clear, but what he was contending for was (1) that the rate of withdrawal might reasonably be regarded as a rate operating at a definite moment of time, instead of as a rate operating over the year; and (2) that they would be able to calculate the rate, if they regarded it in that way, simply and sufficiently accurately from an experience compiled by the Nearest Duration Method.

Mr. JAMES MEIKLE had given the subject of tabulation some consideration, having worked on it in all sorts of ways, including the Institute Method of 1863. He had also thought out a method for himself, which he had called the "Exact Duration Method", though

he did not know whether it was the same method which Mr. Ackland referred to as the *Exact Duration Method*. He had also used the *Nearest Duration Method* of Dr. Sprague on two or three occasions, and found that it was a very nice method. He, however, did not think that Mr. Ackland had touched the difficulties of the point at all. If they had only one life, one entrance, and one exit, it was a beautiful method; but if they had one life and many entrances they perhaps had many exits which did not always harmonize. It was all very well if they were going to tabulate facts for each age at entry, but if they were to amalgamate them all into one aggregate rate of mortality they would not exactly fit in. It was, for instance, proposed to take out the experience of endowment assurances maturing at the several ages 50, 55, &c. But according as they took the ages—that is, according to the relation of the ages at entry to the durations, the policies might mature at ages 49, 50, or 51, or at 54, 55, 56, &c., which was very awkward. Again, if a life had many policies, and that life died in 1893, it might be that, by some cards, the life was living, and by others that the life was dead. This also was very awkward. It seemed to his mind most essential (not that it had much weight on the resultant table of mortality) that the tabulation should be such as would satisfy the mind of the statistician. To tabulate as a fact that the policy had matured at 49, when it was well known that it did not mature till 50, was most unsatisfactory, and it did not cancel that dissatisfaction by being told that it would be balanced by another policy being tabulated as maturing at 51. The facts were that the mortality tabulated was the mortality between an age six months younger and six months older than the integral age. Of all the various methods that have been proposed, he liked the *Exact Duration Method* best, because it is not dependent on any assumptions whatever.

The PRESIDENT said he would ask the members to accord a hearty vote of thanks to Mr. Ackland for the very great trouble he must have taken in preparing the enormous mass of material, formulas, and notations given in the paper. Mr. Ryan, in the course of his observations, had expressed regret that more reports of the valuations of widows' funds, and of funds connected with public institutions of the nature mentioned by Mr. Ackland, had not been brought before the Institute, but it must be remembered that those reports were for the most part confidential, and that unless the express permission of the department which carried on the transactions referred to was obtained it would not be right that they should be disclosed in public, so that the private affairs of the members of these societies might be recognized, and used perhaps in some way which the subscribers, whether rightly or wrongly, would consider detrimental to their interests. Mr. Todhunter had made some very interesting remarks upon the periods when discontinuances would take place. Mr. Ackland had, it would be remembered, referred in his paper to the fact that discontinuances were a function of the duration of the policy, as well as of the age of the person assured. With regard to discontinuances, it would be found, for instance, on investigating a widows' fund in which the annuity to the widow ceased on her marriage, that the rate of re-marriage was distinctly a function of

the duration of the widowhood as well as the age of the widow. A very troublesome result might be discovered from the new investigation, namely, that separate annuity tables would have to be calculated for every age of entry. In this case, the question whether a man held fifty policies or one would be a matter of indifference; they would only have to consolidate those which he took out in the same policy year.

MR. T. G. ACKLAND, in reply, said he was very much indebted to the meeting for the kindly way in which his paper had been received. The President had usefully summarized some views expressed in the paper in saying that he (Mr. Ackland) had a decided preference for the Nearest Duration Method, and also for that method of computing the age at entry which took the nearest age at the actual date of entry. He (Mr. Ackland) might add that the most suitable method of tabulating an experience of assured lives appeared to him to be to deduce the whole of the facts in separate ages at entry and separate years of duration. Mr. Ryan referred to the Exact Duration Method, and no doubt that method was in many respects satisfactory in dealing with a large body of facts; and where the labour, expense, and trouble involved in going through its several processes could be undertaken, the great advantage was secured that no assumptions whatever were made as to the facts and data. There appeared to be a little confusion in the minds of some speakers with regard to Part I. of the paper, dealing with the "period of observation as limited by calendar years." Members, on looking more carefully into the paper, would see that he had not at all gone into the question of investigating and tabulating the experience of assured lives in calendar years. He had assumed from the beginning that they were to be investigated in policy years or years of duration; but the *period of investigation* might be limited by calendar years, and that was an entirely different thing. It was sometimes essential, as, for example, in the case of an investigation of the mortality experience of an office over a quinquennium, to deal with a period of observation covering an integral number of calendar years: but this did not, of course, prevent the observation being made according to the method of policy years, or years of duration. He was glad that the symbols and notation employed throughout the paper had met with the approval of the members. There appeared to be a great deal worthy of notice in Mr. Todhunter's remarks, and he entirely agreed with him that the rate of discontinuance, and the force of discontinuance, were somewhat special in their character, and that they took effect in epochs in the year of duration or policy year, and not as continuous functions. The President had referred to the fact that one reason why they had not had before them more contributions dealing with such subjects as clerks' associations, pension funds, widows' pension funds, and the like, was that the reports made were necessarily confidential in their character. He might say that, in laying before the Institute such facts as he had done, in connection with the particular Association dealt with, he had not indicated any conclusions dealing with its financial matters, and also that he had obtained the courteous consent of the Board of Directors of this Association to the publication of such facts as he had included in his paper.

THE INSTITUTE OF ACTUARIES.

SYLLABUS OF PRIZES, 1896,

OFFERED BY MR. JAMES CHISHOLM.

THE Council of the Institute of Actuaries has the pleasure to announce that Mr. James Chisholm has kindly offered Three Prizes, of the value of *Thirty Pounds*, *Fifteen Pounds*, and *Ten Pounds* respectively, for the three best Essays on the following subject, namely:

“The Relation of the Actuarial Profession to the State.”

The subject should be discussed in connection with:

- (i) The history in the past of the Actuarial Profession in relation to the State, with respect, for example, to the grant of Annuities and Assurances by the Government, and to professional advice upon Legislation affecting Assurance Companies and Friendly Societies;
- (ii) The direction in which professional services to the State might be extended, with reference, for example, to the Preparation and Analysis of Vital Statistics; the consideration of Old Age Pension Schemes; the Insolvency and Reconstruction of Assurance Companies and Friendly Societies; the formation of State, Municipal, and other Sinking Funds; and Currency Proposals.
These examples are to be regarded simply as *illustrations* of the questions which should be considered.
- (iii) The several elements of the subject should be discussed in connection with the Limits of State Control; the Maintenance of Freedom of Enterprise by the Companies and the Public; and the aid and protection which should be accorded by the State to the Profession in the performance of the suggested duties.

CONDITIONS OF THE COMPETITION.

1. Reference must not be made to individual Companies by name.
2. The Essays must be sent to the Honorary Secretaries of the Institute of Actuaries, at Staple Inn Hall, not later than 31 December 1897.
3. The Adjudicators shall be the President and Vice-Presidents of the Institute.
4. Each Competitor must send in his name, under seal, with a Motto corresponding to one to be prefixed to his Essay. Such Motto and Essay must not be in the handwriting of the Competitor, and should be, where practicable, type-written.
5. Successful Essays shall become the property of the Institute.
6. Unsuccessful Essays will be returned, on application at the Institute, with the corresponding envelopes unopened.
7. No Prize will be awarded unless the Adjudicators consider some Essay or Essays worthy of the distinction.
8. The Competition shall be open to all members of the Institute, except members of the Council and members who have already gained Prizes for Essays offered either by or through the Institute.

JOURNAL

OF THE

INSTITUTE OF ACTUARIES.

On some Legal Points arising in Life Assurance Practice. By
ARTHUR RHYS BARRAND, F.I.A., *of the Prudential Assurance*
Company.

[Read before the Institute, 21 December 1896.]

THE subject I have chosen for my paper may seem to call for some apology, on the ground that it is more suitable for discussion at a legal institute than by a body of Actuaries. There would doubtless be some force in such a criticism, and yet I venture to think that in many respects the subject is not an unsuitable one to be brought before this Institute. The time has gone by, if indeed it ever existed, when the ordinary legal incidents arising in connection with life assurance business were submitted to the company's solicitors for advice and direction. In a large life assurance institution it would be impossible, in practice, to submit every question as to title, in the case of claims and surrenders, to a solicitor, on account of the expense and delay thereby occasioned, if for no other reasons, and at least as great practical difficulties would arise if, in order to get over the difficulty of the expense of such a reference, it were sought to saddle the applicant with such legal expenses as might be incurred. Questions of title, unless they be of a peculiarly complicated nature, are therefore usually settled by an official of the company, and since amongst the officials the Actuary will, as a rule, stand almost alone as having, of necessity, some legal knowledge, the ultimate decision on such points will, in most cases, devolve upon

him. I need hardly remind you that this Institute fully recognizes the advisability of a fair amount of legal knowledge in its members. This is abundantly shown in its examinations, its library, and the pages of its *Journal*, and whatever may be thought as to the width of the gulf dividing legal from actuarial matters in theory, I think you will agree with me that few portions of an Actuary's mental stock-in-trade are more useful to him in practice than his knowledge of law in its bearing on life assurance and subjects arising in connection therewith. In saying this I am, of course, regarding an Actuary as not merely a glorified arithmometer, concerned only with the mathematical aspects of life assurance, but, as I take it, he should be, in the opinion of this Institute, as an expert, thoroughly qualified to deal with every question that may arise in life assurance practice.

SCOPE OF THE PAPER.

Without further apology for the subject of my paper, I will proceed to indicate the limits within which I propose to confine myself. In the first place, I need hardly say that these notes are intended rather for the students of the Institute than for the senior members, the latter being, doubtless, perfectly familiar with most of the points to be discussed. They are intended, moreover, not so much to help students in preparing for the legal portion of their examinations, as to give some little assistance and guidance to those who have not many opportunities of gaining such information by practical experience in dealing with the many legal questions that arise in an ordinary assurance office. I need hardly say that I have no intention of discussing, even in the barest outline, the law relating to life assurance in its entirety. Much general information on the better known aspects of the subject can so easily be obtained from text-books that it would be wasting your time to discuss them now. My object is rather to deal with those points on which it is difficult or, in some cases, impossible, to find any direct information in the ordinary legal text-books, but which nevertheless arise, not infrequently, in practice. Even this limitation is not sufficient however, for many points which would fairly come in this category have already been fully discussed at this Institute, notably in the paper read by Mr. William Kent Lemon, in February 1891, and in the discussion which followed. Such points I do not propose to refer to on this occasion, except where fresh light has been thrown upon the matter since, or where, in one or two instances, it has

seemed necessary to do so in order to give a certain completeness to the treatment of some portion of my subject. I feel it necessary that I should call your attention to these limitations that I have imposed upon myself, in anticipation of the criticism that many legal points of great importance have been left wholly untouched by me. I may add, also, that the methods suggested in some few instances will not necessarily be those that might seem to be best from a strictly legal point of view. There are, however, many cases which, if treated from that point of view, would involve an immense amount of trouble and no little expense to the parties concerned, but which can be treated in a far simpler manner if the company is prepared to waive some portion of its legal rights in the matter of title, and this can frequently be done without incurring any appreciable risk. Such questions as these are essentially those which come before the Actuary and Manager for decision, even though, in the first instance, the matter may have been submitted to the company's legal advisers. The latter can only show what title the company is entitled to demand, but it is for the Actuary, balancing the risk run against the gain in reputation to the company for the prompt settlement of its claims, to decide what must be insisted upon and how much may be waived.

A secondary end to be gained by such a paper as this is to bring to light, in the discussion, differences of practice between offices on certain points, and perhaps, as a result, to assist in bringing about that uniformity of practice which is greatly to be desired; and if it should appear that many offices have experienced some grave defect in the present law on any point, such common experience may lead to united action on their part, and so effect a change in the law. A recent case in point is to be found in the *Life Assurance Companies' (Payment into Court) Act, 1896.*

MORTGAGES.

The commonest legal questions that arise in the ordinary routine of the chief office of a life assurance company are probably those in connection with mortgages. It has been before remarked how soon after its issue a policy becomes the subject of an assignment of some form or another, and this tendency, so far from showing any decrease, seems to be increasing even more rapidly than life assurance itself. Such a tendency is not to be wondered at, for, apart from the fact that a policy of assurance forms an admirable security for the amount of its surrender-value, and that the latter is continually increasing, there are many cases

arising in ordinary commercial transactions in which a policy affords exactly the security needed. For instance, a bank may have perfect confidence in the ability and integrity of a customer, and be satisfied that, as long as he lives, his personal security will be sufficient. If, however, as is often the case, the value of the business depends mainly upon its owner, the security may vanish with his life, and, under such circumstances, the personal guarantee and a life policy, together, furnish the bank with a completely satisfactory security. That this is the view taken in many instances may be gathered from the fact that, in perhaps the majority of cases, when the mortgagee surrenders the policy on account of default, it is found that the surrender-value forms but a small portion of the debt due.

**Notice of
Assignment.**

The first question that naturally suggests itself in connection with assignments is that of notice, and one or two practical difficulties that often arise in connection therewith may be referred to. Notice of a deed of assignment is sometimes given to an office, and on subsequent dealings with the policy, it is stated that the transaction was not carried out, and that no such deed was executed. Where the notice is of recent date, no particular difficulty will arise as a rule, it being usually possible to obtain a letter from the party giving such notice certifying that the deed in question never existed. Such a letter could safely be accepted, and the title to the policy treated as if the notice in question had never been given. The matter is not, however, so simple as this in every case, as it sometimes happens that the notice was given twenty or thirty years ago, and it is now impossible to trace the person giving such notice. In such a case, the only safe course open to the company, apart from the payment of the money into Court, is to take a statutory declaration setting forth the facts as to the transaction, and then, on the person otherwise entitled to the policy producing it, and furnishing the company with a good indemnity, the amount of claim or surrender-value can be paid with but little risk, and this is the method which is generally adopted under the circumstances. If, however, the applicant is unable to produce the policy, this plan is hardly advisable, and it will be better to pay the amount into Court.

**Absence of Re-
assignment.**

A closely analogous case to this is that in which notice is given and the policy actually assigned, but on the debt being paid off no re-assignment is taken, and the original mortgagee has died or disappeared. Such a case came under my

notice a short time ago, in which the original notice was given some forty years before, and several transactions with the policy had taken place since. The charge, of which notice had been given, had been cleared off, it was stated, within a year or two of its creation, but no re-assignment had been taken, and no trace of the original mortgagee could be found. In such a case, if the policy can be produced by a person whose title, apart from the transaction referred to, is complete, the matter may be fairly dealt with in the same way as that suggested where no deed had been executed, that is, by taking a statutory declaration and indemnity. In this case, however, the declaration should contain a statement, if, as is usually the case, the original assignment deed cannot be produced, that the missing deed was properly stamped. If the transaction is of recent date, or the missing deed is known to have been unstamped or insufficiently stamped, the safest course would seem to be for the company to pay the amount into Court.

Can notice
of assignment
be cancelled?

Another question that arises in connection with notices of assignment is as to whether, when notice of a deed has once been given, it can be cancelled by the person giving it, or whether such a deed must always remain as part of the title, and be required in any future dealings with the policy. The way in which such a case frequently arises in practice is when the original deed has not been stamped within the proper time, and in order to get over the difficulty, a fresh deed is executed. An instance of this occurred recently, in which notice was given of a mortgage in favour of a bank, and shortly afterwards, a letter was received stating that the deed in question had been cancelled, and withdrawing the notice already given. At the same time, notice was given of a new deed between the same parties, and it was made apparent that the reason of this action was that the first deed had not been stamped within the time allowed. It is clear, in view of the opinion taken by the Life Offices Association, in 1890, that this would be an evasion of the Stamp Act, and that the only proper course to be adopted was to have the original deed duly stamped, and the penalty paid. As, however, a second deed had been executed, and notice thereof given, this latter had also to be stamped, and henceforth both deeds formed part of the title to the policy. It would seem, therefore, that apart from the question as to whether the company would be protected by the Act of 1867, as being without notice of a transaction, of which the notice had been cancelled, it should require to see any deeds of which it had

a record, in order to satisfy itself that they were properly stamped. A different course to that indicated above has, however, sometimes been pursued under similar circumstances. On such a case arising, application was made to the Inland Revenue authorities by the mortgagee, asking that the notice of the mortgage deed first executed might be cancelled by the Assurance Company, as it had not been stamped within the proper time. In reply, a letter was received from Somerset House, stating that, as far as they were concerned, they had no objection to the Company expunging from their books the recorded notice of the assignment, and that no claim for duty, or penalty, in respect of such assignment would be raised. This, of course, is not to be taken as a general sanction to such a practice, but only as showing that in any particular case, upon the presentation of a proper memorial, setting forth the facts, the Inland Revenue authorities may be prepared to forego any penalty that may be incurred by the cancelling of a notice of assignment under such circumstances. In the instance to which I have here referred, the Assurance Company acted upon the authority given, and treated the case as if the original deed had never existed.

**Re-assign-
ments.**

Reference has been made incidentally to the absence of a deed of re-assignment, and this is one of the commonest defects in title occurring in practice, more particularly with policies of comparatively small amount. If the transaction is a large one, it is probable that the matter will pass through the hands of the solicitors of the parties, but where it is only a small transaction this expense is not likely to be incurred, and in such a case, the mortgagor, on payment of the mortgage debt and interest, is usually satisfied with a receipt for the money so paid and the return of the policy. I am not sure that assurance companies are not themselves to blame to some extent, for such defects in title, as, in granting loans on their own policies by way of equitable mortgage, they do not, usually, execute a re-assignment when the loan is paid off, but simply give a receipt for the amount and hand back the policy. When the mortgage is by deed, or the policy has been actually assigned to the mortgagee, it is usual to insist upon a formal re-assignment, though, if from exceptional circumstances, it is difficult or impossible to obtain this, payment would generally be made on the production of satisfactory evidence showing that the debt had been discharged. If the mortgage is only an equitable one, a proper re-assignment would appear to be the exception rather than the rule and

evidence that the debt had been cleared off, such as a receipt and a letter from the mortgagee stating that he had now no claim on the policy, would probably be accepted in every case in lieu of re-assignment. In accepting such evidence in place of a proper deed, the company will not, of course, render itself liable to a penalty under the Stamp Act, as the special clauses referring to the stamping of assignments of policies only apply to documents that have had an existence, and not to deeds which have never been executed and which the company may agree to dispense with.

**Payment to
Mortgagee.**

We can now consider the case in which the mortgagee of a policy applies for the payment of the sum assured, the surrender-value, or the cash value of the bonus. If the mortgage is under seal, and is in order in other respects, no difficulty arises with regard to the payment of the sum assured, as under Section 22 of the Conveyancing Act, 1881, the mortgagee has power to give a good receipt for any money arising under his security, in which category the policy-money manifestly comes. Some alarm was created on the subject a few months ago by the decision of the Court of Appeal in the case of *Jeffrey v. Sayles* (L.R. 1896, 1 Ch. 1), in which it was held that a trustee could not be compelled to pay over the whole fund to the first mortgagee, but could distribute it according to the rights of the various parties entitled to share in the fund. Upon this decision some companies altered their practice, and declined to pay to the mortgagee without the concurrence of the personal representative of the mortgagor in the receipt. The better opinion seems to be, however, that there is no necessity for any such alteration in practice, and that the mortgagee's receipt perfectly discharges the company. It is difficult, indeed, to come to any other conclusion in view of the very clear and explicit declaration in the clause of the Act of 1881, to which I have referred.

**Payment
underequitable
mortgage.**

A point on which there is a greater difference of opinion is as to whether a good discharge can be given for the policy-money by a mortgagee whose security is an equitable mortgage, or an assignment by way of mortgage under hand only. Equitable mortgages arise in one of two ways. Either the mortgagor has nothing but an equitable interest to convey, as in the case of a "cestui que trust", and of a second mortgage when the first is by deed; or the document is in a form recognized only by equity, as when a simple form of charge under hand only is used, or the mortgage is merely by deposit of the policy. It is with the latter class of equitable mortgages that we are now concerned,

and in dealing with such there appears to be a considerable difference both in opinion and practice. Some offices hold that, in the case of a claim under such circumstances, the signature of the personal representative of the mortgagor is necessary, as well as that of the mortgagee, and insist upon having it, but as the powers contained in Section 22 of the Act of 1881 are not expressly limited to mortgages under seal, as is the case in Section 19, many hold that the words "mortgage" and "mortgagee" must be taken in the very wide sense of the definitions contained in Section 2, Sub-section 6. Some offices, taking this view, pay the policy-money to the equitable mortgagee on his receipt alone and appear to be perfectly safe in so doing. The point is of considerable importance owing to the fact that many banks, when making an advance on a policy or accepting it as partial security for a current account, do so by way of equitable mortgage.

Surrender by Mortgagee. Some slightly different points arise when the application is not for the policy-money but for the surrender-value.

If the mortgage is under seal, the mortgagee can exercise his power of sale on default being made, and there can, I think, be no doubt that under Section 21, Sub-section 2, of the Conveyancing Act, 1881, if a proper conveyance were used, there would be no necessity to show in any way that the power of sale was exercisable, except that in the deed it should be expressly stated that the conveyance was made in exercise of the power of sale. As, however, it is not the practice of an assurance company to take anything but a receipt for the surrender-value in such cases, it is usual to require the mortgagee to show, by statutory declaration, that default has been made. It is not easy to see, however, that the position of the company is much improved by the production of such a declaration—at least, as far as the mortgagor is concerned; and if it be decided to dispense with a formal conveyance, it would seem to be quite sufficient that the mortgagee, on applying for the surrender-value, should state that he did so in exercise of the power of sale conferred by the Act. This power of sale arises (a) when the mortgage-money has been called in and remains unpaid for 3 months, (b) when the interest is in arrears for 2 months, or (c) when a breach of some provision, contained in the deed or in the Conveyancing Act, has been committed. It must, however, be borne in mind that where the deed confers powers of sale, more extensive than those conferred by the Act, the former will over-ride the latter. It may be noted

in reference to the third form of default, that when the mortgage of a policy contains a covenant for the payment of premiums by the mortgagor, as is often the case, the non-payment of the premiums will constitute a good ground for the exercise of the power of sale. In such a case, when the policy is lapsed, the Company has, in its own books, sufficient evidence of default, and need not, therefore, call upon the mortgagee to furnish such evidence, even if it is accustomed to do so in other cases.

Statutory
powers of sale
limited to
mortgages
under seal.

The provisions of the Conveyancing Act as to power of sale on default or breach of covenant are, however, expressly limited to mortgages under seal; and when, therefore, application is made for the surrender-value by a mortgagee whose security is under hand only, it cannot be safely paid without the concurrence of the mortgagor, unless the document creating the charge contains an express provision conferring the statutory or other powers of sale. As I have already indicated, equitable mortgages and assignments by way of mortgage under hand only, derive much of their importance, as far as assurance companies are concerned, from the fact that these are forms of security largely used by banks in advancing upon policies; and, although one would naturally think that such institutions would take every possible precaution that common prudence or legal advice could suggest, to completely secure themselves, it is, nevertheless, a fact that a number of banks neglect to insert in their form of charge any provision for sale or surrender on default.

Assignments
absolute in
form but
intended as
mortgages.

In some instances this is due to the fact that the assignment is in the form given in the Schedule to the Act of 1867, which is, in appearance, an absolute assignment. It is the practice of many banks to stamp their security for the face value of the policy, and not for the real consideration. When this is done, and the deed is absolute in form, it is often impossible to tell whether it is intended to be stamped as an absolute assignment for the actual consideration shown, or as a mortgage on the basis of the sum assured, the actual stamp being sufficient for either case. Under such circumstances it might be contended that the assurance company could get a complete discharge for the surrender-value from the bank, on the ground that the latter showed an absolute title to the policy. It seems more probable, however, that a principle, analogous to that involved in the case of the *Earl of Sheffield v. London Joint Stock Bank* (L.R. 13 Ap.,

Ca. 333) would be held to be applicable, and that, from the nature of a bank's business, the assurance company should gather that the assignment was only by way of mortgage and not absolute. It would seem, therefore, that when a bank applies for the surrender-value under a document that is under hand only, and there is no express power of sale, the concurrence of the mortgagor must be obtained whether the document is absolute in form or not.

**Payment of
Bonus to
Mortgagee.**

There remains to be considered the case in which application is made by the mortgagee to surrender the bonus for cash. In the few instances in which the bonus is declared in cash, and is payable as such, unless the policyholder expressly requests that it may be applied as reversion or reduction of premium, it would appear that the mortgagee could give a good discharge for it as money arising under his security, and this, as already indicated, is held by many to apply whether the mortgage is under seal or under hand only. In the more usual case, however, where the bonus is declared as reversion, the surrender of the bonus by the mortgagee would amount to the sale of a portion of his security, and unless default has been made, or his deed confers special powers, he has no right to do this. If, however, his mortgage is under seal, and default has been made, he has power under Section 19 of the Conveyancing Act to sell the mortgaged property or any part thereof, and can therefore surrender the bonuses, leaving the policy still in force. Of course, whatever precautions are adopted by the assurance company as to the exercise of power of sale in the case of the surrender of a policy will also be applicable here. One useful application of this method of surrendering the bonuses will arise when there is a covenant on the part of the mortgagor to pay the premiums. In such a case, the non-payment of the premiums will constitute, as we have seen, a sufficient ground for exercising the power of sale, and the mortgagee, if he wishes to keep the policy on foot, can arrange with the assurance company that if the premium is not paid within the days of grace, then immediately at the expiration of that time, a sufficient portion of the bonus shall be surrendered to pay the premium due, the mortgagee's receipt being, under the circumstances, a sufficient discharge to the company. The mortgagee has the right to pay any premiums necessary to keep the policy in force, and add any amounts so paid to the principal, even though there be no special covenant authorizing him to do so; but, in spite of this power, it will often be found more convenient to adopt

the method of surrendering the bonus above referred to. Apart from these somewhat exceptional cases, the surrender of any portion of the bonus can only be allowed, with safety, on the joint receipt of the mortgagor and mortgagee.

BANKRUPTCY.

The next cases I propose to consider are those arising in connection with bankruptcy. These usually occur in one of three ways: (a) the bankruptcy of the policyholder; (b) the bankruptcy of the mortgagor of a policy; (c) the assignment by the policyholder of his property to a trustee for the benefit of his creditors. All these are common cases in practice, and all, except perhaps the first named, present features worthy of note. In the first case, as you are aware, on the policyholder being adjudged bankrupt, the policy will immediately vest in the Official Receiver, and remain so vested until the appointment of a trustee by the creditors, and, until such appointment, his receipt will be a good discharge to the company for any payments under the policy. It is not usual, however, for the Official Receiver to attempt to realize such securities as life policies in ordinary bankruptcy cases, there being as a rule no reason for immediate realization; and the application will therefore usually come from the trustee. The title to be shown by the latter will consist of the order of adjudication, and the Board of Trade certificate of the appointment of trustee, and office copies of these documents should be retained by the company. It may be noted that the Official Receiver cannot act as trustee, except in the following cases: (1) until a trustee is appointed; (2) during any vacancy in the office of trustee; (3) after the release of the trustee; and (4) in small bankruptcies. In these so-called small bankruptcies, that is, where the estate does not exceed in value, £300, a different form of procedure may be adopted, known as Summary Administration. In such cases, on the Official Receiver reporting to the Court that the property of the debtor is not likely to exceed £300, an order for Summary Administration will be made, under which the Official Receiver will act as trustee, and proceed to wind up the estate as quickly as possible. Under such circumstances, the title to be shown by the Official Receiver, on applying for any payment under the policy, will consist of the order of adjudication and order for Summary Administration. As, however, the creditors have

Summary Administration.

still power, at any time by a special resolution, to appoint some person as trustee, other than the Official Receiver; it is usual to ask for a letter from the latter, stating that no one, other than himself, has been so appointed.

**Courses open
to mortgagee
on bankruptcy
of mortgagor.**

In the case of the bankruptcy of the mortgagor of a policy, four courses are open to the mortgagee. He may (1) rely upon his security, if the surrender-value exceeds the mortgage debt; (2) realize his security and prove for the balance of the debt; (3) give up his security and prove for the whole debt; or (4) he may value his security and prove for the balance. In the first of these cases the situation remains practically unaffected by the bankruptcy, except that where the mortgage deed contains a minimum limit to the duration of the mortgage, such limit will be removed by the bankruptcy of the mortgagor. In the second case the procedure will be the same as that already described with regard to the surrender of a policy by the mortgagee. In the third case the policy will be re-assigned by the mortgagee to the trustee, who will then be able, on the production of the mortgage deed and re-assignment and his title as trustee, to deal with the policy in any way. The fourth case, however, presents somewhat greater difficulty. The Bankruptcy Act, 1883, in the second Schedule, after providing that the trustee may, at any time, redeem the security on payment to the creditor of the assessed value, or may require it to be offered for sale, goes on to state, in Section 12 of that Schedule, Sub-section (c), "Provided "that the creditor may, at any time, by notice in writing, require "the trustee to elect whether he will or will not exercise his power "of redeeming the security or requiring it to be realized; and if "the trustee does not, within six months after receiving the notice, "signify in writing to the creditor his election to exercise the "power, he shall not be entitled to exercise it; and the equity of "redemption, or any other interest in the property comprised in "the security which is vested in the trustee, shall vest in the "creditor, and the amount of his debt shall be reduced by the "amount at which the security has been valued." From this it appears that the trustee will retain his right to redeem the security at the assessed value until he has been challenged to make his election as to whether he will or will not exercise it, and has either directly, or by default, abandoned it. If, therefore, he is not thus challenged, and ultimately the security becomes greatly increased in value, as for instance, by the death of the life

**Trustee's right
of redemption.**

assured, the trustee can redeem it at its assessed value on payment, in addition, of interest, premium, and other necessary expenses incurred by the mortgagee (*ex parte* King, 20 L. R. Eq. 273, *Bolton v. Ferro*, 14 C. D., 171). If, however, the trustee is challenged to state whether he will or will not redeem the security, and he does not exercise the right within six months, the entire interest in the security will vest in the mortgagee, and if the subject of the mortgage is a life policy, the assurance company can safely treat the mortgage as if it were an absolute assignment and the mortgagee as the sole owner of the policy, and this view is acted upon by many offices, if not by all. If the trustee has directly renounced his right of redemption, there is no difficulty, as the only title required in that case by the mortgagee, to enable him to deal in any way with the policy, will be his mortgage deed, office copies of the order of adjudication and appointment of trustee, and a letter from the latter disclaiming

**Abandonment
by trustee of
right to
redeem.**

any intention of redeeming it. When redemption has been abandoned by default, that is, when the mortgagee calls upon the trustee to decide whether he will exercise his power of redemption, and the latter takes no action in the matter for six months, the title would be the same as in the preceding case, except that in lieu of the letter from the trustee, a statutory declaration should be made by the mortgagee setting forth that the requirements of the Bankruptcy Act, referred to above, had been complied with. It may be noted that, under the circumstances here considered, a document only bearing the stamp for a mortgage becomes a valid and properly stamped absolute assignment.

**Assignment
for benefit of
creditors.**

In the case where a policyholder assigns his property to a trustee for the benefit of his creditors, the chief point to be noticed is that under Section 4, Sub-section (a), of the Bankruptcy Act, 1883, such an assignment is an act of bankruptcy, and any creditor who has not concurred can avail himself of it to have the assignor adjudged bankrupt. If this be done, the title of the trustee in bankruptcy who may be appointed will not be limited by the date of his appointment, or even by the date of the receiving order or adjudication, but will relate back to the time of the commission of the act of bankruptcy on which the receiving order or adjudication is made. It is, of course, practically impossible for an assurance company, when called upon to consider the title of a trustee based on such a deed, to ascertain whether or not

every creditor has concurred in it, and under these circumstances it is necessary to postpone any payment to such trustee until three months have elapsed from the date of the deed. Any payment can then safely be made as, by Section 6, Sub-section (c), of the same Act, a bankruptcy petition can only be presented in respect of an act of bankruptcy which has occurred within three months of the presentation of the petition. The doctrine of the relation back of the title of the trustee in bankruptcy does not apply, provided the transaction takes place before a receiving order is made, and the party to the transaction, other than the debtor, has no notice of any available act of bankruptcy committed by the debtor before that time. This exception is, however, of no use in the case we have been considering, as the title produced by the trustee is itself an available act of bankruptcy.

Stamping of
such assign-
ment.

In passing, it may be noticed that such an assignment as I have here referred to, must be registered as a Bill of Sale, and stamped with 2s. 6d. in connection therewith. It must also bear a 10s. stamp, and a further one representing the ad valorem duty of 1s. per £100, payable on the actual value of the estate conveyed. As this latter value is unknown to the assurance company, it is impossible to check the sufficiency of the third portion of the stamp duty paid without making further enquiry, but provided the first and second portions are correct, it is usual to assume that the third portion is also correct, and to pass the deed as properly stamped. It must be remembered that the deed of assignment here discussed is a debtor's private deed of arrangement, and must be distinguished from one made under the sanction of the Court, after a receiving order has been made. In this latter case, if a majority in number and three-fourths in value of all the creditors who have proved, resolve to accept the proposal, it will, when approved by the Court, be binding on all of them.

Administra-
tion in
bankruptcy of
estate of
person dying
insolvent.

Another case to be noticed before leaving the subject of bankruptcy is that in which a man is found to be insolvent at the time of his death. Under the Act of 1883, a creditor of a deceased debtor whose debt would have been sufficient to support a bankruptcy petition against such debtor had he been alive, may now present to the Court a petition for the administration of the estate according to the law of bankruptcy, and the Court, on being satisfied as to the insufficiency of the estate to pay the debts, will make the necessary order. Such an order will operate in the

same way as an order of adjudication, and will over-ride probate or letters of administration if either of these has already been granted. If, therefore, an assurance company has notice that such an order has been made, the title to be produced will not be the ordinary grant of representation, but the order for administration and the certificate of appointment of the trustee, the payment then being made to the latter.

**Exemptions
from stamp
duty in
bankruptcy.**

As you are doubtless aware, there are special exemptions from stamp duty in the case of documents relating to the estate of a bankrupt, under which, amongst other things, conveyances to the bankrupt or his trustee do not require stamping, and this also applies to receipts given by the trustee for any amounts paid to him. It does not, however, apply, apparently, to conveyances by the trustee to other people, as the stamping of such documents devolves on the assignee and the exemption appears to be intended solely to save expense in the administration of the bankrupt's estate.

SETTLEMENTS.

Another subject calling for brief notice is that of settlements, voluntary and otherwise. The chief difficulty arising in connection with a voluntary settlement is its liability to be defeated by the creditors of the settlor under certain conditions. These conditions are that the settlement shall become void as against the trustee in bankruptcy, (a) should the settlor become bankrupt within two years of its execution, or (b) should he be adjudged bankrupt within ten years of its execution, unless it can be shown that at the time of making the settlement he was able to pay all his debts without the aid of the settled property, and that his interest in such property passed to the trustee of the settlement at the time of execution. If application is made, therefore, under a voluntary settlement, for the payment of surrender-value or cash bonus, it is certainly not safe to pay within two years of its execution, and within ten years it is necessary to take a statutory declaration speaking as to the solvency of the settlor without the aid of the policy when the settlement was executed, and also as to the passing of the policy at the time. This latter point is often over-looked, but is necessary. It is sometimes suggested, by way of getting over the difficulty, that the settlor should join with the volunteer or trustee in giving the company a discharge, but it is not easy to

**Payment of
surrender-
value to
volunteer.**

see that this in any way improves matters, as, although a voluntary settlement may be set aside by creditors, yet, when once completed, it is binding on the settlor, who cannot, by any means, undo it, unless, of course, it contains a power of revocation. Under these circumstances, any interest in the settled property, other than that of the volunteer, lies not in the settlor but in his creditors, and his concurrence in the receipt cannot affect their rights.

Payment of claim to volunteer. These difficulties do not, however, arise when the policy has become a claim through the death of the settlor, as the provisions for the setting aside of voluntary settlements by creditors do not apply in the administration in bankruptcy of the insolvent estate of a deceased debtor. (In *re Gould*, 19 Q.B.D., 92). If, therefore, bankruptcy proceedings have not been commenced within the lifetime of the settlor, the volunteer can give a good discharge for the claim. It should also be noticed that the

Title of bonâ fide purchaser from volunteer. title of a bonâ fide purchaser for value from the volunteer will not be displaced by the subsequent bankruptcy of the settlor, even though the purchaser had notice that the settlement was voluntary (*re Vansittart*, 1893, 2 Q.B. 377; *re Brall*, ib. 381). It has been suggested that an assurance company, in paying a surrender-value to a volunteer, is in this position of a purchaser for value, though I do not know of any company acting upon that view.

Marriage Settlements. Policies of assurance frequently form part of the property comprised in a marriage settlement, and a word or two may be said as to questions arising in connection therewith. In the first place, it must be noted with regard to these and all settlements that the trustees have no power to go beyond the terms of the deed; and if a payment is made by an assurance company to the trustees, for purposes not authorized by the deed, it may be held liable to replace such amount, provided it had the means of knowing that a breach of trust was being committed. I may mention, as an illustration of this point, a case that occurred some time ago.

Clause giving power to surrender bonuses. Application was made by the settlor to apply the bonus on the policy towards the payment of the premium then due, and to apply each future bonus, as it vested, to the payment of the premium becoming due next after the date of such vesting. There was a clause in the settlement stating that the bonuses might be applied to the reduction of the future premiums, and it was contended by the settlor that the proposed arrangement came within this provision. The trustees of the settlement were quite

willing to sanction this application of the bonus, but the company, acting on the advice of its solicitors, held that there was no power under the settlement thus to surrender each bonus in part-payment of each future premium, and that the only arrangement that could be made was to apply each bonus to the reduction of the whole of the future premiums. Assuming that the company's legal advisers were right in their interpretation of the provision in the settlement, the company would have been a party to a breach of trust in permitting the proposed arrangement, and might have been held liable for the bonuses thus applied. Another point suggested by this case is the need for careful wording in a clause inserted in a settlement for such a purpose. As a matter of fact, in this instance, the settlement was drawn up by an eminent conveyancer, and it was stated that the clause in question was intended to confer upon the settlor power to do precisely what he now desired to do. It was held, however, that the expression "reduction of the future premiums" had a certain definite significance, and meant the reduction of all the future premiums, and the company therefore declined to accede to the request then made, unless the opinion of the Court was taken as to whether the arrangement came within the scope of the settlement.

Post-nuptial Settlements. A post-nuptial settlement, where there is no consideration but the existing marriage, and the love and affection of the parties, is only a voluntary one, and as such, is subject to the incidents of an ordinary voluntary settlement. If, however, other property is brought into settlement by the wife, on condition that the husband settles the property forming the subject of the deed, or if an allowance is made by the parents or friends of the wife on like conditions, it will cease to be a voluntary settlement, and will be considered as one for valuable consideration.

Stamping of Settlements. Many complicated questions arise as to the stamping of marriage settlements, such as were referred to in the discussion on Mr. Lemon's paper, but it will perhaps suffice here if I call attention to two points on this subject. The first is that when a policy of assurance forms part of the settled property, and there is, as is very frequently the case, a covenant for the payment of the premiums by the settlor, the stamp duty chargeable is the ad valorem duty of 5s. per £100 for the sum assured and existing bonuses. Where, however, no such provision exists, it is only liable for the duty at the same rate for the value of the policy at the time of settlement, but if a deed of this latter

description is submitted to the Inland Revenue authorities for adjudication, they will require a certificate of the surrender-value from the assurance company issuing the policy. The second point to be noted is that a settlement or other instrument containing an assignment of a life assurance policy cannot be passed as in order unless it is properly stamped, not only for the policy, but also for any other operations that it may have.

MARRIED WOMEN'S PROPERTY ACTS.

Questions relating to the Married Women's Property Acts are of frequent occurrence, many companies having a large number of policies issued in connection therewith, in force. It is unnecessary, however, on this occasion, to discuss such questions at any length, the matter having already been dealt with fully by Mr. Hughes, in his paper on the subject, read before this Institute in December 1887. Looking at the matter, however, from the standpoint I have taken in this paper, one or two points may be noticed.

Surrender of
policy taken
out for benefit
of wife.

The question has been raised as to the power of the husband and wife, jointly, to surrender a policy taken out by the former, on his own life for the benefit of the latter, and doubts have been suggested as to the complete safety to the company in allowing such surrender. The difficulty, however, appears to be an imaginary one, at least, as far as policies issued under the Act of 1882 are concerned, for in the words of that Act, "In default of any such appointment of a trustee, such policy, immediately on its being effected, shall vest in the assured, and his or her personal representatives, in trust for the purposes aforesaid." The assured himself is thus made the trustee until one is expressly appointed, and Bunyon, in his *Law of Life Assurance*, referring to this point, says: "If no other trustee is appointed, the policy vests in the assured, in trust for the purposes expressed on the face of the policy. The same provisions, though not in identical words, were extended to Scotland in 1880 (43 & 44 Viet. c. 27). The Scotch judges have held that there is nothing to prevent a husband surrendering a policy so effected, and so putting an end to the trust; they seem to think that he might do so even without the consent of the 'cestui que trust' who in the case before them (*Schumann v. Scottish Widows' Society*, 13 C.S.C., 4th Series, 678) was the wife, but had no doubt that

“he could do so with her concurrence. The words of the Scotch “Act are quite as strong as those in the English Act of 1882, and “the above decision is apparently applicable here.” From this point of view a perfectly good discharge can be obtained, not on the ground that “the wife being alone named would be sole and absolute beneficiary under the policy if she survived her husband”, as has been suggested, but on the ground that the trustee, in the shape of the husband, was a party to the surrender. The same remarks apply, of course, to the surrender of any portion of the

Payment of claim under such a policy. bonus. When a claim arises, if the policy is issued under the Act of 1882, there is no difficulty, as if the assurance company has received no notice of the appointment of a trustee, the receipt of the personal representative of the assured is a complete discharge. If, however, the policy is issued under the Act of 1870, and, as is frequently the case, no trustee has been appointed, the matter is not so simple. Under such circumstances, when a claim arises, application will frequently be made to dispense with the appointment of trustee, and if the amount be comparatively small, it is usual to do so, and pay to the widow, some offices requiring, in such a case, a statutory declaration speaking as to the identity of the person claiming, with the wife described in the policy, the marriage certificate and policy being annexed as exhibits. In view, however, of the uncertainty that undoubtedly exists as to whether the Act of 1882 has repealed the provisions of that of 1870, it would seem advisable that the signature of the personal representative of the assured should also be obtained to the

Appointment of trustee. receipt. If by reason of the large amount assured, or other circumstances connected with the case, it is deemed advisable to insist upon a trustee being appointed, such appointment should be made, in the case of a policy issued before 1883, by the High Court, or by the County Court for the district where the assurance office is situated, and in the case of a policy issued after 1882, by a Court authorized to appoint trustees under ordinary circumstances. It is contended by some, that in the latter case, the County Court for the district where the assured died is not empowered to appoint a trustee, but in this case, and also with policies issued under the Act of 1870, if the County Court of that district is applied to, it will frequently appoint a trustee, and when an appointment is made in this way it is usual to accept it in spite of its apparent irregularity. It is common to allow the surrender of a policy under the 1870 Act,

on the same conditions as if issued under the Act of 1882, though, as the policy does not expressly vest in the assured as trustee, the company does not get so good a discharge. Under either Act it is not advisable to permit the surrender of a policy issued for the benefit of wife and children unless there is a trustee, owing to the possibility of further issue, such possibility being a very real one as long as the husband is alive, whatever may be the age of the wife, owing to the possibility of his surviving his wife and re-marrying. The difficulty could be got over if the names of the children who are to benefit under the policy are inserted therein, and they are all of age and will join in the receipt, and it will not arise with regard to policies issued under the Act of 1882, if the office is prepared to accept the suggestion of the Scotch judges referred to above, but I have not heard of any offices acting upon this latter view. When such a policy becomes a claim, if under the Act of 1882, payment can be safely made to the personal representative of assured in the absence of a trustee, but, if under the Act of 1870, a trustee can only be dispensed with when all the children are of age, and will join in the receipt with the widow, and there is no prospect of posthumous issue. A somewhat exceptional case arose a short time ago. The policy was for the benefit of wife and children, and on the death occurring, it was found that the wife named in the policy was the sister of a previous wife of the assured. On application being made to the Court for the appointment of a trustee, the fact was mentioned, but the judge declined to take notice of it, and left the matter entirely to the trustee, who divided the fund as if the marriage had been a strictly legal one.

The trust
should only
take effect if
the bene-
ficiaries
survive.

In the case of a policy effected by a man for the benefit of his wife, it should be expressed that she shall take it only if she survive him, as otherwise the money will be her separate property, and the right to it will, on her death before him, devolve under her will. It is usual to insert such a provision, but I have seen such policies issued without it, and it is possible that the precaution is not universal. The omission of the precaution here referred to is more common where the policy is issued for the benefit of the wife and children, and it is customary, in such a case, to act on the assumption that the beneficiaries take as joint tenants, and therefore that the share of any one or more of them who may pre-decease the life assured passes to the survivor or survivors of them. This would appear to be a safe assumption

in view of the decision of *Seyton v. Satterthwaite* (34 C. D., 511), unless a contrary intention is indicated in the policy, but the difficulty still remains in the event of the wife and all the children pre-deceasing the life assured, as in that case the policy will pass under the will of the last survivor of them and not to the assured. It would therefore appear to be advisable that in such a policy it should be stipulated that the trust in favour of the wife and children is only to take effect in the event of one or more of them surviving the husband.

Assignment by the beneficiaries of their interest in such policies.

It sometimes happens that in a policy issued for the benefit of the wife and children, some of the beneficiaries assign their rights under a policy, as in a case which occurred recently, where the wife had died and two of the children had assigned their shares to the remaining two, notice thereof being duly given to the company. On the failure of the life assured there was no trustee, and the company insisted on letters of administration to the estate of the deceased being produced, and, the policy being issued under the Act of 1882, they then paid the claim to the administrator so appointed, handing over to him a copy of the notices recorded against the policy. It was properly held that in such a case the assurance company was not concerned with the transfer of interests referred to beyond dealing with the notices in the way described.

Liability of settlement policies to estate duty.

In connection with policies issued under the Married Women's Property Acts, the question has arisen as to whether such policies are liable to estate duty under the Finance Act, 1894, on the death of the life assured. It would seem that policies taken out by the wife on the life of her husband, for her own benefit, would be exempt, though it has been suggested that the Inland Revenue authorities might decline to take this view unless it could be shown that the premiums were paid by the wife out of her separate estate. It might, however, be fairly contended that the premiums, even if paid by the husband, were gifts to the wife, in which case, only premiums paid within twelve months of the death would be liable to duty. If the policy is on the life of the husband for the benefit of the wife, and there is a clause stating that she shall only take the policy-money in the event of her surviving her husband, it must be assumed, I think, that the policy will be liable to estate duty, as the husband has, of course, a reversionary interest in it. If, however, there is

no such clause, he has no such interest, and it would therefore seem to be exempt from duty.

Execution of
assignments
by married
women.

Another subject closely connected with the Married Women's Property Acts is the question as to how assignments by married women should be executed. It is suggested by Mr. Rubinstein, in his notes on the Married Women's Property Act, 1882, that even under that Act, deeds conveying reversionary interests must still be acknowledged in accordance with the provisions of Malin's Act as amended by the Conveyancing Act, 1882. It is difficult, however, to see that there is any ground for the suggestion in view of the explicit words of Section 1 of the Married Women's Property Act, 1882, that a married woman can, under the Act, dispose, by will or otherwise, of any real or personal property as if she were a *feme sole*. There is, I think, no doubt that under this section a married woman can assign a policy of assurance belonging to her as if she were a single woman, provided the marriage took place after 1882, or the policy came into her possession after that date. If, however, neither of these conditions is fulfilled, then her signature to any assignment of the policy should be separately acknowledged in accordance with the provisions of the Conveyancing Act, 1882. This, of course, applies to the granting of a loan on such a policy by the company, and might also be held to apply to the receipt for the surrender-value of the policy or bonus. Before leaving the subject, it may be noted that if, in accordance with the Act of 1882, a trustee is appointed by the assured, the document making such appointment, if under hand only, is not chargeable with any duty, but if under seal is chargeable with a duty of 10s.

STAMPS.

There are many points of importance in connection with Stamp Duties that arise in life assurance practice, some of which have been already referred to. One or two other points may perhaps be noticed here. A mortgage is frequently taken, not in consideration of a definite amount advanced, but to cover a current account, and the question arises as to how the deed should be stamped. Such deeds are frequently stamped to cover the sum assured, but if there is a limit fixed, as, for instance, a stipulation that the advances under it shall not exceed £1,000,

Mortgage to
secure current
account.

the deed must be stamped for the amount so limited. If, however, as is more often the case, no limit is fixed, then the instrument is sufficiently stamped if the actual amount owing when the security is enforced does not exceed the amount for which the deed is stamped, even though greater amounts have been owing during the currency of the account. It is, I believe, the practice of some companies, when the surrender-value is applied for under such a deed, to pass it as in order if the stamp covers such surrender-value ; but the proper course is to ascertain the actual amount owing at the time, and to see that the stamp is sufficient to cover it.

Equitable mortgage.

An equitable mortgage, under hand only, is liable to a stamp duty of 1s. per £100, but this is only intended to apply to the simplest form of security, and if it is framed with the purpose of placing the mortgagee in as good, or nearly as good, a position as if he had taken a legal mortgage, as for example, by giving him all the powers of sale conferred by the Conveyancing Act, the ordinary mortgage duty of 2s. 6d. per £100 must be paid, although the instrument is under hand only. It is open to question whether the form of equitable charge used by many banks, and also that adopted by some assurance companies in lending on their own policies, would not come within this restriction.

Conveyance of property subject to mortgage.

Under Section 57 of the Stamp Act, 1891, the conveyance of property subject to a mortgage must be stamped, not only for the actual consideration, but also for the amount owing on the mortgage for principal and interest. Quite recently, however, the Inland Revenue authorities decided that this did not apply where the mortgage in question was a loan by the assurance company on its own policy, and that in such a case the conveyance need only be stamped with the duty for the consideration actually passing.

Need assignment and re-assignment to mortgagor be stamped?

It has been stated more than once that where a policy has been mortgaged and re-assigned to the mortgagor, such deeds could be passed by an assurance company, though either or both of them were unstamped or incorrectly stamped. I notice, however, in the last edition of Alpe's Law of Stamp Duties, it is stated that this only applies where the instruments in question were executed before 16 May 1888. Even with deeds executed before that date, it is by no means certain that they would be accepted in Court as evidence

of title until they had been properly stamped, though the Inland Revenue Authorities will not enforce any penalty in such cases.

Receipt endorsed on policy. It has been stated that "receipts endorsed on policies, " either on their becoming claims or previously, by way " of surrender, do not require a penny Inland Revenue stamp if the " policy itself be duly stamped"; but this does not apparently hold now, for it is stated by Alpe (*Law of Stamp Duties*, page 187) that the exemptions from stamp duty on receipts do not extend to a receipt endorsed upon a policy of assurance for moneys payable thereunder.

Indemnity clause in receipt. A receipt containing, as is very common, an indemnity clause or a declaration that it is given in discharge of all claims, is liable to a stamp duty of 6*d.* as an agreement, and of one penny as a receipt; but apparently the effect of omitting the sixpenny stamp is not to interfere at all with the receipt portion, but only to render the agreement portion inadmissible as evidence in its unstamped condition.

Receipt for mortgage money endorsed on deed. It may also be noted that a simple receipt for principal and interest, endorsed on a properly stamped mortgage, is not liable to conveyance duty, nor does it require a receipt stamp. If, however, it contains such words as "in full discharge", it is liable to ad valorem reconveyance duty as a discharge.

Deeds executed out of the United Kingdom. If a policy is originally issued in the United Kingdom, all deeds connected with it, wherever executed, must be stamped in accordance with the Stamp Act, 1891. For the purposes of that Act the Channel Islands and the Isle of Man are not within the United Kingdom, so that an unstamped deed, coming from either of those places, can be stamped without a penalty within thirty days after being received in this country.

AGENCY.

Questions relating to the responsibility of a company for the action of its agents, frequently arise, and one or two points may be noticed in connection therewith. It is stated by Bunyon (*Law of Life Assurance*, 3rd edition, page 251) that "In all questions arising upon the acts of agents, it must be remembered that they are only binding upon the principal to the extent of the agency or delegated authority." That this is only fair and reasonable, will be at once admitted; but, unfortunately, experience has

shown that an assurance company has often been held liable for acts on the part of its agents, which, to the unprejudiced lay mind, would certainly seem to have been beyond the scope of their authority, and which were undoubtedly in direct opposition to their instructions. In view of this, it is satisfactory to note the remarks made by the judge in the recent case of *Menhennet v. The*

Is a company
bound by the
acts of its
agents?

Leeds and General Friendly Society. He said "Up to

"a certain point I agree that the company are bound
"by the acts of an agent, but when it comes to a point
"entirely without the scope of his authority, I say the line must be
"drawn. No company would be safe for a single moment, if they
"are held responsible for every irregularity committed by their
"agents." In another recent case, that of *Steers v. The Scottish
Widows' Fund Life Assurance Society*, the same point arose, and
the jury in their answers to the judge's questions, found that there
was a waiver by the Society's agent of certain material information,
and that he had no authority to make such a waiver. Judgment
was, as you are aware, given in favour of the Society; but, as
there were other points involved, it is, perhaps, hardly safe
to give very great weight to its bearing on the question of
agency. Taken, however, with other cases, it is useful as
showing a tendency to take a fairer view than has sometimes
prevailed as to what constitutes an unauthorized act by an agent.

Waiver by
agent of extra
premiums.

Material information may be waived by an agent,
not only as in the case last referred to, in connection
with the proposal, but subsequently, as when the assured
incurs an extra risk by reason of foreign residence, or hazardous
occupation, and it becomes an important question as to whether
waiver by the agent, either of the avoiding of the policy
or of the payment of extra premium, will amount to waiver by the
company. In the case of *Wing v. Harvey* (5 De G.M. & G. 265),
quoted by Bunyon, the assured went abroad without obtaining a
license, but the company's agent subsequently received premiums
on the policy. It was held that as the agent knew that the
payments were made on the faith of the policy continuing valid,
it was his duty and not that of the policyholder to communicate
the facts to the chief office. If this decision is to be taken as the
present law upon the subject, it places an assurance company in a
very difficult position; for, in a great number of cases, the fact of
the extra risk being incurred is almost certain to be known to an
agent living in the same locality as the assured; and, if the

former neglects to inform his principals of the fact, they have no alternative but to undertake a special risk for an ordinary premium. The question is increased in importance by reason of the large number who, at the present time, incur the heavy extra risk involved in a residence in the Transvaal, and also by the fact that the usual extra premiums charged for hazardous occupations, such as the liquor traffic, so far from being in excess, have been shown to be insufficient to meet the extra liability. Under these circumstances, it becomes of great importance that a company should be able to demand the extra premium whenever the risk is run, and the recent cases dealing with agency questions, already referred to, would seem to support the company on this point, though there seems some contradiction between them, and the case of *Wing v. Harvey* just mentioned. It would seem that if there are any cases in which a company should not be held liable for unauthorized acts on the part of its agents, it is in those referring to extra risk premiums; for in these, as a rule, the assured has not the excuse to urge that he was ignorant of having incurred the extra premium, or that he believed the agent had authority to waive it. The fact of such premiums being required, the need of notice to the company when the extra risk is undertaken, and, in default of such notice, the avoiding of the policy, are usually clearly stated in the conditions of the policy, and the assured is therefore left without excuse. In spite, however, of the strong position in which an assurance company appears to stand under such circumstances, many have hesitated to risk a decision on the subject, feeling, I presume, bound by the case referred to above. It is certainly a point on which a modern decision would be of great advantage, provided of course, that it is a favourable one; and in view of the favour, or shall I say, justice being shown to assurance companies by the Courts in this matter of agency, perhaps some public spirited company will take upon itself the enviable task of obtaining such a decision. In this particular matter of extra premiums for special risks, the difficulty of unauthorized waiver upon the part of an agent might, perhaps, be got over by inserting in the conditions of the policy a stipulation to the effect that any such extra risk must be notified direct to the chief office of the company by the assured himself, and that failing such notice, the policy shall become void. There could then be no question as to whether the assured had reasonable ground to believe that the agent had power to waive such information.

POLICIES ISSUED FOR PAYMENT OF ESTATE DUTY.

Though it is somewhat beyond the scope of this paper, as indicated at its commencement, I may perhaps be permitted to say a word or two in reference to policies issued for the payment of the estate duty created by the Finance Act, 1894. The special feature of such policies is the undertaking to pay the sum assured, or some portion of it, without the production of probate or letters of administration. Various methods are adopted by different offices to achieve this end without incurring any risk, but they may be approximately grouped in four classes: (*a*) payment is made of the greater part of the policy-money to the person entitled to obtain probate or letters of administration on producing proof of such title, and on deposit and charge of the policy to the company; (*b*) at the request of the person entitled to obtain legal proof of title, the company will pay the sum assured, or such portion of it as may be required for the purpose, to the Inland Revenue Authorities in payment of the estate duty; (*c*) at the request of the person producing the policy, and on the latter being lodged with the company, the sum assured, or such portion of it as may be necessary, is paid to the Inland Revenue authorities; (*d*) the policy is settled on trustees in trust to pay the estate duty, the balance, if any, of the policy-money being paid over to the personal representative of the assured. The two main points aimed at in the methods here referred to are (1) early payment of the sum assured, and (2) complete discharge to the office; and when such policies were first issued, some considerable discussion took place as to how far the second of these points was attained by the methods adopted.

Referring, first of all, to methods (*a*) and (*b*), we are at once confronted with the difficulty of determining, with certainty, the person who is entitled to probate or letters of administration until the grant is actually made; and the question as to whether a payment made to, or under the direction of, a person apparently so entitled, would hold good against the claim of the person actually obtaining the grant is, to say the least, a doubtful one. If such payment does not hold good, the charging of the policy to the company by the person to whom payment is made will not, of course, give any lien on it. Under these circumstances, it does not appear clear that by paying in either of the two ways here referred to, the assurance company will get that complete discharge to which it is entitled, even when payment is made to the

executors appointed in a will which is produced, there being a possibility of the existence of a subsequent will. If payment is made to the proposed administrator, the company is in a worse position, as such a person has no title whatever to the policy until the grant of letters of administration is made, and there is no guarantee that the grant will be made to any particular person. It is difficult, therefore, to see what is gained by such endorsements or clauses as those referred to, seeing that they depend for their efficiency upon the ability of the assurance company to ascertain, with certainty, the person who will obtain probate or letters of administration before the grant is actually made; and if the company can succeed in doing this, it could, without any such endorsement, pay all its claims without waiting for proof of title, certainly in the case of an executor and, with but little risk, in the case of an administrator. If it be contended that, by the terms of the special clause or endorsement, the company is relieved of all liability on paying to the person who appears to be entitled to obtain legal proof of title, there still exists the difficulty, if the payee does not ultimately obtain such proof of title, as to how far any clause or endorsement to which the assured has assented either directly or tacitly, will bind his executors or administrators. It cannot, I think, be considered as satisfactorily established that when the assured has assented to a provision in the policy that the receipt of some person, other than his legal personal representative, shall be a discharge to the company, such receipt will avail against a claim made by the executor or administrator, and in view of that uncertainty, it seems to me that a company will incur some slight risk in paying under such clauses. The validity of such a power to give a receipt for the policy-money as is here discussed would appear to be depended upon entirely in the third method adopted, in which the company undertakes to pay the sum assured, or any part of it, to the Inland Revenue authorities at the request of the person producing the policy and lodging it with the company. This method appears to combine the disadvantages of the first and second methods without their advantages, the principal thing in its favor being its simplicity. A further objection, more formal than real, common to the second and third methods, lies in the fact that the Inland Revenue authorities will not give a receipt for the amount of duty paid, and the company will therefore be without a receipt for the portion of the policy-money so paid. There remains to be considered the fourth method, that in which the policy is settled on trustees in

trust to pay the death duties to the Inland Revenue authorities and to hand over the residue to the personal representatives of the assured or any other person whom he may nominate. This method has been adopted by at least one office, and it is difficult to understand why the plan has not been more extensively employed. The two points aimed at in all such policies, namely, promptness of payment and a complete discharge, would both appear to be attained, for, in such a settlement, there can be no question of the power of the trustees to give a good discharge for the policy-money, and the amount can therefore be paid over to them immediately on proof of death and production of the deed of settlement. It has been suggested that the executors should be appointed trustees, and no doubt this would, as a rule, be the most convenient arrangement, the chief objection to it being that if a fresh will were made, with different executors, the settlement would have to be altered in accordance, if it were still desired to have the executors as trustees. This difficulty could be got over by inserting in the settlement a power of revocation accompanied by a power of new appointment, by appointing a corporation undertaking such duties, as trustee, or, in the coming bye-and-bye, when we are blessed in this country with a public trustee, by appointing that official to the position. The two latter methods would not, of course, preserve the identity of executors and trustees, but would probably prove equally convenient arrangements. Such a settlement as that here referred to should contain a complete power of revocation reserved to the settlor, as the objection to such a provision in an ordinary settlement, on the ground that it might be exercised by the trustee in bankruptcy to defeat the trust, will not apply here. If the assured should become bankrupt, it will no longer be necessary to provide a considerable sum for death duties, as it is to be presumed that his bankruptcy will leave him practically without property, and there will, therefore, be no reason for the continuance of the settlement. Such a power of revocation will give the settlor complete control over the policy, as, by duly exercising his power, it will become his absolute property again. The same end might be achieved as readily by giving to the settlor a general power of appointment, and if it be desired to guard against the possibility of the power having been exercised without the office being aware of the fact, it might be stipulated that such power should only be exercised by deed after due notice to the assurance company.

In that case, if no such notice had been received by the time of death, the settlement would, of necessity, be in force, and payment could at once be made to the trustees. Even without such a precaution, the company would, I think, be quite safe, under the Act of 1867, in paying under the settlement, unless actual notice of the exercise of the power of appointment had been given. By exercising the power in his own favour, the assured could at once regain control of the policy. If the policy be a with-profit one, power to deal in any way with the bonuses should also be reserved to the settlor. The objects sought to be attained by such a settlement as this might, perhaps, be attained even more simply by inserting in the policy a declaration of trust, stating that the assurance company, on proof of death, should hold the sum assured in trust to pay the death duties, and to pay the balance to the legal personal representatives of the assured. I cannot but think that the method of rendering the policy-money available, immediately on proof of death, by means of some form of settlement, would be more extensively used were its advantages more widely known amongst those who assure their lives, as even where no great amount is required to pay the death duties, the difficulty of obtaining a moderate amount of ready money before the estate can be realized is often keenly felt. By means of a settlement, with the executors as trustees, the policy-money might be placed in their hands at once for the purpose of paying the death duties, funeral expenses, and other like charges, and so save much inconvenience and some expense.

PAYMENT INTO COURT.

Comparatively recent as is the Finance Act, 1894, there is, however, a still more recent Act affecting life assurance companies, in the shape of the Life Assurance Companies (Payment into Court) Act, 1896. This Act is so recent and untried that it would hardly be safe, as yet, to express any decided opinion as to its merits or demerits, and I have made this reference to it mainly in the hope that in the discussion on this paper, the more experienced members of this Institute may be induced to give their views as to the utility of the Act, as limited by the rules of the Supreme Court relating to it, which have now been published. It has long been felt that greater facilities were needed for the payment of money into Court by assurance companies, and the Act of 1896 seemed at first sight to furnish all that could be desired in this

direction, making, as it does, the right to pay into Court depend, not on the actual fact of such a defect in the title as to prevent a proper discharge being given, but upon the opinion of the directors as to the existence of such a defect. This right, however, is stated to be subject to the rules of Court, and the publication of these rules has considerably modified the opinion that had before been entertained as to the usefulness of the Act. The principal points to be noticed in these rules are: (1) that the full amount due must be paid into Court without deduction of anything for costs or expenses, these latter apparently falling on the assurance company in any event; (2) that the company must also submit to pay into Court such further sum for interest or otherwise as the Court or a judge may direct, and to pay any costs which it may be considered should be paid by the company: and (3) that payment into Court shall not be made if any action is pending in the matter, except by permission of the judge. With these restrictions it does not seem likely that the Act will be of great use, as it is to be presumed that, under the powers conferred by the rules, if the judge is of opinion that there was not sufficient ground for payment into Court, the company will have to pay the whole of the costs, and even where such a course is held to be justified, will have to pay its own costs. In addition to this, the delay in payment and the expense incurred by the claimant will, in all probability, be almost as prejudicial to the company as if they had refused to settle the claim. The cases which the Act seems intended to meet are not those in which there is a dispute between rival claimants, but rather those in which there is no dispute as to who should claim, the difficulty lying in the fact that one or more links in the chain of title are missing or defective. Some such cases have already been referred to in the discussion of mortgages.

VARIOUS OTHER POINTS.

Can an
assurance
company insist
on production
of policy?

In conclusion, I will briefly refer to one or two points, which can hardly be classified under any of the subjects previously referred to. When a claim arises, can the person legally entitled to the money be compelled to produce the policy when it is known to be in existence? A case bearing on this point occurred some little time ago. A policyholder in Scotland, not having unlimited confidence in his wife, desired to prevent her from having any control over the policy-money. As, however, there is not, in Scotland,

the complete power of disposition over personal property that exists in England, he assigned the policy to his executors in trust for his children. On the death occurring, a claim was made to the company by the executors in their dual capacity of executors and trustees, but the policy was in the possession of the widow, who gave notice of the fact to the company, and warned them against paying to any one other than herself. The company requested the executors to produce the policy, but they declined to take the necessary steps to obtain it, and threatened an action unless the claim was paid to them without its production. The company took legal advice on the matter, and were informed that it would not be safe to resist payment on the ground of the non-production of the policy. As I have stated, the case occurred in Scotland, but on the particular point here referred to, there does not appear to be any difference between the law of Scotland and that of England.

Surrender of
children's
endowment
assurance
policies.

Another question that may be noticed is as to the advisability of allowing the surrender of endowment assurance policies issued on the lives of children.

In such policies, as under ordinary circumstances and apart from the special provisions of the Friendly Societies Acts, the parents have no insurable interest in the life of the child, it is usual to state in the policy that it is effected for the benefit of the child. The policy thus becomes the subject of a trust, of which the proposer must be considered the trustee. If now, as sometimes happens, application is made by the proposer for the surrender-value and, incidentally, reference is made to the reasons for the application, which show that the proposer intends to use the money for his own benefit, will not the company be a party to a breach of trust in permitting the surrender under such circumstances? Some companies hold that they are in no way concerned with the way in which the surrender-value of such policies may be applied, but though this may possibly be a safe view to take where there is nothing to show the reason for the surrender, I cannot think that it is safe where such an intimation as I have referred to exists, particularly where, as occasionally happens, the surrender-value is to be applied to the payment of premiums on a policy on the proposer's own life. The law on the subject is that a purchaser for value, of trust property from a trustee, has a good title, provided he does not know or has no reason to suspect that the trustee is committing a breach of trust by selling, and from this point of view it is, to say the least,

doubtful whether, under some of the circumstances referred to, the company is sufficiently protected in allowing the surrender. The better and safer plan would seem to be to decline to give surrender-values for such policies, and where the premiums cannot be kept up, to grant instead a free policy subject to the same conditions as the original one.

Payment under probate in which value of estate is incorrectly given. The question has been raised as to whether an assurance company, in paying a claim under probate or letters of administration, should see that the actual amount paid by them does not exceed the gross value of the personal estate named in the grant, or that duty is paid for as large an amount as that paid by the company; and it has been asserted that if the grant is defective in either of these respects, it should be returned for correction. It is difficult, however, to see on what ground this assertion is based, as the duty is not payable on the gross estate, but on the net value after allowance for funeral expenses, debts and incumbrances, so that the company, without the accounts of the estate, is not in a position to check the sufficiency of the duty paid, and would therefore be very ill-advised in refusing payment on such a ground; and this is, I believe, the view taken by most companies, though not by all. I may add that, according to a recent communication, the Inland Revenue authorities, as far as they can be induced to commit themselves, apparently take the same view.

Payment to solicitor or banker. One further point on which a difference in practice exists may be noticed. It is stated by Bunyon (page 515) that "when a policy has been receipted by the claimants, and handed to their agents to exchange for money, it must be accompanied by an authority to the office to make the payment. This should be in writing, and it will be incumbent upon the insurers to see that the authority is genuine. The mere fact that a solicitor or other agent has in his possession a deed executed by his client, or policy receipted by his principal, does not give him authority to receive for his client, the money which is the consideration for the deed or receipt." The practice of many offices does not however agree with this statement, as, although it is usual to ask for a written authority, such as is here referred to, if it is not forthcoming, but the receipted policy is produced by a solicitor or bank, in many instances the money will be paid on that authority alone. Offices taking this course rely upon the Conveyancing Act, 1881, Section 56, where the claimants are not trustees, and where payment is being made to trustees, reliance is placed on the

Trustee Act, 1888, Section 2. The latter Act refers to payment to a solicitor or banker, as agent for the trustees, but the former Act only refers expressly to solicitors, and to cases where the receipt is contained in the body of the deed, or is endorsed thereon. In practice, however, it is common to overlook these latter restrictions, and pay the claim to a solicitor or banker on production of the policy and the receipt, whether the latter be endorsed or on a separate form, and whether the claimants are trustees or not. When, however, payment is being made to a bank, and no express authority to collect the amount is produced, it is not unusual to cross the cheque to the account of the claimants as an additional precaution.

I ought, perhaps, finally to give some explanation of the fact that I have omitted all reference to absolute assignments. My reasons for so doing were that, in the first place, ordinary absolute assignments present no difficulty, being usually quite straightforward and simple, and, therefore, did not seem to need any discussion in such a paper as this. In the second place, the more interesting and complicated questions relating to such assignments, as, for example, the practicability and advisability of converting policies into negotiable instruments by means of an endorsement by the assured or otherwise, would lead me far beyond the limits I laid down for myself at the commencement of this paper, and they have, moreover, been somewhat fully discussed already in connection with Mr. Higham's paper read before this Institute in March 1887. A third reason, the validity and sufficiency of which, I am sure, will at once be admitted by all present, for not discussing this and many other points, is that this paper has already grown to an inordinate length, and would probably, if continued further, defeat its own object by tending to hinder rather than to promote discussion.

DISCUSSION.

The PRESIDENT (Mr. T. E. Young) said he had now the very pleasing duty of giving a welcome in the name of the Institute to a fresh contributor to their knowledge, Mr. Barrand. It was particularly gratifying to him (the President) that the first occasion on which he officially occupied the chair at the ordinary monthly meeting was signalized not merely by the advent of a new author, but also and especially by the very practical character of the subject which he had selected for his first, but not, he hoped, his last, paper. The

paper had been referred by the Council to Mr. Hughes and Mr. Simon, and he called upon them to open the discussion.

Mr. W. HUGHES said he was sure he was expressing the opinion of all the members present in congratulating Mr. Barrand very heartily upon the paper which he had set before them. It was recognized by their constitution that something more than a mere layman's knowledge of law was necessary for the full equipment of an actuary. The President had emphasized that fact in the address which he had delivered last month. Although in practice they had recourse to their legal advisers when any legal point came up, they could not always have the solicitor at their elbow, and even when they did call him in, they required something more than a layman's knowledge to understand the advice which he gave, so as to take due advantage of it, and also, at times, to enable them wisely to neglect it. He would like to point out what he thought was a slip on the part of the author. The very interesting case of the interpretation of the words "bonus in reduction of premiums" illustrated the fact that even eminent conveyancers who had not sufficient knowledge of insurance matters could make mistakes. The conveyancer in that case, no doubt, had not before his mind exactly what was meant by the application of bonus to reduction of premiums. Mr. Barrand said that it had been held that the expression had a certain definite significance, which would seem to indicate that the matter had been submitted to the opinion of a Court. He did not think that was the case. It was held by the best opinion that could be obtained by the office, but not by a Court. Another point was the question of the responsibility of the company for the acts of its agents, referred to on page 229, as an illustration of the advantages that the actuary had of possessing something more than a layman's legal knowledge. An agent too frequently did or said something a little beyond what he ought to say, and the offices were placed in a position of some difficulty. It was of very little use to apply to their legal advisers, because they told them in the most authoritative manner that they were quite safe, that the agent could not bind his principal beyond the scope of his authority. The question of what was the scope of his authority was another point which was not so easy to ascertain, and even if ascertained, the actuary, as an administrator, had to consider, it being admitted that the agent had gone beyond the scope of his authority, how far the company could go and how far it would be safe to adopt the unauthorized action of the agent for the credit of the office, and for the proper settlement of the matter, without unduly prejudicing other cases. Finally, he thought that the paper was a very good example of the usefulness of the practice of noting the cases which came before them in their daily work, and of registering them for future use. He happened to know that a good many illustrations contained in the paper had come before the author in the pursuance of his official work, who had carefully made notes of a great many of them, and had fulfilled the Baconian maxim which was printed on the cover of the *Journal*, and by so doing had attempted to pay off a part of the debt which he owed to his profession.

Mr. L. M. SIMON said that he was the last person to question the value of legal knowledge to the actuary. Acts of Parliament

became interpreted in time, and the settled points of law became the rules under which business was carried on, and a wide and accurate knowledge of the settled points not only enabled the actuary to carry on his work without friction or delay, but also to recognize a doubtful point when it arose and to put it intelligibly before his solicitor. He agreed entirely with what Mr. Barrand said about the forms of policies specially designed for the payment of the estate duty created by the Finance Act of 1894. The particular form Mr. Barrand approved seemed to him the only one which fully met the requirements of both sides—it ensured the immediate payment of the sum assured on proof of death before probate, and enabled the office to get a perfect discharge. Mr. Barrand said that the Inland Revenue authorities would not insist upon the stamping of a deed for which a later deed had been substituted and properly stamped, and there was another case mentioned in the paper in which the Commissioners were not so exacting as might be supposed, namely, where a man had mortgaged his policy and then redeemed it, and afterwards mortgaged it again in another direction, and the latter mortgage was the one under which the sum assured was claimed. In the case where the first deed and its release were unstamped, the Somerset House authorities did not insist upon the penalty or upon their being stamped. Mr. Barrand stated that, in the last edition of Alpe's "Law of Stamp Duties", it was declared that that only applied where the instrument in question was executed before 16 May 1888. He, however, had seen a letter from the Inland Revenue Office, which stated that under section 118 of the Stamp Act they did not require payment of duty upon instruments relating to previous dealings on a life policy which, subsequent to such dealings, had again become vested in the assured, and that if the final assignment were stamped it was not necessary to look behind the assignment. There was no limitation as to date. With regard to another point, he might say that he had seen a great many discharges for claims under life policies, and had never heard that it was considered necessary that they should be stamped when endorsed upon the policy.

Mr. H. R. HARDING considered that their thanks were due to Mr. Barrand both for the industry, and in recognition of the ability, shown in the preparation of the paper read. He said that he agreed with Mr. Barrand as to the desirability, although it was not absolutely necessary, of obtaining the concurrence of the mortgagor to the discharge of a policy when the assignment was a simple form of charge only. He said that Mr. Barrand's references to the power of sale section of the Conveyancing Act would require some additional words, as that section was not retrospective. He pointed out that, in regard to voluntary settlements, Mr. Barrand had used the words "shall become void" and "can be proved." The more emphatic wording of the Act made it the more necessary for them to be cautious in the way they proceeded. In view of Mr. H. B. Buckley's opinion, given to the Life Offices Association in June last, it seems very doubtful whether, under the circumstances referred to by Mr. Barrand, a statutory declaration as to solvency would really protect an office paying within ten years of the date of the settlement. Further,

Mr. Barrand seemed to differ from Mr. Buckley in regard to the title of a *bonâ fide* purchaser for value from a volunteer in the event of the bankruptcy of the settlor. With reference to Mr. Barrand's remarks on the stamp duty payable on marriage settlements being 5s. per-cent on the sum assured and existing bonuses, when there is a covenant by the settlor for the payment of the premiums, although this is completely accurate in regard to settlements executed since the passing of the Act of 1864 (referred to by Alpe), yet it might be mentioned that, if the settlement was executed prior to that date, the Somerset House authorities did not ask for more than the stamp on the value of the policy at date of settlement unless there was actual provision made of a special fund out of which the premiums could be paid. When referring to a policy effected by a man for the benefit of his wife, Mr. Barrand said it should be expressed therein that the wife only takes it if she survives her husband, as otherwise the money will be her separate property, and the right to it will, on her death before him, devolve under her will. Was this really so? It seemed to him, (Mr. Harding) that in such a case on the death of the wife the trust was absolutely extinguished, and the policy reverted to the husband, and became part of his estate. The policy is not the wife's separate property. It is vested in the husband, or his nominee, or in his executors or administrators on his death, in trust, and may at any time during the husband's lifetime be made subject to a trust deed containing such provisions as would take away from the wife all power whatever of alienating or dealing with the policy-moneys, even if she survived her husband. Another thing which made him think Mr. Barrand was wrong on this point was the wording of the Married Women's Property Act itself. This was that such a policy should "enure and be deemed a trust for the benefit of the wife for her separate use . . . and shall not, so long as any object of the trust remains, be subject to the control of the husband, or form part of his estate." Clearly the intention is, that if the wife die first, the trust becomes extinguished, and the policy reverts to the husband's control. Mr. Barrand stated that if a policy is originally issued in the United Kingdom, all deeds connected with it, wherever executed, must be stamped in accordance with the Stamp Act, 1891. In the case of a life who died domiciled abroad, and on whose death the policy-moneys were payable, this, however, would not be so, as the title is then a foreign or colonial one, and the office could pay in accordance with local law.

The first case under the Payment into Court Act had recently come to the front. A few days ago the money due under a policy had been paid in under that Act. The necessity for it arose from the fact that the office, 41 years ago, had notice that the policy was assigned in trust to certain parties. When the claim occurred, not only was that deed not forthcoming, but all the parties to it had disappeared and could not be traced, and, to add to the complication, the policy also was missing. Of course, the personal representatives of the assured were anxious to give a discharge if the office would accept their signatures, but on the difficulty being pointed out to them, they were quite ready to agree to the money being paid into Court, and, accordingly, that was done. What it would cost them to get it out he did not know. That was one of the points of the Act

which was not altogether satisfactory. It did not cost the office very much to pay into Court, but still it cost them more than it would have done to pay their solicitor's bill if it had been simply a question of ordinary title, and he did not see why the office should be saddled with that additional expense. When there was more than one claimant the Act would be very useful, but something more simple was required when there was no opposition.

Mr. J. E. FAULKES said it would be interesting to know whether the deeds mentioned on page 209, notice of which was allowed by Somerset House to be removed from the register, were dated before or after May 1888. The author referred to the power of sale under an equitable mortgage. It seemed doubtful whether, if power of sale were inserted, it did not make the mortgage for the purposes of stamp duty a legal mortgage, and probably that was the reason of the action of the banks. The case mentioned of "*Sheffield v. the London Joint Stock Bank*" had, of course, been overruled by "*Simmons v. the London Joint Stock Bank*", and even if that were not so, he doubted whether an office would be justified in assuming that the ordinary business of a banker was to lend on policies only and not to purchase them. He had made some enquiry on that head recently and had found that one bank had purchased a considerable number of policies, so that the point could hardly be insisted upon. With regard to stamping marriage settlements, it was interesting to note that the covenant to pay premiums need not be in the settlement at all: it might be an entirely separate deed. Secondly, it had been contended by Somerset House that a mere discretionary power to the trustees to apply the income of other settled property in payment of the premiums on the policy, rendered the settlement liable to stamp duty on the full amount of the policy, although that was merely a discretionary power of the trustees to apply other moneys in payment of the premiums—not a covenant to pay premiums. He doubted whether that would be upheld, but that was the contention of Somerset House in a recent instance. He thought Mr. Barrand would find that the appointment of trustee under section 11 of "*The Married Women's Property Act, 1882*", was free from stamp only where the trustee was appointed by the terms of the policy or by an endorsement thereon not under seal at the time the policy was effected. If, however, the appointment was made by a separate memorandum or by an endorsement on the policy subsequent to the date of issue of the policy, then, he thought, a 10s. stamp would be required, it being considered to be the appointment of a *new* trustee. The practice of his office had been for some years past to require a penny stamp in cases of receipts for claims and surrenders. That practice had, he understood, been adopted after consultation between the solicitors of the office and the Somerset House authorities. With regard to payment of policy-moneys to a solicitor or banker, section 56 of the Conveyancing Act of 1881 referred to deeds and to deeds only, and in but very few instances could the receipt on a policy come under that section. The Trustee Act of 1888 had been repealed and re-enacted by the Trustee Act of 1893, which did not apparently require that the receipt should be endorsed on the policy, but gave an implied authority to the solicitor or banker producing the policy and

receipt to receive the policy-money. That applied also to the ease of legal personal representatives by section 50. When paying to an assignee other than a trustee, they must have an express authority in the receipt to pay to anyone other than the actual claimant.

MR. GEORGE KING said that when the Finance Act of 1894 was passed he had interviewed the Somerset House authorities for possible procedure under it. Somerset House would not accept money to be placed to the credit of an estate on account of probate duty—they must have the exact amount and at the right time, and therefore one could not pay money into Somerset House out of the policy-money for a will about to be proved—they could only do so when the affidavit went in, that was when the executors or administrators, as the case might be, were finally decided upon. Somerset House would accept the cheque of an insurance company, so that the company taking an affidavit themselves and lodging it at Somerset House with a cheque of their own, would have the will proved, and, he thought, get a perfect receipt. They paid in the exact amount and had the cancelled cheque, and could get a receipt on the policy itself from the proper parties, so that if the policy was unencumbered, and bore the proper clause inserted in it during the lifetime of the assured, no risk could be run. He thought it would be more convenient to carry out the arrangement in that way than by the system of trustees, which must be very troublesome and to a certain extent expensive. As to the appointment of trustees under the Married Women's Property Act, he thought Mr. Faulks was not quite accurate in stating that in order to avoid stamp duty it was required to be made at the date of the policy. He had the authority of Somerset House, given a year or two ago, for saying that if the appointment was the first, even if not made at the time of the issue of the policy, it did not require a stamp, but any change of trustees would require a stamp. However, it did not always follow that the Inland Revenue authorities were the highest or most trustworthy authorities in case of stamps, because he had found them contradict themselves within a few weeks, and they were always liable to take a different view at some future time.

MR. J. E. FAULKS said that what he had stated had been taken from a letter written last year by the Board of Inland Revenue.

DR. T. B. SPRAGUE, in a communication to the President, said that when a mortgage of a policy had been executed and notified to the insurance office, he held that the office might be satisfied with evidence that the charge on the policies had been paid off, and that it was unnecessary to insist on having a formal re-assignment of the policy. What risk would the office run if it dispensed with a re-assignment? The case seemed to him to come under the head of the strictly legal requirements which the office might waive for the purpose of saving trouble and expense to the claimants. When a policy had been thus mortgaged, and the loan paid off, it was not necessary for the office to see that the mortgage was properly stamped, and that was true whether the mortgage deed was forthcoming or missing. The case considered by Mr. Barrand, where a mortgage of a policy was not stamped within the proper time, and, in order to remedy that, a second assignment in the same terms was

executed and duly stamped, occurred not uncommonly. Some legal authorities take the view that, the policy having been conveyed by the first assignment, there was nothing left to be conveyed by the second, which, therefore, was wholly inoperative. It was not usual to require evidence that default had been made by the mortgagor when the mortgagee applied to surrender a policy. In the case of a bank which held what purported to be an absolute assignment of a policy, it sometimes happened that the mortgagor was taken bound to pay the premiums, and this raised a strong presumption that the assignment was only intended as a security. As to the discharge to be taken for a policy under the Married Women's Property Act, perhaps the strongest objection to the office paying direct to the beneficiaries was, that it rendered itself liable to see that the proper duties in respect of the policy-money were paid to Government. In connection with the case where the wife named in the policy was the sister of a previous wife of the assured, perhaps the most material fact was that, the wife being legally not a wife, the policy was not protected from creditors.

The PRESIDENT said that he had been very much impressed by the practical and business-like manner in which the author had dealt with the subject. In the application of legal principles to their ordinary work it must always be remembered that they had to deal with them as business men, who must not merely be prepared to face reasonable business risks, but to incur them, even though they were compelled to traverse legal technicalities. One of the wisest pieces of advice he received at the beginning of his career was furnished by the solicitor of one of the oldest and most respected companies, who, when a case, consisting largely of legal niceties, was submitted, advised that, as a lawyer, he would require the office to enter into a series of requisitions; but, speaking as a business man, he would distinctly counsel them to pass them by.

Mr. BARRAND, in reply, said that as he originally wrote the paper he had stated that certain acts on the part of the agent were in direct opposition to their instructions, and therefore *à fortiori* beyond the scope of their authority, but in talking over the matter with a barrister who had special experience in agency questions, he was told that an act might be in direct opposition to an agent's instructions, and yet by no means without the scope of his authority. The agent's instructions were meant for his guidance, and the assured was not necessarily concerned with them. With regard to assignment and re-assignment, he had taken the opinion of Alpe, as being the best authority, who took the view that the application of the exemption referred to was only to deeds executed before 16 May 1888. He had been told that Mr. Buckley took an opposite view to himself on a certain point in reference to voluntary settlements, but his statement had been taken almost word for word from the latest edition of Williams on "Personal Property", who quoted two very recent cases in support of his view. He had taken the statement that it should be stated in a "benefit of wife" policy that the wife should only take in the event of her surviving her husband, as otherwise it would pass under her will, from Mr. Wolstonholme's remarks on the Married Women's Property Act. He was prepared to admit there

was not much danger in reference to some of the methods of dealing with Finance Act policies, but his contention was that companies should not run *any* risk when there was a method at hand which would answer every possible purpose. A question had been raised as to the point mentioned in the paper with regard to one deed not having been stamped within the proper time, and another deed being executed in order to get over paying the penalty on the first deed. In the particular case where Somerset House stated they had no objection to the company expunging from their books recorded notice of the first assignment, those assignments were dated this year, so that it might be considered as quite free from limitations as to the passing of the Inland Revenue Act of 1888. In the offices of which he had experience, it was usual to take evidence of default. He thought the strongest objection to the payment to beneficiaries was that the company might not get a proper discharge, and might have to pay again.

MORTALITY IN THE LIQUOR TRADE.

INVESTIGATION BY THE ASSOCIATED SCOTTISH LIFE OFFICES.

THE ASSOCIATED SCOTTISH LIFE OFFICES have recently concluded an important investigation into the rate of mortality among persons engaged in the sale of intoxicating liquors, and the results are embodied in two valuable reports. These, through the courtesy of the Associated Offices, we are now able to present to our readers. They are prefaced by the memorandum of instructions for the filling up of the cards, it having been thought desirable to preserve in permanent form a complete history of the enquiry.

Mr. George M. Low, a member of the Committee of Investigation, read a paper on "Extra Risks" on 11 February 1897, before the Actuarial Society of Edinburgh, in which, *inter alia*, he dealt very fully with the subject matter of these reports, and gave mortality tables and monetary values, deduced from the statistics. This paper forms No. 5, Vol. iv, of the Transactions of the Society, and to it we refer our readers.

ED. J.I.A.

FIRST REPORT *of the* COMMITTEE ON THE MORTALITY *among* *Persons engaged in the Sale of Intoxicating Liquors.*

The Managers, at their meeting on 9 July 1890, appointed us a Committee to report as to the necessary arrangements for conducting an investigation of the mortality experience of the Offices in regard to persons engaged in the sale of intoxicating liquor.

The object of such an investigation is to determine the rates of extra premium, if any, that should be charged for insuring the lives of Hotel-keepers, Publicans, and others engaged in the sale of intoxicating liquor by retail; the mortality that has been experienced among such persons being for this purpose compared with the general mortality experience of assured lives.

It is obvious that, if the investigation is to be of practical value, the observations to be embraced in it ought not to apply to a set of circumstances materially differing from those which are likely to prevail in the future. Considerable changes have taken place in recent times in the state of the laws regulating the sale of intoxicants, and some of these may reasonably be supposed to have favourably influenced the health and longevity of persons engaged in the trade.

In Scotland the Licensing Laws have remained practically unaltered in their essential features since the passing of the Forbes Mackenzie Act in 1853, which put a stop to the sale of intoxicating drink by Grocers for consumption on the premises; and, by the terms of its certificates, shut up all public-houses on Sundays, and enforced their being closed from eleven o'clock at night until eight o'clock in the morning on other days. Recently the Licensing authorities in many districts have availed themselves of the discretion given by an Act of 1887 to close public-houses at ten o'clock.

Any restriction of the hours of sale—particularly of the night hours—is likely to have a beneficial effect, from the insurance point of view. The last hour of the day's work is probably more prejudicial than any other to men engaged in public-houses. Their physical energies are then at the lowest point, while the state of the atmosphere they have to breathe is probably at its maximum of impurity—heated by gas, and laden with accumulated exhalations and fumes of liquor. Those are precisely the conditions most likely to excite a craving for stimulants, and to favour the development of disease. In view of the apparent tendency to close the public-houses at an earlier hour, it may be assumed that, if no new unfavourable conditions arise, an extra premium calculated on the basis of past experience will be at least sufficient for the future.

In England, the Licensing Laws have undergone more recent modification. The Wine and Beer Houses Act of 1869, and the Licensing Acts of 1872 and 1874, introduced important features, among them the existing regulations for closing, and the establishing a standard of value for licensed premises, a measure adopted apparently with the object of keeping the trade in the hands of a better class of men. These Acts also give very wide powers to the police, of entering and inspecting licensed premises at all times.

Licensed premises in England must now be closed in rural districts at ten o'clock and in towns at eleven o'clock; the sale of liquor after those hours being prohibited, save in the case of Hotels and Inns, to persons lodging in the premises. In the Metropolis, licensed premises may be kept open until midnight. It would, accordingly, be useful to ascertain, if possible, whether the mortality among Publicans and their assistants is greater in London than in the Provinces. It seems to us, however, that a positive opinion cannot be formed as to the

practicability of this branch of enquiry, until it is seen what is the extent and exact nature of the experience that can be contributed by the Offices taking part in the investigation.

Permission to sell liquor within certain hours on Sundays is a peculiarity of the English law.

In Ireland, an important change was introduced when the Licensing Act of 1874 was passed, which had the effect, among other things, of closing all licensed premises at ten p.m. A further change, perhaps of less importance, was introduced in 1877 by Mr. Meldon's Act, which shut up public-houses in Ireland on Sundays. A peculiarity of Irish Licensing, as distinguished from English and Scotch, appears to be that public-house licenses are generally held by shopkeepers, the bar being an adjunct of the ordinary business premises. On the other hand, there are in Ireland a class of "Spirit Grocers", selling liquor for consumption off the premises only, and corresponding in some degree to the "Licensed Grocers" in Scotland.

So far as Scotland is concerned, it would appear that the year 1853 is a suitable date from which to commence the investigation. In England, circumstances have so altered since 1872, that it would seem most suitable to commence from that date; and, similarly, in Ireland, the year 1874 would seem to be the proper date to start from.

We therefore recommend that the investigation begin as from the year 1853; that is to say, that the Offices be requested to furnish particulars regarding lives insured in all three countries since that year, the intention being that the experience since 1872 in England, and 1874 in Ireland, shall be afterwards investigated separately.

Undoubtedly, the experience would be all the more valuable if a sufficient body of statistics could be got relating to persons who have *entered the trade* since the dates above mentioned. If the information were available, it would be important also to ascertain how the extra mortality varies according to the age at which individuals have entered the trade, and the length of time they have been engaged in it. It seems highly probable that men who have grown up in the business, or who have been for a long time accustomed to dispense liquor, and have passed through that experience without injury to their health, and without acquiring the habit of over-indulgence, are less likely afterwards to succumb to the adverse influences and temptations by which they are surrounded. If this was clearly established, a distinction might be made, in the matter of extra premium, between such cases and those of *persons who have newly entered the trade*. We feel that the data necessary for the complete investigation of this point are not to be found in the records of Insurance Offices; but we wish to take the opinion of the Managers as to whether information can be got respecting the length of time the lives have been engaged in the business of selling liquor.

The data for the investigation should be collected, as usual, on cards. We think that those Offices which have already taken out their experience of this class of lives should be invited, in the first instance, to send in their own cards, as already prepared, and it will then be seen whether it is possible to utilize these without re-writing. A form of card to be furnished to those offices whose experience has

not already been taken out, will be adjusted by the Committee. It is proposed that the cards shall contain the following particulars:—

Name.	Date of Birth (or Age at Entry).
Designation.	Date of Entry.
Place.	Date of Exit.
English, Scotch, or Irish.	Mode of Exit.
Sum Assured.	Cause of Death.

Designation.—This will be stated on the card, in each case, as it appears in the Company's books.

We think it desirable that, in arranging the material for the investigation, the lives should be divided into appropriate groups, according to the nature of their connection with the sale of liquor. The distinctions drawn by the Forbes Mackenzie Act suggest a convenient form of classification, namely: (1) Hotel-keepers and Inn-keepers; (2) Publicans; and (3) Licensed Grocers. This classification, however, is not in all respects satisfactory. For example, an Inn is, in many cases, hardly distinguishable from an Hotel of fairly good class; but in many places the Inn is to all intents and purposes merely a public-house. In this view, the effect of grouping Inn-keepers and Hotel-keepers, therefore, would be to credit the Hotel-keepers with a portion of the mortality that ought rather to go to the account of the Publicans. It will have to be considered whether Hotel-keepers and Inn-keepers should be embraced in one group, or whether the latter should be placed in a class by themselves, or grouped with the Publicans.

Under the head of "Publicans" will be included Licensed Victuallers, Wine and Spirit Merchants (assumed to be retail dealers), and generally all persons whose occupation consists in selling drink for consumption on the premises. Some difficulty may be anticipated in determining, from the designations furnished by the assured, who are and who are not "Licensed Grocers." In Scotland, the majority of Grocers do take licenses, so that a person described simply as a "Grocer" is more likely than not to be a "Licensed Grocer", but there is the possibility that he may not be. In Ireland, a person of the same designation is not unlikely to be also a Publican. In England, on the other hand, comparatively few Grocers seem to hold licenses of any kind to sell liquor. After getting the cards, we may perhaps see some way of overcoming this difficulty. But this is a point on which we should be glad to have the opinion of the Managers. In the meantime, however, we suggest that the enquiry should extend to *all* Grocers in Scotland and Ireland insured with the Offices since 1853, but should not embrace Grocers in England, unless where these are known to have been licensed.

When the lives of Publicans, &c., have been extra-rated, on account of family or personal defects, we think that this fact should be noted on the card, so that the mortality among this class of lives may be separately investigated, if thought desirable.

Date of Birth.—We shall be glad to have the opinion of the Managers as to whether it will be more convenient to furnish the "Date of Birth" or the "Age at Entry."

We think that Offices should not be asked to state whether or not extra premium for occupation has been required in particular cases, nor as to the rate of extra charged. But, for completeness, information should be given as to the cases in which the extra premium charged has been taken off on the occupation being given up; and we wish for the opinion of the Managers as to whether this information may be expected from the Offices.

T. B. SPRAGUE.
DAVID DEUCHAR.
GEO. M. LOW.

20 November 1890.

At the Special Meeting of Managers, held on 26 November 1890, the above Report was read and adopted, and it was resolved that it be printed and circulated among the Offices.

The following questions suggested by the Report were considered:—

1. Can information be got respecting the length of time that the various lives have been engaged in the business of selling liquor?
2. Can any means be taken to determine whether a person described as a "Grocer" is, or is not, a *Licensed Grocer*?
3. Will it be more convenient to the Offices to furnish the Age at Entry, rather than the Date of Birth?
4. Can the Offices give information as to the cases where an extra premium charged for the occupation of a Liquor Seller has been taken off on the trade being given up?

It appeared from statements made by several of the Managers that their proposal papers contain information on some or all of these points, but other Offices would not be able to give information without making special inquiry of their agents, and the result of this would be doubtful. It was agreed that the contributing Offices be requested to give such information on the above points 1, 2, and 4, as is contained in the proposal papers, and that the Dates of Birth should be given, and not merely the Ages at Entry.

It was further agreed that the investigation should include Travellers in the Wine, Spirit, and Beer trades.

It was then remitted to the Committee to take the necessary steps to have the investigation commenced.

NOTES FOR THE GUIDANCE OF PERSONS ENTRUSTED WITH FILLING UP THE CARDS *for the Investigation of the Mortality among Persons engaged in the Sale of Intoxicating Liquors.*

1. The Investigation is to cover the period between 1 January 1854 and 31 December 1890, and to be confined to Assurances effected between those two dates.

2. It is to include all Assurances on the lives of persons engaged in the following occupations, whether an extra premium on account of occupation has been charged or not, namely:—

Hotel-Keepers, Inn-Keepers, Publicans, Licensed Victuallers, Wine and Spirit Merchants (Retail);

and, generally, all persons whose occupation consists in selling drink for consumption on the premises: also

Grocers in Scotland and Ireland (unless known to be unlicensed Grocers), Licensed Grocers in England (when known to be such), Travellers in the Wine, Spirit and Beer Trades.

3. In cases where a life has been insured more than once, a card will be written for each Policy.

4. *Name*.—The surname to be written first, followed by the Christian name in full.

5. *Designation*.—This to be given as it appears in the Company's papers. In the case of Grocers, known to be licensed for the sale of liquor, the word "Licensed" to be added, within brackets, after the designation "Grocer."

6. *Place*.—The name of the town, village, or locality, to be given, without further particulars; but as this may not always indicate in what division of the Kingdom the life resides,—

7. The letter "E", "S", or "I", as the case may be, to be written after the words "English, Scotch, or Irish."

8. *Age at Entry, Age at Exit, and Duration of Policy*, are not to be filled in by the Contributing Offices.

9. *Mode of Exit*.—Insert "D" for dead, "T" for surrendered, lapsed, expired, or terminated otherwise than by death. If in force at 31 December 1890, make a horizontal stroke —.

10. *Cause of Death*.—This to be given in full, as it appears in the Registration Certificate.

11. *How long in Liquor Trade when Assured?*—It is not expected that this information will be available in the majority of cases; but it is requested that the information may be given when contained in the proposal papers.

12. *If left the Trade and Extra remitted*.—The object here is to note those cases in which an extra rate charged for the occupation

has been taken off on account of the assured giving up his connection with the trade. This, however, is not to interfere with the filling-up of the card as to the subsequent death, exit, or existence of the life.

13. *If extra-rated for other causes than occupation.*—Note here the rate of extra (or addition to the age) and the cause.

14 *January* 1891.

Form of Card used in this Investigation.

FRONT.

No.	£
Name.....	
Designation.....	
Place.....	
English, Scotch, or Irish	
Date of Birth	18.....
.. Entry	18.....
.. Exit	18.....
Age at Entry.....	
.. Exit	
Duration of Policy	
How written off.....	
If extra-rated for other causes than occupation	
Cause of Death	

BACK.

How long in Liquor Trade when assured

If left the Trade, and extra remitted

SECOND REPORT *of the* COMMITTEE ON THE MORTALITY *among*
Persons engaged in the Sale of Intoxicating Liquors.

The Committee were appointed on 9 July 1890, and, at a meeting of the Association on 26 November of the same year, they submitted their first Report, which was afterwards printed and circulated among the Members of the Association. Appended to the print

is a statement as to certain questions that were considered at the meeting referred to.

The Investigation having now been completed, the Committee beg to present their Report upon its results.

Except one Office, whose practice had been to decline insurances on the lives of persons retailing liquor, all the Offices in the Association contributed their experience.

The Investigation covered the period between 1 January 1854 and 31 December 1890, and was confined to Assurances effected between those dates.

The Committee prepared and issued in printed form "Notes for the guidance of persons entrusted with filling up the cards." They beg to refer to those Notes for details as to the manner in which the experience was collected.

With regard to the questions considered by the Association on 26 November 1890, the Committee found that the Offices could not in any substantial number of cases give information as to how long the lives had been engaged in the business of selling liquor, nor as to the remission of extra premium on the trade being given up. The Committee state further on in this Report how they have dealt with lives described as "Grocers."

In preparing the "Notes" above mentioned, the Committee found it necessary to extend the classification of occupations suggested in their first Report; and, when the cards came to be arranged, they found it necessary to adopt a still wider range of designations, in order to deal satisfactorily with the lives embraced in the observations.

For the purpose of comparing the mortality in the various groups with that of a standard Table, calculations were made of the "Expected Deaths" by the H^M Table of Mortality. The following are the results arrived at, the various classes (except the "Miscellaneous") being arranged in the order in which their rates of mortality exceed those shown by the standard Table:—

MALE LIVES	No. of Lives.	Exposed to Risk	Average Duration of Risk	DIED		ANNUAL MORTALITY PER-CENT		
				Actual	Expected	Actual	Expected	Difference
1. Beer Sellers . . .	63	462	7·3	17	7·33	3·68	1·59	2·09
2. Publicans . . .	1,761	15,009	8·5	430	235·37	2·86	1·56	1·30
3. Inn-keepers . . .	911	10,757	11·4	306	198·44	2·84	1·81	1·00
4. Hotel-keepers . . .	979	8,033	8·2	204	125·34	2·54	1·56	·98
5. Wine and Spirit Merchants, Eng- land	149	1,578	10·6	40	25·17	2·53	1·59	·94
6. Licensed Grocers, Ireland	255	2,019	7·9	46	29·13	2·28	1·44	·84
7. Licensed Grocers, Scotland	713	5,915	8·3	102	74·93	1·72	1·26	·46
8. Wine Merchants, Scotland	130	1,362	10·5	23	17·80	1·69	1·31	·38
9. Grocers, Ireland . . .	502	5,723	11·4	96	90·95	1·68	1·59	·09
10. Grocers, Scotland . .	2,077	20,771	10·0	250	258·66	1·20	1·25	— ·05
11. Wine Merchants, England	140	1,496	10·7	23	24·33	1·54	1·63	— ·09
12. Travellers in Liquor Trades . . .	133	862	6·5	7	10·19	·81	1·18	— ·37
13. Miscellaneous . . .	575	5,228	9·1	80	71·36	1·53	1·37	·16
Total Male Lives	8,418	79,215	9·4	1,624	1,169·00	2·05	1·47	0·58
FEMALE LIVES.								
1. Hotel-keepers . . .	154	1,309	8·5	35	25·52	2·67	1·95	·72
2. Inn-keepers . . .	131	1,410	10·8	42	35·80	2·98	2·54	·44
3. Wives of Liquor Sellers	141	1,234	8·8	21	18·02	1·70	1·46	·24
4. Publicans	116	1,035	8·9	18	20·60	1·74	1·99	— ·25
5. Grocers	92	868	9·4	16	18·61	1·84	2·14	— ·30
Total Female Lives	634	5,856	9·2	132	118·55	2·25	2·02	0·23

With reference to the various classes in this Table, the Committee have to make the following observations.

No. 1. "Beer Sellers" were grouped by themselves, as it was thought possible that this class might show a rate of mortality different from that of ordinary Publicans, and this is seen to be the case. The death-rate as here shown is very high, being more than double the tabular rate; and although the group is a small one—only 63 lives, among whom were 17 deaths—there can be no doubt that lives of this class are extra hazardous.

No. 2. Under the head of "Publicans" are grouped Licensed Victuallers, "Wine and Spirit" Merchants, known or assumed (as mentioned below) to be retail dealers, and,

generally, all persons whose occupation appeared to consist in selling drink for consumption on their premises. It was not found practicable to ascertain, separately, the rates of mortality among Publicans in London and in the Provinces, as suggested in the Committee's first Report.

No. 3. "Inn-keepers."—This group embraces Inn-keepers in England and Scotland only, the designation being, apparently, not used in Ireland.

No. 4. "Hotel-keepers."—This includes all persons so described, whether in England, Scotland, or Ireland.

Nos. 5, 8, and 11.—The designation "Wine and Spirit Merchant" being often used by retail dealers (though not so frequently in England as in Scotland), the Committee had to consider which of the persons so describing themselves should be regarded as wholesale, and which as retail, dealers. They decided to place the English "Wine and Spirit" Merchants in a group by themselves, but to put the Scotch lives bearing that designation among the general class of "Publicans", with this exception, that persons insured for £1000 or upwards, in either country, were included in the class of "Wine Merchants", as being, probably, wholesale dealers.

The Committee think it may be fairly inferred from the results shown in the Table—(1) That a wholesale Wine Merchant is not exposed to extra hazard by his occupation; (2) That the bulk of the English "Wine and Spirit" Merchants, included in these observations, were probably retail dealers; and (3) That among the 130 Wine Merchants in Scotland were probably included a number of retail dealers.

Nos. 6, 7, 9, and 10.—In many cases it was not possible to ascertain whether a person described as a "Grocer" was, or was not, licensed to sell intoxicating liquor by retail. It was therefore thought advisable to put the "Licensed Grocers" and the "Grocers" in different groups, so that the mortality among them might be separately ascertained. Apparently licensed grocers in Scotland, selling liquor for consumption off the premises only, are subject to an extra rate of mortality; and licensed grocers in Ireland, who sell for consumption on the premises, are subject to a rate of mortality higher than that of the Scotch licensed grocer, but lower than that of the retail dealer who sells liquor only. The grocer who does not sell liquor is, apparently, an average risk.

No. 12. It is an unexpected result that the number of deaths among the 133 "Travellers" engaged in the Liquor Trade, has proved to be below the expected number by the Hrd Table; but the whole number exposed to risk is so small, that no practical conclusion can safely be deduced from the figures.

No. 13. There were a number of lives which, on consideration, the Committee did not see their way to place in groups by

themselves for the purpose of investigation, nor to class among other lives so grouped, namely:—

English Grocers (Licensed and Unlicensed)	156
Restaurant-keepers	124
Waiters	75
Brewers and Distillers	73
Wine and Spirit Merchants (Ireland)	26
Wine Merchants (Ireland)	24
Proprietors of Licensed Houses	19
Canteen Stewards	18
Brewers' Agents	16
Brewers' Clerks and Collectors	11
Bottlers	6
Cellarmen	5
Ships' Stewards	5
Other designations	17
	<hr/> 575 <hr/>

These were all included under the head of "Miscellaneous." In this class the mortality is about 10 per-cent in excess of the tabular rate.

It is remarkable that Female Publicans and Female Grocers show a very favourable rate of mortality, while the wives of Liquor Sellers show a small extra mortality; but in each of these classes the number exposed to risk is small. The mortality among Female Inn-keepers and Female Hotel-keepers, compares favourably with that among Male lives engaged in those occupations. Taking a general view of the figures, there seems to be no doubt that the extra risk in the case of Females, engaged directly or indirectly in the sale of liquor, is not so great as in the case of Males of corresponding occupations.

For the purpose of considering Rates of Extra Premium, Classes 9 to 13 in the above Table may be put out of consideration. Class 8 may also be left out of account, if the views suggested above as to the mortality among Wholesale Wine Merchants are considered to be well founded. In the absence of more extended information, all that can be said as to Class 1 (Beer Sellers) is that special caution should be exercised in admitting such lives, if they are not charged a higher rate of extra Premium than Publicans.

The figures in the above Table relating to the three important groups of Publicans, Inn-keepers, and Hotel-keepers, are as follows:

	ANNUAL MORTALITY PER-CENT		Extra Mortality per-cent
	Actual	Expected	
Publicans	2·86	1·56	1·30
Inn-keepers	2·84	1·84	1·00
Hotel-keepers	2·54	1·56	·98
Those relating to Licensed Grocers are:			
In Scotland	1·72	1·26	·46
In Ireland	2·28	1·44	·84

It is to be observed, however, that the extra risk is not the same at all periods of life. It appears, from an analysis of the figures, that under the age of 25 (where, however, the numbers exposed to risk are very small), and again from the age of 65 onwards, when the special risks of occupation have been safely gone through, the death-rate is practically normal. During the active period of life, from 25 to 65, the extra rates of mortality, calculated for decennial periods, are as follows:

Ages	Publicans	Inn-keepers	Hotel-keepers
25-34	·62	·59	·84
35-44	1·48	1·13	1·48
45-54	1·55	·89	·90
55-64	1·54	·70	·71

Considering the close agreement between the rates of mortality of Inn-keepers and Hotel-keepers, as shown in the foregoing Tables, and considering that there is no clear distinction between the two occupations, the Committee are of opinion that, for practical purposes, the lives of persons described as "Inn-keepers" and as "Hotel-keepers" may be grouped together. They have accordingly, in framing further Tables, formed the three classes above mentioned into two, namely: (1) Publicans, and (2) Inn-keepers and Hotel-keepers. Mortality Tables have been constructed for those two classes, and have been roughly graduated by a graphic process. The following is a comparison of the resulting Rates of Mortality (q_x), at quinquennial ages, with the corresponding rates of the H^M Table:

Age	H^M	ANNUAL MORTALITY PER-CENT (i_x)			
		Publicans		Inn-keepers and Hotel-keepers	
(1)	(2)	(3)	(3) - (2)	(4)	(4) - (2)
25	·66	1·18	·52	1·25	·59
30	·77	1·48	·71	1·60	·83
35	·88	2·08	1·20	1·90	1·02
40	1·03	2·59	1·56	2·21	1·18
45	1·22	2·86	1·64	2·51	1·29
50	1·60	3·08	1·48	2·72	1·12
55	2·10	3·67	1·57	3·14	1·04
60	2·97	4·59	1·62	3·76	·79
65	4·34	5·30	·96	4·80	·46
70	6·22	6·09	- ·13	6·52	·30

The variation in the Rates of Mortality has been investigated in another aspect, namely, as regards the length of time the lives had been under observation. The following Table shows the total mortality in the above two classes, divided into sections according to the duration of the Policies:—

Publicans.

Duration (Years)	Exposed to Risk	Died	Expected Deaths	Extra Mortality per-cent
0- 5	6,511	121	69.0	.80
5-10	3,721	115	49.6	1.76
10-20	3,620	138	71.3	1.84
20 and over	1,157	56	45.6	.90
Total	15,009	430	235.5	1.30

Inn-keepers and Hotel-keepers

0- 5	7,468	126	82.8	.58
5-10	4,570	136	64.2	1.51
10-20	4,890	162	100.7	1.25
20 and over	1,862	86	76.0	.54
Total	18,790	510	323.7	.99

It will be noted that, in this comparison, as well as in the preceding comparisons by Ages without reference to the duration of the Policies, the extra rate of mortality increases up to a certain point and then diminishes. As shown in the comparison by Ages, it finally disappears. These variations appear strongly to support the view suggested in the Committee's first Report, that the extra risk is materially dependent on the length of time for which the lives have been exposed to the adverse influences and temptations of their business. Probably in reference to this, as well as to age and to duration of Assurance, the extra risk increases to a maximum and then diminishes.

In their first Report the Committee also suggested the possibility that the legislation of 1872 might be found to have had some effect in diminishing the extra rate of mortality among Liquor Sellers in England. To determine this, the mortality among English Publicans, Inn-keepers, and Hotel-keepers, was separately ascertained for the period before and the period subsequent to the date mentioned. The following are the results :—

ENGLAND only	1854-1872					1873-1890				
	No. of Lives	Exposed to risk	Died	Expected Deaths	Extra Mortality per-cent	No. of Lives	Exposed to risk	Died	Expected Deaths	Extra Mortality per-cent
Publicans . .	330	1,959	54	25.8	1.44	502	2,758	70	34.8	1.27
Inn-keepers .	429	2,881	65	40.5	.85	267	1,688	40	21.2	1.11
Hotel-keepers.	139	635	18	8.0	1.57	352	1,974	52	24.3	1.40
Total . .	898	5,475	137	74.3	1.14	1,121	6,420	162	80.3	1.27

The Committee do not consider the difference between the results of the two periods sufficiently decided to affect materially the conclusions that may be drawn from the statistics taken as a whole. The comparatively low rate of mortality among Inn-keepers in the earlier period arrests attention. The Committee do not know how far the figures may have been affected by a different use of the term "Inn-keeper" from that which is now prevalent. They observe that the relative proportions of "Inn-keepers" and "Hotel-keepers" differ greatly in the two periods. Of 568 Inn-keepers and Hotel-keepers assured in 1854-1872, as many as 76 per-cent were described as Inn-keepers, and only 24 per-cent as Hotel-keepers; whereas, of 619 lives of both classes assured in 1873-1890, only 43 per-cent were described as Inn-keepers and 57 per-cent as Hotel-keepers. The proportion of Publicans, in the total numbers embraced in the Table, had increased from 37 per-cent in the earlier to 45 per-cent in the later period.

In view of the results arrived at with regard to the English lives, the Committee have not thought it necessary to carry out their original intention to investigate separately the Irish lives assured since 1874.

In attempting to arrive at some practical conclusion as to the rates of extra premium to be charged for this class of risks, the Committee have naturally been led to compare the rates of premium deduced from the Mortality Tables already referred to with the rates of the H^M Table on which the foregoing comparisons have been based. The following are the rates at quinquennial ages, interest being taken at 3 per-cent in each case; and the respective differences as compared with the H^M Table:—

AGE	H ^M	PUBLICANS		INN-KEEPERS AND HOTEL-KEEPERS	
		P' _x	Excess	P' _x	Excess
20	1.427	2.119	.692	2.042	.615
25	1.625	2.447	.822	2.343	.718
30	1.880	2.843	.963	2.662	.782
35	2.193	3.270	1.077	3.003	.810
40	2.589	3.676	1.087	3.387	.798
45	3.114	4.116	1.002	3.832	.718
50	3.801	4.693	.892	4.403	.602
55	4.725	5.471	.746	5.189	.464
60	5.987	6.385	.398	6.276	.289

The extra rates of premium shown by the above Table differ materially from those obtained by the rough-and-ready method of taking the percentage by which the aggregate number of deaths in each class exceeded the expected number. The percentages calculated in this way would be (as before shown)—

For Publicans	.	.	.	1.30
For Inn-keepers and Hotel-keepers99

The reason of the difference lies in the fact that the true extra premium for the risk falls to be calculated, not on the whole amount assured, but on the amount actually at risk: namely, the excess of the sum assured over the Policy-value or Reserve for the liability under the Policy. Where there are Bonus Additions, the extra rate should in strictness be applied also to the excess of the Bonus over the Reserve made in respect of it.

It is to be remembered that the extra rates shown above are net rates—that is to say, they contain no addition for expenses or other purposes. Also, that the Mortality Tables from which they are deduced are not “Select” Tables; but all lives passing through a given age have been enumerated together, without reference to the length of time they have been under observation.

Assuming that the Offices desire to maintain the system of charging for each class of risks a uniform extra rate, calculated on the original sum assured, the Committee are disposed to think that the following scale of extra premiums is warranted by the experience now reported upon, namely:—

	Per-cent per annum.
For Publicans	25s.
For Inn-keepers and Hotel-keepers	20s.
For Grocers, &c., in Ireland, licensed to retail liquor	15s.
For Licensed Grocers in Scotland	10s.

Apart from the question of the extra rates to be charged, the following considerations have occurred to the Committee in the course of this investigation:—

1. As different classes of Traders in liquor, “Wholesale” and “Retail” dealers for example, are subject to different rates of mortality, it is important to ascertain clearly at the time of an Insurance being proposed, to what class the life belongs.
2. As the Grocer who sells liquor, although for consumption off the premises only, is exposed to an appreciable extra risk which does not exist in the case of the Unlicensed Grocer, it seems necessary, when an Assurance is being proposed on the life of a person described as a “Grocer”, to ascertain whether he is, or is not, a Licensed Grocer.
3. The Committee would further suggest that Policies issued to persons whose occupation or circumstances may probably lead to their becoming retailers of liquor, should contain an express condition that the Assured shall not engage in the sale of liquor by retail without first obtaining license from the Company.
4. It seems advisable that when a remission of extra rate is proposed in consequence of the Assured having given up the sale of liquor, evidence should be obtained both as to health and habits.

In conclusion, the Committee think it right to say that the Association are under great obligations to Mr. Fletcher, the Secretary

in Edinburgh of the *City of Glasgow* Life Assurance Company, who offered his services for the arrangement of the cards and tabulation of the data contained in them, and devoted to this work a great deal of care and attention. The subsequent manipulation of the figures, and construction of the Mortality Tables and Rates of Premium, was undertaken by Mr. Low, who desires to say that he has been greatly assisted by Mr. Alexander Fraser, A.F.A., of the *Edinburgh Life* Office, in making the necessary calculations.

D. DEUCHAR.

T. B. SPRAGUE.

GEO. M. LOW.

Edinburgh, 15 July 1896.

NOTE BY THE COMMITTEE.

It has been suggested that there is an inconsistency between the observations as to "Beer Sellers" on page 254 of the Committee's Report, as printed, and the observations as to "Travellers" on page 255, the Committee apparently basing an opinion in one case upon data even more limited than those which in the other case were held insufficient to support a practical conclusion.

The Committee would point out that the existence of an extra hazard in the case of "Beer Sellers" may be inferred from the figures not only of that particular class, but also of the class of "Publicans", to which it is closely related. If, as the Committee believe, the Beer Seller is practically an inferior kind of Publican, the coincidence of experience (small as the figures are), with reasonable expectation, seems to leave no room for doubt that he belongs to an extra hazardous class.

On the other hand, the Committee had no such support for a conclusion, one way or the other, as to the risk involved in the occupation of a Traveller in the Liquor Trade. They felt that any conclusions they drew must be on the side of safety, and having only the evidence of 133 cases, with an average duration of $6\frac{1}{2}$ years, their view was that no practical conclusion could *safely* be deduced from the figures.

Edinburgh, 27 January 1897.

On the Rate of Mortality amongst the Female Nominees of the General Annuity Trust Fund, during the period from 1 January 1869 to 31 December 1895. By GEORGE KING, F.I.A., F.F.A.

THE Institute of Actuaries and the Faculty of Actuaries are at present jointly engaged on an investigation into the mortality experience of Life Assurance Companies, which will include an examination of the rates prevailing amongst Annuity Nominees. Nevertheless it is thought that any other investigation throwing light on this important subject may not be without interest.

For the purpose of the recent valuation of the General Annuity Trust Fund, the mortality experience of the female nominees was extracted, and the resulting rates give a much lighter mortality than that shown by any existing table. Moreover, the constitution of the Fund is such that the force known as Selection is practically excluded, and therefore a short note descriptive of the processes followed, and giving the rates deduced, is now submitted, in the hope that it may prove of service pending the publication of the much more extensive investigation above referred to.

The history of the Fund need not be recapitulated. Suffice it to say that for many years it has ceased to transact new business, and that each nominee has come on the Fund by the failure of another life. In fact, the benefits offered by the Fund were Reversionary Annuities to nominees, falling into possession on the deaths of members. In a few contracts there was a proviso that in the event of the nominee dying before the member, a second nominee might be appointed; but during the period covered by the present investigation the number of such cases was very small. In the event of the marriage of a nominee the annuity became suspended, but was liable again to become current on the death of the husband.

Only the experience of annuities during the time they were actually current was taken out; and therefore the entrants consisted of (1) those who for the first time became annuitants by the death of a member, or of a husband, and (2) those whose annuities, after being in suspense, again came into possession by the death of a husband other than the assured life. Lives passed from observation (1) by the marriage of annuitants, and (2) in the ordinary way by death. There were no surrenders. Therefore the only place where selection could come in was in the

withdrawals by marriage, it being probable that the healthy lives on the average would be more likely to marry than the unhealthy, and that therefore the rate of mortality might be increased by such withdrawals. There were, however, during the period embraced in the investigation, very few marriages, only 64 in all; and of these only 28 occurred after the age of 48. Below that age the experience is too limited to produce trustworthy results, and, in the annuity tables constructed, has been neglected; so that it may be assumed that such selection as may have occurred on marriage can have produced no appreciable effect on the annuity-values.

The experience is one of Annuities, and not of Lives. In a number of cases more than one annuity existed on the life of the same nominee, but it was found that such cases were distributed with some regularity over the experience. The labour would have been greatly increased by eliminating the duplicate annuities, and it was evident that by so doing no practical difference would have been produced in the final results.

The particulars were taken out on cards in the usual way, and when the ages had been filled in, the cards were sorted and counted, and the numbers entered in the schedule.

The ages which had to be noted in the enquiry were (1) the age of the nominee on the date of first becoming an annuitant; (2) the age on 1 January 1869, in the event of the nominee being an annuitant at that date; (3) the age on re-marriage after 1 January 1869; (4) the age of re-widowhood after 1 January 1869; (5) the age at death; and (6) the age if existing on 31 December 1895.

The age was taken under (1) as the nearest integral age, say y . Under (2) y + the nearest integral duration; under (3), (4), and (5) y + the curtate duration; and under (6) y + the nearest integral duration. In respect of (3) and (4), the curtate duration was taken in view of certain further investigations, which were contemplated, but not carried out. On account of (3) and (4) being thus differently dealt with from (2) and (6), a necessary difference in treatment between ω and m , and ϵ and η respectively, appears in the succeeding formulas for Exposed to Risk. Transferring now the ages to the respective epochs between 1 January 1869 and 31 December 1895, and using x as the symbol of age, and writing E_x for the Exposed to Risk from the year of age x to $x+1$, ϵ_x for the Existing on 1 January 1869, η_x for the Existing on 31 December 1895, n_x for the Entrants by the death of a member, ω_x for the Entrants by re-widowhood, m_x for the

Withdrawals by marriage, and d_x for the Deaths, we have the formula

$$E_x = E_{x-1} + \epsilon_x + n_x + \frac{1}{2}(\omega_{x-1} + \omega_x) - \frac{1}{2}(m_{x-1} + m_x) - d_{x-1} - \eta_x$$

or, giving the formula in summation form,

$$E_x = \sum_0^x (\epsilon_x + n_x + \omega_x - \eta_x - m_x - d_{x-1}) - \frac{1}{2}(\omega_x - m_x).$$

The following Table No. 1 gives the data as extracted from the cards, together with the Exposed to Risk deduced by the formulas above mentioned,—both formulas having been used for the purpose of check,—and the ungraduated rates of mortality.

TABLE NO. 1.

x Age	ϵ_x Existing on 1 January 1869	n_x Entrants by the Death of Members	ω_x Entrants by Re-widow- hood	m_x With- drawals by Marriage	d_x Deaths	η_x Existing on 31 Dec. 1895	E_x Exposed to Risk	q_x Rate of Mortality Ungradu- ated
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
14	1	1	...
15	...	4	5	...
16	5	...
17	5	...
18	1	6	...
19	6	...
20	1	7	...
21	1	6·5	...
22	6	...
23	...	2	8	...
24	...	3	11	...
25	11	...
26	3	1	13·5	...
27	13	...
28	2	15	...
29	1	...	1	16·5	...
30	1	1	...	6	16	...
31	2	2	17	...
32	2	12	31	...
33	11	6	...	1	1	...	47·5	·0211
34	16	11	...	1	72·5	...
35	7	4	1	1	83	...
36	4	5	2	90	...
37	10	2	3	...	102	·0294
38	14	7	1	6	117·5	...
39	11	2	1	2	2	...	127·5	·0157
40	6	9	1	2	139·5	...
41	11	9	6	1	2	...	161·5	·0124
42	20	14	3	2	1	...	196·5	·0051
43	13	14	1	...	223	·0045
44	20	9	1	6	4	1	247·5	·0162
45	21	14	2	3	274	...
46	13	24	3	...	1	...	313·5	·0032
47	22	13	3	3	2	1	348	·0057
48	19	25	3	3	1	10	380	·0026
49	43	20	4	...	8	2	442	·0181
50	33	26	1	3	2	1	493	·0041
51	16	24	3	7	6	2	526	·0114
52	13	21	3	6	6	2	548·5	·0109

TABLE NO. 1—(continued).

x Age	ϵ_x Existing on 1 January 1869	n_x Entrants by the Death of Members	ω_x Entrants by Re-widow- hood	m_x With- drawals by Marriage	d_x Deaths	η_x Existing on 31 Dec. 1895	E_x Exposed to Risk	q_x Rate of Mortality Ungrad- uated
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
53	34	32	4	...	7	4	605	·0116
54	31	32	3	...	7	1	663·5	·0106
55	46	31	1	2	16	13	721·5	·0222
56	39	33	2	...	11	10	768	·0143
57	47	32	2	1	23	19	818·5	·0279
58	68	29	6	...	13	8	888	·0146
59	56	33	1	1	19	22	945	·0201
60	51	27	6	...	22	12	995	·0221
61	63	41	5	2	25	22	1,059·5	·0236
62	51	55	3	...	26	15	1,128·5	·0230
63	75	25	6	1	34	10	1,196·5	·0284
64	64	51	7	...	21	27	1,256·5	·0167
65	65	40	6	1	40	22	1,324·5	·0302
66	87	52	5	...	33	26	1,402·5	·0235
67	48	31	...	1	54	16	1,434·5	·0376
68	58	50	5	...	64	16	1,474·5	·0434
69	48	59	5	3	62	29	1,492	·0416
70	36	51	51	31	1,487	·0343
71	46	54	6	...	85	37	1,502	·0566
72	65	45	1	...	56	41	1,489·5	·0375
73	39	50	2	...	80	22	1,502	·0533
74	28	36	70	28	1,459	·0480
75	32	23	99	47	1,397	·0709
76	20	39	1	...	94	54	1,303·5	·0721
77	25	29	94	46	1,218	·0772
78	28	18	113	41	1,129	·1000
79	23	14	125	38	1,015	·1231
80	14	31	95	21	914	·1039
81	16	8	1	...	98	30	813·5	·1205
82	12	19	74	39	708	·1045
83	5	11	74	29	621	·1192
84	7	4	97	44	514	·1887
85	3	4	79	16	408	·1936
86	6	51	25	310	·1645
87	1	67	14	246	·2724
88	4	29	10	173	·1676
89	3	2	39	7	142	·2746
90	25	13	90	·2778
91	10	3	62	·1613
92	...	3	12	7	48	·2500
93	2	4	32	·0625
94	4	2	28	·1429
95	3	...	24	·1250
96	8	2	19	·4211
97	1	...	11	·0909
98	3	...	10	·3000
99	3	...	7	·4286
100	3	...	4	·7500
101	1	...
102	1	...	1	1·000
	1,681	1,377	115	64	2,162	947	41,994·5	...

TABLE NO. 2.

Age x	l_x	d_x	q_x	p_x	${}_0e_x$	${}_x a_x$ at $2\frac{1}{2}^\circ, {}_0 \text{ int.}$	Age x
49	100,000	1,019	0102	9898	24.4	16.929	49
50	98,981	1,050	0106	9894	23.6	16.531	50
51	97,931	1,097	0112	9888	22.9	16.126	51
52	96,834	1,161	0120	9880	22.1	15.716	52
53	95,673	1,243	0130	9870	21.4	15.305	53
54	94,430	1,332	0141	9859	20.6	14.893	54
55	93,098	1,425	0153	9847	19.9	14.484	55
56	91,673	1,522	0166	9834	19.3	14.077	56
57	90,151	1,623	0180	9820	18.6	13.673	57
58	88,528	1,726	0195	9805	17.9	13.272	58
59	86,802	1,823	0210	9790	17.2	12.874	59
60	84,979	1,912	0225	9775	16.6	12.479	60
61	83,067	1,993	0240	9760	16.0	12.085	61
62	81,074	2,075	0256	9744	15.4	11.691	62
63	78,999	2,150	0272	9728	14.7	11.299	63
64	76,849	2,213	0288	9712	14.1	10.906	64
65	74,636	2,276	0305	9695	13.5	10.509	65
66	72,360	2,337	0323	9677	12.9	10.111	66
67	70,023	2,394	0342	9658	12.4	9.710	67
68	67,629	2,448	0362	9638	11.8	9.305	68
69	65,181	2,504	0384	9616	11.2	8.896	69
70	62,677	2,582	0412	9588	10.6	8.482	70
71	60,095	2,656	0442	9558	10.1	8.068	71
72	57,439	2,716	0473	9527	9.5	7.652	72
73	54,723	2,802	0512	9488	9.0	7.232	73
74	51,921	2,902	0559	9441	8.4	6.813	74
75	49,019	3,035	0619	9381	7.9	6.397	75
76	45,984	3,219	0700	9300	7.4	5.990	76
77	42,765	3,400	0795	9205	6.9	5.602	77
78	39,365	3,543	0900	9100	6.4	5.238	78
79	35,822	3,618	101	8990	6.0	4.900	79
80	32,204	3,639	113	887	5.7	4.586	80
81	28,565	3,571	125	875	5.3	4.300	81
82	24,994	3,424	137	863	5.0	4.037	82
83	21,570	3,235	150	850	4.7	3.795	83
84	18,335	2,989	163	837	4.5	3.576	84
85	15,346	2,701	176	824	4.2	3.379	85
86	12,645	2,377	188	812	4.0	3.204	86
87	10,268	2,054	200	800	3.8	3.044	87
88	8,214	1,741	212	788	3.7	2.900	88
89	6,473	1,450	224	776	3.5	2.772	89
90	5,023	1,175	234	766	3.4	2.662	90
91	3,848	935	243	757	3.3	2.562	91
92	2,913	731	251	749	3.2	2.469	92
93	2,182	561	257	743	3.1	2.379	93
94	1,621	425	262	738	3.0	2.282	94
95	1,196	318	266	734	2.8	2.169	95
96	878	237	270	730	2.7	2.029	96
97	641	176	275	725	2.5	1.848	97
98	465	131	282	718	2.2	1.613	98
99	334	96	288	712	1.9	1.303	99
100	238	87	364	636	1.4	876	100
101	151	87	578	422	1.0	412	101
102	64	64	1.000	000	.5	...	102

It will be noted that the total number of years of life was 41,994.5 with 2,162 deaths. The experience below age 50 was too limited to be trustworthy, but that above age 50 could be relied on. At age 50 and over there were 38,429.5 years of life with 2,136 deaths.

Starting from age 49, graduation by a graphic process was effected, q_x being the function used. On multiplying the resulting graduated q_x into the Exposed to Risk so as to obtain the expected deaths, and on comparing these with the actual deaths, it was found that for ages 49 and upwards the expected deaths were 2,143.6, as compared with the actual deaths, 2,144, and that at no point was the accumulated error of any magnitude. The graduation may therefore be accepted as perfectly satisfactory.

In the preceding Table No. 2 (page 266) are given the usual columns of l_x , d_x , q_x , p_x , \hat{e}_x , and a_x at $2\frac{1}{2}$ per-cent interest, the radix selected being 100,000 living at age 49.

Selection not appearing in this experience, it is reasonable to assume that the most suitable existing annuity table for purposes of comparison is that of the Government Female Nominees after selection has passed off, that is, the table embracing the Government Experience four years and later after the grant of the annuity. The Exposed to Risk at each age was therefore multiplied by the q_x of the Government Table, and the following Table No. 3 gives a summary of the results.

TABLE NO. 3.

Ages	Years of Life (Exposed to Risk)	Actual Deaths	Expected Deaths, Government Experience	Percentage of Actual Deaths to Expected
50- 54	2,836	28	43.6	64.2
55- 59	4,141	82	81.2	101.0
60- 64	5,636	128	143.0	89.5
65- 69	7,128	233	269.1	94.0
70- 74	7,439.5	342	426.7	80.2
75- 79	6,062.5	525	552.4	95.1
80- 84	3,570.5	438	490.0	89.4
85- 89	1,279	265	262.8	100.8
90- 94	260	53	76.9	68.9
95- 99	71	18	31.1	57.9
100-102	6	4	5.1	78.4
...	38,429.5	2,136	2,381.9	89.7

It will be observed that, except in two quinquenniums of life, namely ages 55 to 59, and 85 to 89, where the General Annuity

experience is fractionally heavier than that of the Government, the rates of mortality now exhibited are remarkably low, and on the average fall short of the Government rates by 10·3 per-cent. This characteristic has of course material effect on annuity-values, as is shown in the following comparative Table No. 4. Not only are the annuity-values by this new experience higher than those given by the Government experience after selection has become exhausted, but, except at very advanced ages, they exceed the Government select values. The importance of this fact in the granting of annuities at the present day does not require to be emphasized.

TABLE No. 4.

Comparison of Annuity-Values at $2\frac{1}{2}$ per-cent Interest.

Age	General Annuity Fund a_x	Government, Non-Select $a_{[x-4]+4}$	Government, Select $a_{[x]}$
50	16·531	15·960	16·190
55	14·484	14·059	14·329
60	12·479	12·067	12·333
65	10·509	9·946	10·296
70	8·482	7·902	8·260
75	6·397	5·985	6·537
80	4·586	4·405	5·044

REVIEW.

*Sickness and Mortality Experience of Friendly Societies in England and Wales.**

THE Blue Book, comprising Mr. William Sutton's Report upon the sickness and mortality experience of Friendly Societies from 1856-80, contains a vast store of facts, the publication of which is of the greatest interest and importance to all concerned in the financial position of Friendly Societies in this country.

The extent and importance of the new materials dealt with in the Report, may be gathered from the following statement taken from Mr. Sutton's preface, the figures relating to the Manchester Unity,

*1. Blue Book, containing Special Report on Sickness and Mortality in Registered Friendly Societies, together with Tables, &c., by W. Sutton, Actuary to the Friendly Societies (Central Office), Eyre & Spottiswoode, London; John Menzies & Co., Edinburgh and Glasgow; and Hodges, Figgis & Co., Ltd., Dublin.

2. Friendly Societies' Valuation and other Tables, by Wilfred A. Bowser. Charles & Edwin Layton, London.

and Ancient Order of Foresters, being added for the purpose of comparison:

Experience	Exposed to risk of Sickness	Total weeks' Sickness	Exposed to risk of Death	Total Deaths
Males England (1856-60)	722,338	1,166,208	788,891	8,698
" " (1861-70)	1,789,532	3,199,139	1,789,532	24,530
" Wales (1856-75)	167,255	357,458	177,897	2,619
Females England and Wales (1856-75)	139,122	325,613	146,793	2,042
Males England and Wales (1876-80)	1,662,562	3,147,044	1,662,562	23,048
Total	4,480,809	8,195,462	4,565,675	60,937
M. U. 1866-70	1,321,048	1,975,033	1,321,048	16,680
A. O. F. 1871-75	1,302,166	1,769,036	1,302,166	15,815

The relative weight of the new experience is, indeed, somewhat greater than would appear at first sight from the above figures, as, in order to avoid the uncertainty and complication arising from the varying periods at which members became free to benefit in different Societies, the years of membership, 0, 1, and 2 (practically the first two-and-a-half years) were, in the case of both the Male 1861-70 and 1876-80 sections, excluded from the experience.

Of the five sections referred to in the above table, that to which Actuaries will turn with most interest is the Male experience for the Quinquennium 1876-1880, representing as it does the latest body of facts, having reference to rates of sickness and mortality in Friendly Societies.

This portion of experience was divided into five groups, according to the population of the locality in which each Society was registered, the classification being as under:

1. Places with under 2,000 inhabitants.	Exposed to Risk, 364,324
2. " " between 2,000 and 7,000 inhabitants.	" " 377,345
3. " " " 7,000 and 25,000	" " 300,206
4. " " " 25,000 and 100,000	" " 363,205
5. " " " upwards of 100,000 inhabitants.	" " 257,482

the number exposed to risk in each class having been supplied by Mr. A. W. Watson.

The facts relating to each of these five groups are stated in considerable detail for each age at entry and each year of membership; the first twelve months of sickness being divided into twelve periods of four or five weeks each, and that for the second year being separated into two periods of six months each.

As the form of return, upon which the experience is based, provided definite information as to the commencement of each attack, it will be seen that we have now, probably for the first time, a thoroughly satisfactory analysis of sickness claims in respect to the duration of illness. Unfortunately, the less complete returns furnished by the Societies prior to 1875, from which the Female and

Welsh Experience were drawn, have only permitted a classification of the sickness into "under two years" and "over two years" duration respectively.

Our readers will, probably, be mainly interested in the comparison of the new data with that which they have hitherto been accustomed to use. The nearly simultaneous appearance of Mr. W. A. Bowser's elaborate tables, based upon the M.U. Experience, enables this comparison to be made with facility. Mr. Bowser, having graduated both the sickness and mortality tables by Woolhouse's formula, has kept much nearer to the original facts than did the earlier tables constructed by Ratcliffe.

The following table gives the rates of sickness for quinquennial ages for the first six months, second six months, and remainder of illness, together with the rates of mortality according to Mr. Sutton's Tables and the M.U. respectively. It will be seen at once that, at all ages, the New Experience gives a greater aggregate sickness rate than the "Manchester Unity", the difference being mainly due to the larger amount of sickness after the first twelve months.

COMPARATIVE SICKNESS AND MORTALITY RATES.

Ages	1ST 6 MONTHS OF ILLNESS		2ND 6 MONTHS OF ILLNESS		REMAINDER OF ILLNESS		MORTALITY PER-CENT.	
	Government Experience Males 1876-80	M. U. 1866-70	Government Experience Males 1876-80	M. U. 1866-70	Government Experience Males 1876-80	M. U. 1866-70	Government Experience Males 1876-70	M. U. 1866-70
20	·775	·608	·045	·026	·067	·011	·702	·581
30	·779	·740	·055	·060	·123	·064	·709	·799
40	·997	·891	·086	·094	·285	·167	1·066	1·116
50	1·396	1·278	·160	·209	·622	·477	1·730	1·670
60	2·116	2·113	·402	·501	1·803	1·362	3·402	3·114
70	3·441	3·789	1·221	1·464	7·576	5·129	7·233	6·741

It has been, we believe, a general opinion that the "Manchester Unity" Experience under-estimates this portion of the sickness liability in the case, at least, of the average Friendly Society, and the results of the New Experience confirm this opinion.

The difference between the mortality rates in the two sets of data is not so marked, the new rates being generally lower up to about 45 years of age, and, thereafter, higher than the M.U. rates, the difference increasing as age advances. The net result, however, of these divergencies is to leave a larger reserve under Mr. Sutton's Tables than under the M.U., the difference depending, of course, partly on the age at which sickness is assumed to cease and pensions to begin, and partly on the nature of the reductions made in the sickness allowance during prolonged illness. The nature of the differences will best be seen by the following comparison of the values of the sickness benefit, up to ages 65 and 80 respectively, with pensions thereafter, the figures for the "Manchester Unity" being taken from Mr. Bowser's work, while those for the Government

Experience are based on the monetary tables forming the concluding part of Mr. Sutton's Report :—

Value of Sickness and Pension Allowances of £1 per week, at 3 per-cent.

Ages	SICKNESS CEASING AND PENSION COMMENCING AT AGE 65									
	1ST 6 MONTHS OF ILLNESS		2ND 6 MONTHS OF ILLNESS		REMAINDER OF ILLNESS		PENSION		FULL PAY 1ST 6 MONTHS, $\frac{1}{2}$ PAY THEREAFTER, PENSION OF $\frac{1}{4}$ PAY	
	Govt. Males, 1876-80	M.U.	Govt. Males, 1876-80	M.U.	Govt. Males, 1876-80	M.U.	Govt. Males, 1876-80	M.U.	Govt. Males, 1876-80	M.U.
20	21·70	20·14	2·25	2·53	7·55	5·09	55·9	59·6	40·7	38·9
30	22·01	20·62	2·66	3·12	9·84	6·95	79·9	55·8	48·2	47·1
40	21·68	20·44	3·10	3·69	12·13	8·92	117·3	126·4	58·6	58·3
50	18·83	18·50	3·28	3·96	13·55	10·31	180·1	194·6	72·3	71·3
60	10·20	10·26	2·32	2·69	10·10	7·57	307·3	326·9	93·2	97·1

Ages	SICKNESS CEASING AND PENSION COMMENCING AT AGE 80									
	1ST 6 MONTHS OF ILLNESS		2ND 6 MONTHS OF ILLNESS		REMAINDER OF ILLNESS		PENSION		FULL PAY 1ST 6 MONTHS, $\frac{1}{2}$ PAY 2ND 6 MONTHS, $\frac{1}{4}$ PAY THEREAFTER, PENSION OF $\frac{1}{4}$ PAY	
	Govt. Males, 1876-80	M.U.	Govt. Males, 1876-80	M.U.	Govt. Males, 1876-80	M.U.	Govt. Males, 1876-80	M.U.	Govt. Males, 1876-80	M.U.
20	25·13	24·05	3·44	4·12	15·72	11·13	4·4	5·2	31·9	30·2
30	26·92	26·24	4·36	5·42	21·54	15·65	6·3	7·5	36·0	34·7
40	28·88	28·73	5·59	7·07	29·30	21·73	9·2	11·1	41·3	40·5
50	29·89	31·26	7·11	9·17	39·91	30·04	14·1	17·1	47·0	47·7
60	29·07	31·69	8·85	11·42	55·07	40·71	24·1	28·6	53·3	54·7

It will be seen that the value of the sickness benefit after the first twelve months is about 40 per-cent greater by the new tables. On the other hand, the value of the deferred pensions is somewhat less, so that, if we assume reductions in the sickness allowances as shown in the last two columns of the table, the difference between the values given by the new tables and those of the "Manchester Unity" is not very great. In estimating the relative reserves brought out at a valuation, account would, of course, have to be taken of the smaller value of the future contributions according to Mr. Sutton's tables.

In addition to the main table, a separate table is given, representing the experience of the Welsh Male Societies from 1856 to 1875. The report states that the majority of members dealt with in the Welsh returns were engaged in what are known as unhealthy occupations, being employed mainly in collieries and iron works. The rates of sickness and mortality, however, do not show those

divergencies from the general table which will perhaps be expected. It is true that, up to about age 45, the Welsh rates, both of sickness and mortality, are higher than those of the Males (1876-80), and again, with respect to sickness rates, beyond age 65; the mortality rates, however, being lower for ages over 45.

The following table gives the Welsh sickness and mortality rates (1856-75), those for the Male (1876-1880) experience, and for the Manchester Unity Miners, 1866-1870 (Rateliffé, p. 103), from which it will be seen that the latter data would give a much higher value for the sickness liability for all ages up to 60. The smaller rates of mortality shown at the older ages in the Welsh experience are to some extent apparent in the "Manchester Unity" figures.

Age	SICKNESS PER ANNUM IN WEEKS			MORTALITY PER-CENT		
	Males, 1876-80	Welsh, 1856-75	Miners, M. U., 1866-70	Males, 1876-80	Welsh, 1856-75	Miners, M. U., 1866-70
20	·887	1·069	1·018	·702	·794	·846
30	·957	1·078	1·246	·709	·946	·721
40	1·368	1·376	1·732	1·066	1·076	1·150
50	2·178	2·055	2·798	1·730	1·657	1·716
60	4·321	4·172	5·388	3·402	2·900	3·254
70	12·238	14·738	12·811	7·233	6·284	6·407

Among the many interesting points upon which the new experience throws light, is the effect of selection on the sickness rates. This varies somewhat strangely with the age. On the whole, the effect would appear to be very slight, judging by a comparison of the unadjusted rates at the different ages for years of membership three to ten, and those for years 11 and upwards. It would appear, however, on a somewhat cursory examination of the figures, that the rates of sickness among the lives entering at the younger ages, especially about age 18, remain permanently lower than the average, which may be due to the fact that members entering at these ages belong to a certain special type of Society or of locality.

The magnitude of the returns, no doubt, precluded the possibility of dealing separately with the different population districts. A rough examination of the unadjusted mortality rates, however, for the rural districts (population under 2,000) shows them to be considerably—approximately about 20 per-cent—below the rates for all districts combined, without, so far as one can see, any similar abatement in the rates of sickness. This would accord, on the whole, with one's experience of Rural Societies, which will usually be found to require special treatment as regards mortality.

No tables that we are aware of, dealing with the experience of Female Friendly Societies, have been published since those of Mr. Finlaison's, in 1854. A certain special interest therefore attaches to the Female tables included in Mr. Sutton's report, although, from the smaller extent of the data and their more limited application, they cannot, of course, compare in importance with the Male table.

The most noticeable feature of the Female experience is, as might

be expected, the higher sickness and mortality rates at the younger ages, and the lower rates beyond middle life. While the sickness rates, however, cross about age 54, the mortality curve intersects about twelve years earlier at age 42, the rates for ages beyond 50 being remarkably low.

These differences will be sufficiently indicated in the following comparison of the Male and Female rates.

Age	MALES			FEMALES		
	Sickness in Weeks		Mortality per-cent	Sickness in Weeks		Mortality per-cent
	First 2 years	After 2 years		First 2 years	After 2 years	
20	·862	·025	·702	1·005	·005	·772
30	·884	·073	·709	1·148	·071	·860
40	1·167	·201	1·066	1·470	·274	1·089
50	1·734	·444	1·730	1·630	·795	1·199
60	2·962	1·359	3·402	2·265	1·227	2·451
70	6·254	5·984	7·233	3·453	8·965	6·144

The Report contains ample materials for investigating the financial effect of secessions upon contributions, or upon reserves, and concludes with a complete series of monetary tables, showing the value of the benefits for the whole of life, at rates of interest proceeding by intervals of $\frac{1}{4}$ per-cent from $2\frac{1}{2}$ per-cent to 4 per-cent, and tables of the logarithms of the commutation functions, allowing the calculation of temporary or deferred benefits to be made with facility.

G. F. H.

CORRESPONDENCE.

MR. ACKLAND'S PAPER ON RATES OF MORTALITY AND WITHDRAWAL.

"DISCONTINUANCES."

To the Editor of the Journal of the Institute of Actuaries.

SIR,—In the course of the debate on the paper read before the Institute, last session, by Mr. Ackland, I ventured to suggest that discontinuances are due to a force essentially different in character from the force which causes deaths, that the former force would be more appropriately measured by rates of non-renewal than by rates or forces of withdrawal (as ordinarily understood), and that the facts of an experience of assured lives, as exhibited by the Nearest Duration Method are directly applicable to the calculation of rates of non-renewal. I now write, with the object of expressing my views more clearly.

I suggest, in the first place, that discontinuances in ordinary business are attributable to a force essentially different in character from the force which operates to produce mortality. Death claims are caused by a force which operates *continuously* throughout the history of policies, whereas discontinuances are caused—so far as regards the large majority of policies—by a *discontinuous* force coming into operation at certain recurring epochs. A policyholder does not exercise his option of withdrawing *continuously* in the same sense that he is continuously subject to the risk of death; he exercises it periodically, in most cases on the occasions of his receiving a renewal notice, and having to decide whether to pay or not to pay. This applies, I think, to nearly all lapses, the only exceptions that occur to me being the few cases in which a policyholder pays a series of fines to extend the days of grace for successive short periods; and lapses constitute a very large proportion of the whole number of discontinuances. It applies, also, to all surrenders carried out at or about the renewal date. The proportion which these form of the entire body of surrenders, will no doubt vary in different classes of business; in a small experience of two years, I found that 55 out of a total of 94 surrenders took place during the days of grace, 11 took place within a month before the renewal date (probably on receipt of renewal notices), and the remaining 28 were scattered. Having regard to the small proportion that the number of scattered surrenders forms of the entire number of lapses and surrenders, I think it may fairly be stated as a general proposition, that discontinuances are mainly due to the exercise of a periodical option at or about the renewal date.

If this proposition be admitted, it follows that the force which causes discontinuances would be more appropriately measured by rates of non-renewal, than by rates or forces of withdrawal—in other words, by the ratios that the withdrawals at definite epochs bear to the exposed to risk of withdrawal at those epochs, than by the ratios that the withdrawals in given periods bear to certain numbers supposed to be continuously exposed to the risk of withdrawal throughout those periods. As applied to a collected experience, this second proposition pre-supposes a policy-year tabulation of the observed facts. A tabulation by calendar-years, or years of life, will, of course, have the effect of spreading the discontinuances over the years of observation, and will thus exhibit something of the nature of a continuous force of discontinuance. If it be admitted that such a force has no real existence, the fact that it is artificially created by any method of tabulation other than one that follows the years of assurance may be considered another argument in favour of the Policy-year Method.

I proceed now to consider the applicability of the facts of an experience, as exhibited by the Nearest Duration Method, to the calculation of rates of non-renewal. For the first two years of assurance, during which the exposed to risk of death and the rate of mortality are rapidly changing—the former by discontinuances, and the latter by the lapse of time since selection—it may, perhaps, be considered desirable to exhibit a large experience, such as that in course of compilation by the Institute, by quarters of a year. The

discontinuances during that period will consist almost entirely of lapses. Even assuming that the instructions given by the New Experience Committee have achieved the difficult task of securing absolute uniformity of treatment by the contributing offices in regard to such matters as dating-back and days of grace, the effect of an attempt to classify these discontinuances *in* the quarters—with the object of calculating rates of withdrawal on the analogy of rates of mortality—will be to locate in, and to represent as spread over, say, the second quarter, a large number of lapses which occurred by nonpayment of the second quarterly premium; the effect of a Nearest Duration tabulation, on the other hand, will be to convey what seems to me to be the real meaning of these discontinuances by representing them as having occurred at the *end* of the first quarter. If absolute uniformity of treatment has *not* been secured, the first-mentioned method of tabulation will stereotype the divergencies by locating some of the discontinuances in the first quarter and some in the second, whereas the Nearest Duration Method will neutralize them. Passing on to the general effect of the Nearest Duration Method, as applied to a tabulation by integral years, I cannot do better than refer to the result of Mr. Ackland's investigation of the average error resulting from the operation of the method (vol. xxxiii, pp. 144-5). Upon the assumption that yearly, half-yearly, and quarterly cases enter into a general experience in the proportion of $62\frac{1}{2}$, $32\frac{1}{2}$, and 5 per-cent respectively, and that surrenders take place *one month* on the average before the renewal date, Mr. Ackland shows that the Nearest Duration Method will dislocate the average date of surrender by only about $1\frac{1}{2}$ days. The conclusion to which this result leads is enforced by the considerations that the proportion of yearly cases persisting long enough to be entitled to a surrender-value is relatively larger than the proportion coming on the books, and that the majority of surrenders take place (according to my observation), not before the renewal date, but during the currency of the days of grace. A fact of importance, moreover, is that the Nearest Duration Method not only locates the whole body of surrenders correctly *on the average*, but also locates the majority of them *exactly*. Taking Mr. Ackland's proportions of yearly, half-yearly, and quarterly cases, and assuming that all are equally likely to discontinue, 80 per-cent of the renewal-date discontinuances (*i.e.*, lapses and such surrenders as occur at the renewal date), represented by the Nearest Duration Method as having occurred at the end of the year, will have actually occurred there; the remaining 20 per-cent will have been about equally drawn from the preceding and succeeding quarters or half-year. Hence, as applied at annual intervals, the Nearest Duration Method accurately represents the rates of non-renewal for a majority of the cases under observation, and makes an approximate allowance for the rates experienced at adjacent quarters. Objection may be taken to the dislocation of the discontinuances occurring at the ends of half-years—one-half of which would have to be collected at the end of the year, and the other half at the end of the preceding year—but this objection does not appear so serious as the objection that may be urged to tabulating as having occurred *during* the years the far

larger number of discontinuances occurring exactly at the *end* of the year.

The general conclusions at which I arrive are that the Nearest Duration Method locates the discontinuances fairly, and, to a large extent, exactly, for the purpose of an experience compiled by *years*, and that, if the experience for the first year or two be compiled by quarters, the Method will, during that period, represent the discontinuances with absolute accuracy. I venture to suggest, moreover, not only that the Nearest Duration Method collects the discontinuances in such a way as to enable the force which causes them to be appropriately measured, but also that the curtate duration method collects them in such a way as to lead to a misrepresentation of the force, and that the Exact Duration Method offers no advantages over the Nearest Duration Method for the calculation of rates of non-renewal.

Although it is my immediate object to show that the way in which discontinuances are dealt with by the Nearest Duration Method is appropriate to the accurate measurement of the force which causes discontinuances and does not, therefore, constitute any objection to the employment of that method, it may be of interest to consider the application of the resulting functions to one or two of the problems that arise in connection with the question of withdrawal. The most important of these problems relates to the influence of withdrawals on the rate of mortality, and, for its solution, rates of non-renewal—exhibiting the force of discontinuance as operating at the ends of years—appear likely to be more useful than rates of withdrawal, the effects of which have to be looked for partly in the year to which they relate and partly in subsequent years. If on comparison of two experiences a materially higher rate of non-renewal is exhibited by one, at the end, say, of the third year of assurance, than by the other, a more confident conclusion could, I think, be drawn from the relative progression of the rates of mortality from the third to the fourth year than would be possible from a comparison of rates of withdrawal. In this connection I may refer to Mr. Frank Sanderson's recently-published exhibition of the Canada Life Experience, the facts of which are tabulated by the Nearest Duration Method; the following is an extract from a section relating to the rate of discontinuance in Mr. Sanderson's Report:

“In obtaining the rates of mortality by years of assurance, we have seen that it is a necessary condition of the policy-year method that the deaths should be allocated to the policy-year in which death takes place.

“If it were thought necessary to obtain with equal precision the rate of discontinuance, it would have been necessary to tabulate the discontinuances in a manner similar to the deaths, *i.e.*, in the exact policy-year of discontinuance. But, in view of the fact that the rate of discontinuance is less regular than that of mortality, differing according to different companies' different plans of assurance, and other circumstances, it was thought that for the present purpose, at least, the tabulation of the withdrawals, according to the Nearest Duration Method, would give results sufficiently approximate for all practical purposes.

“From the explanation given on page 10, it will be remembered that the withdrawals are made to pass from observation at the end of the policy-year. In consequence of this, the rate of discontinuance is determined as at the end of the year, and not in the year. The function tabulated, therefore, is not exactly the same as in some other experiences.”

It will be seen that the function Mr. Sanderson has tabulated is the rate of non-renewal, and this rate appears to me to exhibit the experience in regard to discontinuances with greater precision than would have been obtained by tabulating the discontinuances in a manner similar to the deaths. With the object of seeing whether any evidence as to the influence of the discontinuances upon the rate of mortality could be obtained by comparison of different sections of the experience, I grouped the figures for ages 22, 24, 26, 27 and 31 at entry (for each of which the total discontinuances in the first five years were over 25 per cent. of the entrants), and the figures for ages 21, 23, 25, 28, 29, 30 and 32 (for each of which the total discontinuances in the first five years were under 25 per cent. of the entrants), with the following results:

Ages at Entry 22, 24, 26, 27, 31.

Year of Assurance	Exposed to Risk of Death	<i>d</i>	<i>q</i>	Exposed to Risk of Non-Renewal	<i>w</i>	Rate of Non-Renewal
1	8,016	30	·0037	7,986	1,288	·161
2	6,313	36	·0057	6,277	476	·076
3	5,535	25	·0045	5,510	297	·054
4	4,945	29	·0059	4,916	119	·030
5	4,404	25	·0057	4,379	176	·040
6	3,958	15	·0038	3,943	95	·025
Total	33,171	160	·0048	33,011	2,481	·075

Ages at Entry 21, 23, 25, 28, 29, 30, 32.

Year of Assurance	Exposed to Risk of Death	<i>d</i>	<i>q</i>	Exposed to Risk of Non-Renewal	<i>w</i>	Rate of Non-Renewal
1	11,053	22	·0020	11,031	1,649	·150
2	8,845	56	·0063	8,789	557	·063
3	7,869	47	·0060	7,822	379	·048
4	7,085	32	·0045	7,053	257	·036
5	6,281	28	·0045	6,253	233	·037
6	5,695	35	·0062	5,660	165	·029
Total	46,828	220	·0047	46,608	3,240	·070

The deaths are not sufficiently numerous, nor is there a sufficient divergence between the rates of non-renewal, to admit of any conclusion being drawn from the figures; but the results may, perhaps, be of interest as an example of the concurrent calculation of the probabilities of dying and the rates of non-renewal in successive

years of insurance,* and as an illustration of a method of comparison which in a larger experience might afford some indication as to the effect of withdrawals upon the average vitality of a body of assured lives.

The problem of the calculation of annual premiums for term assurances may be briefly referred to. If it is a correct assumption that a body of assured lives is deteriorated by withdrawals, it is obvious that select tables compiled from an experience chiefly consisting of whole-term assurances are not directly applicable to the calculations of annual premiums for more numerous discontinued assurances. It is possible, however, to provide for more numerous withdrawals of healthy lives than are covered by the tables by using the formula

$$p_1 = \frac{M_{[x]} - M_{[x]+n} - p(M_{[x+1]} - M_{[x+1]+n-1}) - q(1-p)(M_{[x+2]} - M_{[x+2]+n-2}) - \&c.}{M_{[x]} - M_{[x]+n} - p(M_{[x+1]} - M_{[x+1]+n-1}) - q(1-p)(M_{[x+2]} - M_{[x+2]+n-2}) - \&c.}$$

where, p , q , &c., represent the extra proportions of healthy lives withdrawing at the end of the first, second, &c., years.

Now it is questionable whether the term-assurance experience of life offices is sufficient to admit of the construction of a special set of select tables for the calculation of premiums, and whether, in any case, it would be worth while to construct such tables, but the experience will, of course, readily lend itself to the calculation of rates of non-renewal. The excess of these rates over the rates of non-renewal shown by the whole-term table, would then afford an indication of the values to be given in the above formula to the quantities p , q , &c.

In conclusion, I may just mention one other problem to the solution of which rates of non-renewal appear to be directly applicable—the determination of a superior limit to the amount allowable in commutation of an annual commission.

I am, Sir,

Your obedient Servant,

R. TODHUNTER.

39, King Street, Cheapside, E.C.,

20 February 1897.

MR. KING'S INTER-VALUATION FORMULA FOR "EXPOSED TO RISK."

To the Editor of the Journal of the Institute of Actuaries.

SIR.—In reading Mr. Ackland's interesting paper on "Methods for deducing the Rates of Mortality and Withdrawal", I notice that, in referring to Mr. King's Inter-Valuation Method, Mr. Ackland states that there is a possible error of twelve months in the age at

* To obtain the exposed to risk of non-renewal, I have followed Mr. Sanderson in deducting the deaths from the exposed to risk of death. It is assumed that lives which were assured up to the close of the observations, but withdrew immediately afterwards, are included in the "withdrawals", and not in the "existing."

entry, and concludes by saying: "It will probably, however, give "on the whole good results; but is clearly inferior to the Method of "Nearest Ages."

As this point has already given rise to some discussion, I was surprised to find that no further mention was made of it after the reading of the paper at the Institute.

The subject is certainly of interest to a large number of Students, and I therefore venture to make a few suggestions which, I hope, may prove of some assistance to them.

In volume xxvii, p. 218, Mr. King states that his formula was devised more particularly for taking out the mortality experience of a company between two valuation epochs; and he explains how this may be done with a minimum of trouble by making use of the classification which will already have been prepared for valuation purposes.

As explained by Mr. King, the age at entry is taken as the nearest age at the nearest 31 December, so that all entries are considered as taking place at the end of a year. Thus, if a life is actually exposed for less than six months in the year of entry, it is *not* tabulated as an exposure for that year, and if for more than six months, then it is tabulated as a full year's exposure: therefore the limit of error, as to the period of exposure in the initial year of entry, is six months.

To take an example. Suppose a man born 30 June 1820, enters in July 1860, his age will be taken as 41. but by the formula he is not assumed to enter until 31 December 1860, when 41 is really his nearest age, and throughout the remaining history of the policy, the age tabulated for each year of exposure coincides with the nearest age of the life, and there can never be an error of more than six months in the age.

The formula, like others, involves an error both as regards the duration in the initial year of exposure and the age at which the policy is tabulated; but it is only where these errors are in the same direction that there is the possibility of an apparent, but not real, error of twelve months, and it is, I think, decidedly misleading to saddle the whole of this error on to the age, without making any mention whatever of the duration.

Perhaps the most striking feature of the method is the automatic way in which it meets the difficulty of the unequal distribution of entrants. Thus, suppose, as is often the case, there is a considerable rush of entrants towards the close of the year, so that the average date of entry is, for instance, 1 September instead of 1 July, as it would be if entries were equally distributed. In this case, if the Institute method were applied, the period of exposure in the initial year would be taken as six months instead of four, thus causing the mortality shown for the year of entry to be lighter than is really the case. By Mr. King's method, however, this difficulty is provided for, since the larger proportion of entrants at the close of the year, the larger will be the proportion of lives taken at the succeeding age, and consequently the fewer will there be credited with exposure in the initial year. In this way, the average period tabulated is, undoubtedly, a very close approximation.

The method of treating the withdrawals is similar to that used for the entrants; and there is, consequently, the same limit of error; but, since most of the withdrawals will occur on an anniversary of the date of entry, the effect will be to make the period of exposure tabulated to exactly coincide with the actual experience.

In cases of death, the age is necessarily taken as the nearest at the *previous* 31 December, thus, as pointed out by Mr. Whittall, giving rise to the apparent possible anomaly of a policy taken at age x at entry and dying at age $x-1$.

In considering this point, it must not be forgotten that the method is intended for the formation of aggregate tables, and that there will be a balance of errors from age to age. It may also be pointed out, that whatever the youngest age may be at which premiums are quoted by the office, the mortality experience will necessarily commence at one year younger; and, although a life may be tabulated as dying at a younger age than that at which it is assumed to enter, yet there will always be a proportion of the entrants also referred back to that previous age to form the exposures from which the death will be assumed to have occurred.

With Endowment Assurance Policies, the method of classification adopted at the valuation would, probably, prevent the results being used in the same way as with the Whole-Life Policies; but even in those cases, if at any valuation, all the Endowment Policies in force were retabulated (for mortality investigation cases only) in a similar manner to the Whole-Life Policies, it would then be a very simple matter to keep a record of all entrants and withdrawals, and so obtain the results required.

For the purposes for which Mr. King's method was intended, the saving of labour is very considerable. When once the policies have been classified at nearest ages at a valuation, we shall be able to trace the greater proportion of these for *exact* periods of exposure with a very small amount of trouble; and, looking at the method from this point of view, I think it will be admitted that it is undoubtedly the best which has yet been suggested.

I am,

Yours obediently,

J. BURN.

*Prudential Assurance Company,
Holborn Bars,
June 1896.*

INTERNATIONAL CONGRESS OF ACTUARIES, 1898.

AS announced by the then President of the Institute of Actuaries (Mr. A. J. Finlaison), in his address of 25 November 1895, the Second International Congress of Actuaries is to be held in London, in 1898. The meetings have now been fixed for May 16, 17, 18, 19 and 20, and the Council of the Institute has appointed an Organizing Committee to make the necessary arrangements.

It has been resolved that membership of the Congress shall be limited to

(1) Members of the Brussels Congress of 1895, and Members of the Permanent Committee of Actuarial Congresses.

(2) Fellows, Associates, and Corresponding Members of the Institute of Actuaries.

(3) Fellows of the Faculty of Actuaries in Scotland, and members of that body of analogous rank to the Associates of the Institute.

(4) Members of Actuarial Institutions in the Colonies and Foreign Countries of analogous rank to Fellows and Associates of the Institute of Actuaries.

(5) Actuaries and Assistant Actuaries of all Life Offices, and Government Actuaries and Statisticians, in the United Kingdom, the Colonies, and Foreign Countries.

(6) Members of the Council of the Royal Statistical Society.

While membership is to be restricted in accordance with the above rules, yet, that no one with a real claim to membership should be excluded, it has been arranged that the Organizing Committee will specially consider the application of any person well recommended by one qualified under the rules to be a member.

The subscription for membership has been fixed at £1.

The Organizing Committee invite Colonial and Foreign Governments, and Actuarial Institutions in the Colonies and Foreign Countries, to nominate delegates, qualified under the rules, as their official representatives at the Congress. They also invite the contribution of papers, suitable to be read and discussed at the Congress, such papers to be sent to the Honorary Secretaries of the Institute of Actuaries not later than the end of February 1898.

The official language of the Congress will be English, but papers may be written in French or German. So far as possible, papers should be type-written.

At the first Congress, held at Brussels, in September 1895, a Permanent Committee was established, one of the principal functions of which was intended to be to co-operate with the Organizing Committees in the preparation of the work of future Congresses. The following are the Constitution and Bye-Laws of the Permanent Committee, together with the names of the present office-bearers and members of the Executive Council:

CONSTITUTION AND BYE-LAWS OF THE PERMANENT COMMITTEE OF INTERNATIONAL ACTUARIAL CONGRESSES.

ART. 1.

The Permanent Committee of International Actuarial Congresses is established to be a bond between the Actuaries and also the Associations of Actuaries of various countries. Its head-quarters are at Brussels.

ART. 2.

It has for objects:

1. To originate and to follow out all works or researches interesting in the science or in the practice of the Actuary.

2. To publish periodically a Journal: *a.* bringing together all information, technical, legislative, statistical, or juridical, bearing upon Actuarial Science; *b.* summarizing the publications and the works, which appear in the different countries, bearing upon Actuarial matters; *c.* giving space, moreover, to original contributions of general interest to Actuaries.

3. To co-operate with the *Organizing Committees* in the preparation of the work of International Congresses, and in the publication of their Proceedings.

ART. 3.

The *Associate* members of the Permanent Committee shall be all those who are admitted by the *Executive Council*, hereinafter defined, and who undertake to pay an annual minimum contribution of 10 francs.

The *Donation* members shall be those Associate members who undertake to pay a minimum annual contribution of 50 francs. Their names shall be published each year in the Journal with a memorandum of the sums paid by them.

Associate Life members shall be those of the Associate members who have compounded for their annual subscriptions by a minimum payment of 200 francs. made in instalments during one or two years at the most.

Donation Life members shall be those Donation members who have compounded for their annual subscriptions by a minimum payment of 1,000 francs., made in instalments during one or two years at the most.

Public (Governmental) Departments of the different countries, Assurance Companies, and generally all Institutions, Societies, or Associations, having for their object the study, or the practical application of thrift, and Social Economy, may become Associate members, or Donation members, of the Permanent Committee.

The Associate members, and the Donation members shall receive free the Journal of the Permanent Committee.

ART. 4.

The Permanent Committee shall be governed by an International *Executive Council* consisting of 40 members, and including the 33 members who were appointed by the first International Congress of Actuaries at its meeting of 6 September 1895, and such members as shall be in future elected by the Executive Council itself, proceeding by co-option.

Each country possessing Actuaries shall be, as far as possible, represented on the Executive Council by one or several members.

The members of the Executive Council shall be elected for 4 years and shall be eligible for re-election.

The re-election shall take place each year, and by one-fourth of the members of the Council. During the transitional state the members to retire in 1897, 1898 and 1899 shall be decided by lot.

The election shall be made by those members not retiring, and it can take place by correspondence.

ART. 5.

Each year the Executive Council shall choose from among its own members a *Bureau* consisting of a President, 7 Vice-Presidents, a General Secretary, a Treasurer, and 7 Secretaries, distributed, as far as possible, among the members representing the different countries.

The Executive Council may moreover nominate correspondents whose duty it shall be to represent it in those countries where it is not represented by any member of the Bureau. The election of the Bureau may take place by correspondence.

ART. 6.

The Executive Council shall meet at the head-quarters of the Permanent Committee, and shall be convened by the President, or, failing him, by the General Secretary.

Notice of meeting accompanied by a detailed agenda shall be sent to each member of the Executive Council at least one month before the date fixed for the meeting. Those members who find themselves unable to attend at a meeting may send to the President their views on the questions on the agenda.

In case of urgency the President and the General Secretary may take such steps as may be found necessary, provided always that they submit them at the earliest possible moment for ratification by the Executive Council.

Decisions of secondary importance may be taken without convening a meeting of the Council, on the vote of an absolute majority of the members of the Executive Council, given by correspondence on the invitation of the President.

Ten members of the Executive Council shall form a quorum. If this number be not present at a meeting, the President may take a vote by correspondence as explained in the last preceding paragraph.

The Executive Council may authorize those members, who, on account of the distance of their residence, are unable to send in their views by correspondence within one month, to be represented, for the purpose of voting, by European members of the Council.

In case of equality of votes, the President shall have a casting vote.

ART. 7.

The place and date of each International Congress shall be fixed either by the last preceding Congress, or, failing that, by the Executive Council, which in such case shall be guided by the convenience of the Actuaries of the different countries interested, and by the advice of the Societies of Actuaries established in those countries. The Executive Council shall establish in the country designated, an *Organizing Committee*. The Organizing Committee shall prepare a programme of the business in concert with the Executive Council. It shall defray all expenses of the Congress, and of publishing its proceedings.

During the period in which the International Congress is being organized two members of the Organizing Committee of such Congress shall be temporarily added to the Executive Council, and shall have the same powers as the ordinary members.

ART. 8.

The income of the Permanent Committee shall consist :

1. Of the contributions of its members ;
2. Of the subsidies that may be granted to it by public (Governmental) departments of the different countries ;
3. Of the subsidies which may be granted to it by the different Institutions, Societies, or Associations ;
4. Of the interest on its invested funds ; and
5. Of sundry receipts.

ART. 9.

There shall be formed a Reserve Fund, into which shall be paid :

1. The sums received in commutation of subscriptions ;
2. The contributions and subsidies made on the condition that they shall be placed to the Reserve Fund ; and
3. Such sums as the Executive Council shall think wise to carry to the Reserve Fund.

The Reserve Fund shall be invested in the public debt of Belgium. It cannot be trencned upon except by a formal decision of an International Congress taken on the motion of the Executive Council. The income derived therefrom may be applied to defray the ordinary expenses.

ART. 10.

The present regulations cannot be altered except by a future International Congress on the motion of the Executive Council or at the request of 25 members of the Congress. In this latter case the proposal must be communicated to the Executive Council at least one month before the date fixed for the opening of the Congress.

ART. 11.

With the formalities prescribed in the last preceding article, any future International Congress may, by a majority of two-thirds at least of the members present, dissolve the Executive Council, and dispose of the available capital, as also of all documents, works, and sundry archives, belonging to the Committee.

OFFICE BEARERS.

<i>President</i>	Mr. O. LEPREUX.	
<i>General Secretary</i>	„ AM. BÉGAULT.	
<i>Treasurer</i>	„ L. DUBOISDENGHIEN.	
<i>Vice-Presidents.</i>					<i>Secretaries.</i>	
<i>Germany</i>	Mr. J. KARUP.				<i>Germany</i>	Mr. GROSSE.
<i>Australasia</i> ...	„ R. TEECE.				<i>Australasia</i> ...	„ J. H. RICHARDSON.
<i>United States</i> }	„ E. MCCLINTOCK.				<i>United States</i> }	„ I. C. PIERSON.
<i>and Canada</i> }					<i>and Canada</i> }	„ LÉON MARIE.
<i>France</i>	„ P. GUIEYSSE.				<i>France</i>	„ F. B. WYATT.
<i>United Kingdom</i> }	„ T. E. YOUNG.				<i>United Kingdom</i> }	„ G. TOJA.
<i>Holland</i>	„ M. C. PARAIRA.				<i>Italy</i>	

MEMBERS OF THE EXECUTIVE COUNCIL.

<i>Germany</i> ...	{	Mr. GERKRATH.		<i>United Kingdom</i>	{	Mr. A. J. FINLAISON.
	{	„ GROSSE.			{	„ G. KING.
	{	„ J. KARUP.			{	„ J. MEIKLE.
	{	„ K. SAMWER.			{	„ G. H. RYAN.
<i>Australasia</i> {		„ J. H. RICHARDSON.			{	Dr. T. B. SPRAGUE.
	{	„ R. TEECE.			{	Mr. T. E. YOUNG.
	{	„ H. ADAN.			{	„ F. B. WYATT.
<i>Belgium</i> ...	{	„ AM. BÉGAULT.		<i>Italy</i>	{	„ L. PEROZZO.
	{	„ L. DUBOISDENGHIEN.			{	„ G. TOJA.
	{	„ O. LEPREUX.			{	„ M. C. PARAIRA.
<i>Canada</i> ...		„ T. B. MACAULAY.		<i>Holland</i> ...	{	„ E. W. SCOTT.
	{	„ E. MCCLINTOCK.			{	„ L. WOLTERBEEK.
<i>United States</i> {		„ I. C. PIERSON.			{	„ S. DE SAVITCH.
	{	„ H. W. ST. JOHN.		<i>Russia</i>	{	„ R. PENL.
	{	„ E. CHEYSSON.			{	„ A. LINDSTEDT.
<i>France</i> ...	{	„ P. GUIEYSSE.		<i>Sweden</i> ...		„ H. DE CÉRENVILLE.
	{	„ LÉON MARIE.				
	{	„ J. MARTIN-DUPRAY.		<i>Switzerland</i>		

JOURNAL

OF THE

INSTITUTE OF ACTUARIES.

On the Rates of Mortality in Certain Parts of Africa. By
ALFRED E. SPRAGUE, M.A. Cantab., B.Sc. Edin., F.I.A.,
F.F.A., of the *Scottish Equitable Life Assurance Society*.

[Read before the Institute, 25 January 1897.]

AS the published facts concerning the rate of mortality in tropical Africa are very scanty, and Insurance Offices are therefore often compelled to fix the extra premiums for African risks in an almost purely arbitrary manner, I venture to lay the following investigation before the members of the Institute as a contribution to our knowledge of the subject. So far as I am aware, the only publications dealing with the subject are a note by my father, Dr. T. B. Sprague, on the Rate of Mortality on the Congo (*J.I.A.* xxv, 437); a note read by myself before the Actuarial Society of Edinburgh on the Rate of Mortality in Sierra Leone (*Transactions*, iii, 365); a paper* by Mr. J. R. Hart on the Mortality on the West Coast of Africa, which appeared in the *Insurance Record* of 2 October 1896; and a paper by Dr. T. Glover Lyon on the Mortality amongst Europeans in certain Unhealthy Districts (*J.I.A.*, xxix, 541). In the first three of these, the number of lives under observation is very small, and in all of them the information given concerning the

* The substance of this paper is given in a communication from Mr. Hart in the present number of the *Journal*, p. 307.

lives is very scanty. For example, in none of these experiences are the ages of the lives stated, and therefore only average rates of mortality, irrespective of age, could be deduced from them. I have now, through the kindness of Dr. Felkin, had access to statistics embodying a fairly large and much more complete body of facts, concerning Government Officials, Traders, and Missionaries, employed in various parts of the Continent; and as these statistics appear to have been on the whole very carefully compiled, I think the results which I have deduced from them are worthy of attention, and carry some little weight.

The statistics in question relate to (a) 971 employees of the Belgian Government on the Congo; (b) 178 employees of a Dutch Trading Company on the South-West Coast; and (c) 654 missionaries sent out by a number of Missionary Societies to various places, some to stations on the Congo, others to the West Coast, and others to different parts of Central Africa. The total number of lives under observation is thus 1,803. The date on which each life went to Africa is stated, and the date of death or return to Europe; with dates showing for what periods each life was away from Africa on furlough. The locality where each life was stationed is indicated more or less definitely; and in the majority of cases, the ages also are given. A small proportion of the lives are females, and these are included among the others, in order to obtain as large a body of facts as possible. They are not sufficiently numerous to yield any valuable results if placed in a group by themselves. Sufficient information is thus furnished to show the average rates of mortality in the various districts, irrespective of age; to trace the variation in these rates according as the length of residence increases; and also to form rough mortality tables showing approximately how the mortality varies with the age. In many instances, only the calendar years are given, instead of the exact dates; and in these cases I have taken the duration of the risk to be the difference between the years of entry and exit. Where the exact dates are given, I have calculated the duration to the nearest integral number of years; except in the case of those who have died, where the year of death has been treated as a complete year. Having calculated the duration of the risk in this way, I find the total number of years of life comprised in the observations to be 5,849, and the number of deaths 378, so that the average rate of mortality is 6·5 per-cent. As the rate among the whole population of England (according to English Life Table No. 3) is only 2·2 per-cent, it is at once seen that the

climate of Tropical Africa must be extremely unhealthy in the districts in question; but, as the climate varies to so large an extent in different parts, and the statistics relate to so large an area, this result only shows that there is a heavy risk for residence in certain parts of Africa, and does not enable us to measure it sufficiently accurately to fix suitable extra premiums. I have, therefore, attempted to divide the statistics into groups corresponding to various districts, each of which has more or less the same climatology throughout, so as to trace the rates of mortality in each such district. In doing this I have been greatly assisted by Dr. Felkin's work "*On the Geographical Distribution of Tropical Diseases in Africa*" (*Reviewed J.I.A.*, xxxii, 65), from which I have extracted much information. All the lives resided in one or other of the following districts, each of which appears, so far as I can judge, to possess certain distinctive characteristics of climate; and I have accordingly divided the statistics into these four groups—

- (a) Places situated on the Congo;
- (b) Central Africa (excluding the neighbourhood of the Congo);
- (c) The West Coast from Sierra Leone to Gaboon;
- (d) The Coast of German South-West Africa.

The two first groups I would have included in one, were it not that the Congo appears to exercise a marked effect on the climate of places in its immediate vicinity, rendering them decidedly more unhealthy than other parts of Central Africa. All along its course malaria and dysentery seem to be more prevalent than in the country at some distance from the river; and in this region there is also found the so-called blackwater fever, which is not understood, and from which there is a heavy mortality.

The West Coast has the reputation of possessing a climate more deadly to Europeans than almost any other place on the globe; and undoubtedly the mortality there in past years has been frightful. This has not, however, been entirely the fault of the climate; but may, in great part, be attributed to ignorance, neglect of sanitary precautions, and irregular living; and at the present day the mortality is markedly less than in former years. Mr. Hart notices this fact in his paper, and throws some light on the cause. Dr. Felkin also mentions it, and indicates that the lower mortality at present experienced is largely due to better sanitation. Thus he says, "Very much may be done to render even the worst climate in

“Tropical Africa more salubrious, and the sanitary precautions to which attention will be called in the sequel will do a great deal to raise the health of the community. For instance, attention to the water supply at Sierra Leone has had a very marked effect in lowering the death rate.” German South-West Africa has undoubtedly a far better climate than any of the preceding districts, and in particular it is comparatively free from malaria. Neither does dysentery appear to be found in this region to any great extent.

The following table gives a rough measure of the rate of mortality in each of these four districts, and shows that the unhealthiness of each is in the order named :

District	Lives	Years of Life	Deaths	Mortality per-cent
Congo	1,099	2,096	198	9·4
Central Africa . .	214	604	47	7·8
West Coast . . .	312	1,926	90	4·7
S.W. Coast . . .	178	1,223	43	3·5
Totals	1,803	5,849	378	6·5

For purposes of comparison, I add the following table as a summary of the principal results arrived at in the four papers mentioned above :

District	Date of Experience	Years of Life	Deaths	Mortality per-cent
Congo (Dr. Sprague)	1879-84	262·5	27	10·3
W. Coast (J. R. Hart)	1881-89	1,031·5	54	5·2
Lagos (Dr. Lyon)	1879-88	1,080	76	7·0
Gold Coast—Officials (Dr. Lyon)	„	661	34	5·1
„ —Non-Officials	„	1,261	106	8·4
Gambia (Dr. Lyon)	1878-90	702	37	5·3
Sierra Leone „	1879-88	2,710	75	2·8
Totals for W. Coast (Dr. Lyon)	...	6,417	328	5·1
Sierra Leone (A. E. Sprague) . .	from 1804	704	49	7·0

Although Dr. Lyon's figures are the most extensive, it is not clear that they carry the greatest weight, as much will depend on the source of the information and its accuracy. The “years of life” for Gambia and Sierra Leone are, I observe, based on the Census of 1881; while the deaths are taken for thirteen years in the former case, and ten years in the latter, during which periods

the population may have varied considerably in number. This is, of course, a possible source of error, and makes the figures less reliable.

The average rate of mortality on the Congo, of 9·4 per-cent, seems enormous, but I do not think it is an exaggerated estimate of the risk. Dr. Sprague deduced an even higher rate, namely 10·3 per-cent, among persons resident at stations on that river.

In Central Africa the rate is decidedly lower, being only 7·8 per-cent, and this in spite of the fact that all the lives in this group were missionaries. If, as some people suppose, it be a fact that these latter are worse lives than Government Officials, it seems clear that the Congo exercises a very pernicious influence on the health of Europeans in its vicinity, as otherwise we might expect the Central African experience to be the worse of the two. The rate of 4·7 per-cent on the West Coast may be compared with Mr. Hart's rate of 5·2 per-cent in the same locality; and the two results are seen to be fairly consistent. The rate formerly deduced by myself for Sierra Leone was 7·0 per-cent, which is considerably higher; no doubt because the data on which it was based related largely to the early part of the century, when the death-rate was heavier than it is at present.

The South-West Coast is obviously by far the healthiest of all the four districts, having a mortality of only 3·5 per-cent.

The supposition just mentioned, namely, that the lives of Government Officials are better for insurance purposes than those of missionaries, is on the face of it by no means improbable; one of the reasons for this view being that the former have greater facilities for recruiting their health by leave of absence, and more extensive privileges in the way of furlough for other causes than health; and another reason is the physical examination which Government Officials are required to pass, so that they are really "selected" lives. I have therefore divided the Congo Experience into two portions, so as to show the rates of mortality in each class. The following are the figures:

Class of Lives	Lives	Years of Life	Deaths	Mortality per-cent
Government Officials .	971	1,705	159	9·3
Missionaries . . .	128	391	39	10·0
Totals . . .	1,099	2,096	198	9·4

The number of missionaries is too small to yield a very trustworthy result; but there does seem to be a traceable difference between the rate of mortality among Government Officials and that among missionaries resident in the same climate; the difference being in favour of the former class. A somewhat similar feature will be noticed in Dr. Lyon's figures regarding the Gold Coast, the mortality among the Officials in that Colony being 5.1 per-cent as against 8.4 per-cent among the non-official population. In this case the Government Officials appear to possess a well-marked superiority. On the other hand, I observe that Mr. Hart deduces a rate of mortality of 5.2 per-cent among Government Officials on the West Coast, while my present figures (which relate solely to missionaries) show a mortality in that district of only 4.7 per-cent.

I next took the cases where the age at entry was stated; and, omitting all the other lives, worked out the years of life, exposed to risk, and the deaths, for each age. As the numbers are small, the resulting rates of mortality are necessarily irregular; but if several ages are grouped together, it is seen that the rate varies according to a regular law. In order to bring out this law clearly I found it necessary to make many trial groupings of the facts; and after doing this, and studying the results carefully, I adopted the following groupings as indicating most plainly the law of variation in the rate of mortality:

CONGO				CENTRAL AFRICA				WEST COAST				SOUTH-WEST COAST			
Ages	Years of Life	Deaths	Mortality per-cent	Ages	Years of Life	Deaths	Mortality per-cent	Ages	Years of Life	Deaths	Mortality per-cent	Ages	Years of Life	Deaths	Mortality per-cent
17-25	408	29	7.1	19-25	84	5	6.0	18-25	218	9	4.1	16-27	508	9	1.8
26-28	288	23	8.0	26-29	163	11	6.7	26-27	204	10	4.9	28-33	425	15	3.5
29-33	362	29	8.0	30-34	117	9	7.7	28-32	404	31	7.7	34-38	183	11	6.0
34-37	151	16	10.6	35 & up.	88	7	8.0	33-35	115	9	7.8	39 & up.	107	8	7.5
38 & up.	138	16	11.6	36 & up.	351	5	1.4
Total	1,347	113	8.3	Total	452	32	7.1	Total	1,292	64	5.0	Total	1,223	43	3.5

It will be observed that the mortality among those lives on the Congo whose ages are stated, is 8.3 per-cent, which is considerably less than the mortality among all the lives in that district taken together, that is to say, including those whose ages are not stated. From this I infer that the lives whose ages are not

stated, are, on the average, older than the others. In the Central African and West Coast districts there are also slight differences in the rates compared with those deduced for all the lives; but as the data are smaller in extent, and the difference is not so large, these may be simply accidental irregularities. The ages of all the lives on the South-West Coast are given; and it should be mentioned that this portion of the statistics and the greater part of those relating to the Congo, appear to have been very carefully compiled, and accordingly carry special weight.

It will be seen that the mortality in each district presents special features. In the Congo region the rate is at all ages higher than in the other three districts, and steadily increases from about 7 per-cent at the youngest ages, to over $11\frac{1}{2}$ per-cent at ages 38 and over. In Central Africa the rate varies in a similar manner; but throughout the whole of life it is noticeably lighter than in the Congo district. On the West Coast the peculiar feature is met with, of a rate of mortality increasing from about 4 per-cent to nearly 8 per-cent at ages 33-35, and thereafter diminishing again. I at first thought this might be a mere irregularity arising from the smallness of the data; but, having carefully examined the figures, I think it is fairly clear that, after the lives have passed the age of about 35, they can continue to reside on the West Coast without experiencing so heavy a mortality as is felt at the earlier ages. This feature is peculiar to the West Coast alone, and I am unable to suggest any explanation of it. On the South-West Coast the rate at the younger ages is very low compared with the other districts, being under 2 per-cent; but it increases with the age much more rapidly than in those districts, until, at ages 39 and over, the mortality is almost as heavy as the maximum mortality on the West Coast.

The next question to be considered is the phenomenon of acclimatization, the effect of which on the rate of mortality is shown in a decrease in the rate, among those who have been subject to the climate for some years, as compared with the rate among persons who are exposed to it for the first time. The popularly accepted explanation of this phenomenon is that a European on going to a tropical climate is liable to fevers and other disorders, which are most acute in their first attacks; that, after recovering from these, the subsequent attacks are milder and less frequent, the life having become, as it were, inoculated, and therefore less subject to these diseases. This explanation

may be partially correct, and I understand that it applies to a certain extent to some diseases, such as the bilious remittent fever which occurs chiefly in new comers to tropical Africa.

I find that, in a paper on acclimatization, read before the British Association in 1891, Dr. Felkin says: "A lengthened residence gives strangers a certain immunity from such diseases as dyspepsia, and all that wide class of disease which is induced by plethora. It may be, too, although the fact is denied by some authorities, that the longer a European resides in some portions of the Tropics where yellow fever obtains, the less likely is he to suffer from that fever." He goes on to say, however, that "The probability of strangers suffering from certain diseases is heightened by the length of residence. For instance, a person is far more likely to suffer from malarial fever, and all those chains of symptoms induced by malarial intoxication, the longer he resides in a district where malaria is rife." As a large proportion of the deaths (probably about 25 per-cent in the case of the Congo experience) were caused by malaria, I think that this popular explanation of acclimatization is clearly insufficient to account for the decrease in the rate of mortality, and I believe that this decrease is chiefly due to another cause, which may be described as follows:—Out of a large body of men selected at random, some are more susceptible than others to the diseases peculiar to the climate. These fall early victims, leaving as survivors the less susceptible lives, who, of course, suffer less from disease, and in consequence experience a smaller death rate. In other words, there is a process of selection, or weeding out, of the more susceptible lives; and when these have all been eliminated, we may expect to find the mortality reduced to its ultimate, or minimum, rate. This will occur after some years, when the so-called acclimatization has ceased to cause any further diminution in the rate of mortality. In order to study this phenomenon completely, the lives should be arranged according to the length of time they have been resident in Africa; the entrants at each age being kept separate, and their mortality traced throughout their whole life. Were the data numerous enough to admit of this, we should be able to ascertain exactly the true effects of acclimatization on the rate of mortality, and learn at what ages it is most powerful. But, unfortunately, the present data are not sufficiently numerous, and in default of more extensive statistics, I therefore adopted the plan of grouping together, irrespective of age, all those lives who have been resident

the same number of years. As most of the lives had been allowed furlough on one or more occasions, it was some considerable trouble to arrange the facts properly. At first I tried grouping them according to the total number of years spent in Africa by each life, leaving altogether out of account the years at home; that is to say, supposing a life had been resident in all for four years in Africa, I treated him as an "exit" at the end of four years, as if his term of residence had been continuous. On further consideration this seemed an unsatisfactory method of dealing with the case, and I therefore made a fresh arrangement, grouping the lives according to the time elapsed since first going to Africa, and excluding from the exposed to risk, those years spent at home. Thus, if a life was out for two years, then had a year's furlough, and then went out a second time for another two years, I treated him as an "exit" at the end of two years, and as entering again on the fourth year of residence (so that during the third year he was not included among the exposed to risk), and finally, as an "exit" at the end of the fifth year. I was thus enabled to find the rates of mortality for each year since the life first went to Africa; but, as in the former case, the figures were too few in number to show sufficient regularity, and required grouping to some extent, in order to show clearly the law of progression. The following are the groups I finally adopted, after many trials, as showing the greatest regularity:

CONGO				CENTRAL AFRICA				WEST COAST				SOUTH-WEST COAST			
Year of Residence	Years of Life	Deaths	Mortality per-cent	Year of Residence	Years of Life	Deaths	Mortality per-cent	Year of Residence	Years of Life	Deaths	Mortality per-cent	Year of Residence	Years of Life	Deaths	Mortality per-cent
1	355	110	11.5	1	184	21	11.4	1	303	16	5.2	1	174	4	2.3
2	515	44	8.5	2	120	13	10.8	2	261	26	10.0	2	154	4	2.6
3-4	394	29	7.4	3	76	5	6.6	3	193	15	7.8	3-5	357	12	3.4
5 & over	232	15	6.5	4 & over	224	8	3.6	4	154	7	4.5	6-12	423	16	3.8
...	5-6	234	9	3.8	13 & over	115	7	6.1
...	7 & over	781	17	2.2
Total	2,096	198	9.4	Total	604	47	7.8	Total	1,926	90	4.7	Total	1,223	43	3.5

These may be compared with the following table taken from the results formerly given by Mr. Hart and myself:

WEST COAST—GOVERNMENT OFFICIALS (J. R. HART)				SIERRA LEONE—MISSIONARIES (A. E. SPRAGUE)			
Years of Residence	Years of Life	Deaths	Mortality per-cent	Years of Residence	Years of Life	Deaths	Mortality per-cent
$\frac{1}{2}$	105 $\frac{1}{2}$	3	2·8	1	102	22	21·6
$1\frac{1}{2}$	184	14	7·6	2	70	5	7·1
$2\frac{1}{2}$	151	8	5·3	3	54	3	5·6
$3\frac{1}{2}$ — $4\frac{1}{2}$	204	9	4·4	4	47	1	2·1
$5\frac{1}{2}$ — $6\frac{1}{2}$	133	7	5·3	5	45	1	2·2
Afterwards	254	13	5·1	Afterwards	386	17	4·4
Totals	1,031 $\frac{1}{2}$	51	5·2	Totals	704	49	7·0

The first two of these groups show the effect of acclimatization very clearly, the mortality being at its maximum in the first year of residence and steadily decreasing with each successive year thereafter. The mortality for the first two years taken together, on the Congo and in Central Africa, is much about the same, being about 10·5 per-cent and 11·2 per-cent respectively; but in subsequent years the mortality in Central Africa diminishes much more rapidly. On the West Coast, leaving the first year out of account for a moment, there is a similar progressive diminution in the mortality; but in that year there is the strange feature of a comparatively low rate of mortality. On first noticing this peculiarity, I thought it might be possible that the diseases prevalent on the West Coast required, say, six months or a year to take effect on a healthy European exposed to their influence for the first time; but if this were the case we might not unreasonably expect to find a similar feature on the Congo and in Central Africa. One obvious explanation is to assume that the low rate is merely an accidental irregularity arising from the small extent of the data; but as Mr. Hart deduces the similar feature of a low rate of mortality on the West Coast during the first six months, which is followed by a heavy rate in the succeeding year, and steadily diminishes thereafter; and as the lives in his experience belong to a different class of persons from the above, it does not appear probable that the peculiarity is accidental. Mr. Hart suggests that it may be caused by the large number of lives invalidated during the first six months; but unfortunately the figures at my disposal are not precise enough, as regards the invalidated lives, to enable me to investigate this point.

The South-West Coast presents a most marked contrast to all

the other districts. Here the rate, instead of diminishing with the length of residence, is at its minimum in the first year, and steadily increases with each successive year. This increase is, as nearly as I can estimate, at about the same rate as that which would be caused by the increasing age of the lives, and may therefore be entirely accounted for by the increase in the average age. In other words, it seems as if acclimatization had little or no effect on the rate of mortality in this district.

I made an attempt to ascertain the effects of acclimatization at different ages, by dividing the Congo experience into four groups, the first being entrants under the age of 25; the second, entrants between that age and 30, and so on. The following table shows the mortality in each of these groups, during the first year of residence, and in subsequent years, as well as the average mortality for each group:

CONGO DISTRICT.

Ages at Entry	13 to 24			25 to 29			30 to 34			35 and upwards			25 to 34		
Years of Residence	Years of Life	Deaths	Mortality per-cent	Years of Life	Deaths	Mortality per-cent	Years of Life	Deaths	Mortality per-cent	Years of Life	Deaths	Mortality per-cent	Years of Life	Deaths	Mortality per-cent
1st year .	220	22	10.0	231	19	8.2	127	10	7.9	86	11	12.8	358	29	8.1
Subsequent years .	176	14	8.0	262	20	7.6	161	10	6.2	84	7	8.3	423	30	7.1
Totals .	396	36	9.1	493	39	7.9	288	20	6.9	170	18	10.6	781	59	7.6

It will be seen that the heaviest mortality occurs among those who were aged 35 and upwards when they first went to Africa; and the next heaviest, among the entrants aged less than 25 years. In both these groups the effect of acclimatization is well marked. In the intermediate groups the mortality is less, and the effect of acclimatization not so large, being a minimum between ages 25 and 30. These figures seem to indicate that middle-aged and very young men suffer more severely than those who are aged between 25 and 35 at entry. I find that this is, as regards the young lives, corroborated by Dr. Felkin, who says, "No one should go to Africa under the age of 25. Persons under this age are bound to suffer more from typhoid fever, from the severer forms of malarial fever, and from dysentery, than those who are older."

The figures relating to Central Africa are not numerous enough to be dealt with in this manner; and nearly all the entrants in the West Coast experience were about the same age, namely, between 25 and 30, so that no comparison could be made for different ages at entry.

It may be mentioned that, in those returns where the ages were stated, the average age at entry is between 28 and 29 in all four of the above districts.

Having made several rough adjustments of the foregoing figures and constructed mortality tables therefrom, I calculated the annual premiums at quinquennial ages from 20 to 50, at 4 per-cent interest; and comparing these with the 4 per-cent tables in the Institute of Actuaries' Text-Book, I am inclined to think that the following rates of extra premium represent the very least that an insurance office can prudently charge for the best lives resident in the four districts in question; and, owing to acclimatization, I think the rates in the first three cases should be increased, at all events during the early years of assurance.

- (1) Congo . . . from £6 to £7 per-cent per annum;
- (2) Central Africa from £5 to £6 per-cent per annum;
- (3) West Coast . . . from £3 to £4 per-cent per annum;

(but the figures seem to indicate that a smaller extra would be sufficient at ages above, say 35, provided the lives have then been resident some years in the district).

- (4) South-West Coast from £2. 10s. to £3. 10s. for lives under 30, say, and about £4 for older lives.

These rates are to be regarded only as first approximations of a very rough nature. The statistics are too scanty to admit of very accurate results being deduced from them, and they contain practically no information on at least one very important point—namely, the number of lives who return to Europe on account of bad health, and the subsequent experience of those lives. The rates of mortality deduced are, therefore, independent of those invalided lives, while the extra premiums to be charged will naturally depend to some extent on them. In other words, as soon as a life left Africa he passed out of observation, whereas the extra premiums we require to find, should be calculated to cover the extra risk run by the office, not only while the life is resident in Africa, but also after his return to Europe during the continuance of the policy. In this connection it should be borne

in mind that a man sent home invalided would be a damaged life, and therefore would have a strong inducement to keep his policy in force at all costs.

A few of the returns sent in by the Missionary Societies contained remarks showing which lives were invalided, and among these the average rate of invaliding was about $8\frac{1}{2}$ per-cent per annum; but this can scarcely be taken as a criterion of the rate prevailing among the whole experience. Mr. Hart deduces a rate of only 3·6 per-cent per annum among Government Officials on the West Coast, and we may therefore perhaps regard the above figure of $8\frac{1}{2}$ per-cent as a somewhat exaggerated estimate of the true rate of invaliding; but in any event it is clear that the point requires further investigation, and that caution should be exercised before taking the foregoing tables as a measure of the full risk of the African climate, especially as the remarks appended to some of the returns show that a number of the invalided lives died either on the voyage home or shortly after reaching Europe. In particular, it seems that offices would act prudently in stipulating that no return of a portion of the extra premiums should be made for time spent in Europe on furlough, unless strict medical evidence of health be furnished.

DISCUSSION.

The PRESIDENT (Mr. T. E. Young) in inviting discussion, said it was of special interest and hope that the first two papers of the present session should be the production of fresh contributors, and he trusted their excellent example would stimulate many of the competent younger men in their ranks to follow it. The essayist on the present occasion bore the passport of an honoured name, but although he was thus happily endowed, he did not require any credentials of acceptance, as they also welcomed him for his own sake and for the well-considered paper he had contributed to their stores.

Mr. C. D. HIGHAM said that there was heredity in talent, and they welcomed the son the more, as they remembered their obligations to the father. The very excellence of the paper and the little that was known of West African mortality made it almost impossible to say very much in the way of criticism. If he said there was anything wanting in the paper he fully recognized that it was not the fault of Mr. Sprague, but caused by the paucity of data with which he had to deal. In Mr. Sprague's bibliography he noticed that he had not mentioned a small table in the first volume of the *Journal*, showing the mortality on Her Majesty's ships on the coast of Africa. In that table the mortality from 1840 to 1848 (inclusive) ranged from 2·1 per-cent to 7·9 per-cent, or a mean of 3·9 per-cent; and

bearing in mind that this mortality related to those at sea (which was recognized almost as a sanatorium), he hoped (unless there were many deaths in action included) that some improvement had taken place as he compared these rates with certain rates given in the paper relating to the present time. One of the great drawbacks in connection with the subject was that there could be so little chronological dealing with it to see to what extent the mortality had been lessened in recent years, since it was only of late that there had been any intelligent grouping of the facts. Another book which would afford information was Miss Kingsley's "*Travels in West Africa*", just published, and well worth reading. At the end of that book there was a note, based greatly on Dr. Plehn's opinions, on the diseases of West Africa, which was of great interest in connection with the subject of the paper. Mr. Sprague said little about the occupation of the lives, but he gathered that most of them were either Government officials or employed in commerce, but if any had been engaged in the expeditions which had from time to time taken place, that would make a difference in the mortality, for they might not be unharmed though unarmed. Missionaries, however, were mentioned as a class, and he thought Mr. Sprague was in error when he took for granted that they were not medically examined before being sent out, for it was the practice, he believed, at any rate of the great missionary societies, to very strictly examine every candidate who submitted himself. The book he had above referred to gave an entirely different reason for the heavier mortality of missionaries, for that put it down to their being mostly teetotallers. The deadly climate on the immediate coast was equally bad for the negroes as for Europeans; and whole tribes, as they moved down to the marshes and lagoons, seem to have in time absolutely died out. There appeared to be little yellow fever there—at any rate it was not endemic—but malarial disease seemed most serious on the actual West Coast. Higher up remittent and intermittent fevers were prevalent, and lower down there was much hæmaturia. In the South West, as Mr. Sprague pointed out, the mortality was less, which might probably be partially due to the fact that it was further from the equator. With regard to Mr. Sprague's groupings, the members would all accept as accurate any groupings Mr. Sprague had done; but personally he always looked with some suspicion when a writer spoke of having had to group facts several times to get a regular law, for there was the fear that in his anxiety for regularity he might overlook the peculiarities of the mortality. In the book to which he had made reference it was stated that, except for men who could be counted on the fingers, every one was bound to have fever within six months of landing, and if not careful they would probably get it in a fortnight, and very likely once for all; so that the light mortality for young ages given by Mr. Sprague was strange, particularly after what was said later on as to lives under 25, and especially when it was borne in mind that the younger men were probably less attentive to their mode of life, and would also doubtless be subject to the higher death rate of new comers. It was very interesting, after Dr. Sprague's papers on selection, to find his son tracing a sort of selection in this mortality, and finding an improvement when selection had done its work, much

the same as in the case of the assured's selection against the office. It might possibly also be, as it were, some inversion of this improvement that on the South West Coast there was the steadily increasing mortality, in that the lives were not killed off in the earlier years. He would like to know more about acclimatization, because, though the paper seemed to trace acclimatization, Dr. Plehn maintained there was none whatever. But whatever doctors might say, he could not help thinking the members would all maintain that there must be something in the way of acclimatization, as a man's experience taught him how to take care of himself, and he learned the proper rules of life for the particular climate to which he was exposed. If so, it showed that the practice of charging an equal extra premium every year must be wrong. Many years ago, when the ordinary West Indian rate was 3 per-cent, he greatly desired to charge 5, 4, 3, 2, 1, per-cent respectively during five years, instead of five premiums of 3 per-cent, which would, he thought, have been more satisfactory. Moreover, any acclimatization seemed to give a sort of rough justification for the very common practice of charging an extra premium only during the first few years of the assurance, though obviously it failed when a man went out some time after the grant of the policy. The system, however, of issuing the policy "whole-world" from the beginning and charging, when necessary, a fixed addition to the ordinary premium was better; and it provided a reserve for the risk, to which Mr. Sprague had called attention, of death at home, but from disease acquired in the colony. He was not quite at one with Mr. Sprague as to the need of examining a man before any return of extra premium was made, for if it was agreed to charge the extra only during absence, questions of health should not come in, and the premium should be sufficient for this additional risk. Nothing was more likely to induce quarrelling and bickerings as a difference between doctors whether a man had returned well or not, but of course it all depended on the terms of the contract. He hoped Mr. Sprague would give more of the abbreviated table of premiums which he had calculated. It would be interesting to observe his deductions from it. It had been truly said that nothing hinders a man's work like his dying, and if in anything they could do they could find out any rules or get any information which would help the English race to safely colonize what was admittedly a productive country, they would feel that Mr. Sprague's labours had not been in vain.

MR. GEORGE KING entirely concurred in the eulogistic expressions which had fallen from Mr. Higham. To a certain extent, he felt himself personally responsible for the paper being read at the Institute. Mr. Sprague had sent it to him for the purpose of insertion in the *Journal*, but after reading it over, he came to the conclusion that it would be very useful if it were discussed at a sessional meeting. One reason he had for that, beyond the merits of the paper itself, was that he would like to obtain some more facts, and he hoped that there might be members present who would be able to give them some facts connected with the mortality, from the point of view of risks actually accepted by insurance companies. If such facts could be obtained, they would be more useful than all the experience which Mr. Sprague had so carefully, and so industriously, and so ably collated. Although

this was the first occasion on which they had had the pleasure of having a paper from Mr. Sprague, it was not his first contribution to the *Journal*. It was from his pen that the very interesting review of Dr. Felkin's book proceeded, and Mr. Sprague had also read a paper at another place on a similar subject. The subject of the paper was one of considerable difficulty. In the first place, Africa was a small word, but it represented an immense district, and the geographical area to be considered was very large. Africa, as was well known, was not homogeneous as regards climate and mortality. There were certain districts dealt with in the paper that were tolerably well defined—for instance, the Congo district and the West Coast, from Sierra Leone to Gaboon; but he would like to have a clearer definition of Central Africa, and to know what portion of Central Africa was more particularly referred to in the statistics Mr. Sprague put forward. Another difficulty was the necessary paucity of figures, and he was afraid that for a long time to come there would not be much relief in that direction. They would have to take the indications which the few figures supplied and work them out to the best of their ability. The figures were terrible in their import, the rate of mortality shown being enormous. It was really larger in proportion to the home mortality than Mr. Sprague brought out. Mr. Sprague showed that the average rate of African mortality was 6·5 per-cent, and compared it with the rate for the whole population of England, according to the English Life Table, which was only 2·2. But the rate of 2·2 per-cent, according to the English Table, was scarcely applicable, for the simple reason of the great difference in age distribution. To get a true comparison, a population should be taken of roughly between ages 25 and 50, and in that case the English death-rate would only be about ·8, not 2·2. So that the comparison should be 6·5 with ·8, and then it would be seen that the rate of mortality in the districts analyzed by Mr. Sprague, was just about eight times the rate in Great Britain among persons of corresponding ages. The question arose,—were those figures inevitable? There was a certain tendency to fatalism in these matters, but that he would deprecate. The reading of a paper such as that, should tend towards curing the great evil, for to call public attention to it through a body of experts was to help to remedy the mischief. The mortality in the British army in India, in the first half of the century, was something like 7 per-cent—that was, it was comparable very closely with the mortality brought out by Mr. Sprague for lives in Africa of corresponding age. A Royal Commission was appointed in 1854 to inquire into that mortality, and they reported, and great improvements were introduced, and the result was that, partly through the labours of that Commission, and partly through the labours that had gone on since, the mortality of the British army in India was reduced to about 1·2 per-cent. If so much had been done in India, why should not a great deal be done in Africa too. The cases were not quite parallel, because in India there was a great deal more massing of the population, and the army was much more under control, and its health could be better looked after. But the measures applied to masses, individuals should apply to themselves; and, if individuals going to Africa were careful to ascertain in advance what was the

proper course to pursue, the mortality would be immensely reduced. In Sierra Leone, by attending to the water supply, the mortality was very much diminished. Wherever there was a sufficient population to carry through such works, that should be done; but he believed that the mortality in Africa was very much due to carelessness and want of knowledge on the part of the individuals who went out there. It was not the heat that produced it; it was the heat combined with moisture which fostered micro-organisms fatal to life. But people were now getting to understand much more the ways of micro-organisms. Science at the present day was making rapid strides in discovering their life-history and their effect on the human frame, and in this country, at all events, their injurious effects were being to a large extent counteracted. If so, still greater things might be hoped in connection with Africa. Those who went to Africa neglected precautions which they ought to take. They carried English habits with them, instead of learning the habits of the African microbes, and if they would only learn the habits of African microbes they would in great measure escape the terrible mortality. About a fortnight ago he had had the pleasure of meeting a missionary who had recently returned from Central Africa, who told him very much what he had been saying that evening, that it was entirely a question of manner of living. Very soon after that missionary reached Africa he had to be invalided home. He recovered and went out again with superior knowledge, and he remained some time and did not suffer in the least degree in health, not because he had been acclimatized, but simply because he knew how to take more care of himself. Among other little hints, he said that he never touched alcohol during the day, but always took a tablespoonful before going to bed at night. He also mentioned that one great danger was to go to bed too fatigued, and in that way lay oneself open to the evil influences of malaria. It was a mistake to go out as a total abstainer, but a man should be a total abstainer all through the heat of the day, but just before going to bed to take a very little stimulant. Another most important precaution was to avoid chills. Looking at the paper from the practical point of view, he was glad to see Mr. Sprague's suggestions as to the premiums he had deduced, that they were the very least that should be charged. His (Mr. King's) own personal opinion was that they were decidedly not high enough, and that the really sufficient rates would be practically prohibitive. He believed that the risk of a man returning and dying at home was a very heavy one, which was not in any way provided for in Mr. Sprague's rates. Except from the records of insurance offices, which he hoped might be forthcoming, they could not have any measure of that risk.

Dr. T. GLOVER LYON said there were one or two points in the paper which he should like to touch upon. First of all, he endorsed Mr. Sprague's praises of the book of Dr. Felkin, for it was one of the very best books he had ever read. Mr. Sprague mentioned that the Congo appeared to exercise a marked effect on the climate of places in its immediate vicinity. The same thing happened with regard to most rivers in tropical climates, because the conditions in which malaria raged were dampness, heat, and great variation of temperature during the day. Near a river the first two were certainly obtained, and he

was not surprised to find that right up the Congo into the middle of what was termed Central Africa there were a large number of fatal diseases, more especially malaria. There was a marked difference in the mortality between parts of the Congo territory away from the river and those in its vicinity, and that was quite in accordance with ordinary medical experience. The next point was the matter of sanitation, and on that Mr. King had said much of great interest; but he rather thought he had been a little too much of an optimist. The diseases which had to be dealt with in tropical South Africa were not such as were found in England, and were not nearly so amenable to prophylactic treatment. It was scarcely known, for instance, how malaria was spread. Yellow fever was not understood, and it was not known how to isolate it in the same way that other fevers were isolated. It was not difficult to see how the mortality in India had been much reduced. Formerly that mortality was largely due to cholera and typhoid fever, and those were diseases that were under absolute control if only the precautions were taken which were well known at the present day. Mention had been made by Mr. Sprague of his (Dr. Lyon's) paper. Just before he wrote it several men came to the company of which he was medical officer, who were going to the West Coast, and who wanted to be insured. He thought it was only fair in giving an opinion upon the point to find out all he could about the question, and he went to the Colonial Office and obtained some figures. Mr. Sprague had well pointed out that there was a great deal of doubt as to the lives at risk in Gambia and Sierra Leone, but with regard to the West Coast and Lagos the papers seemed to be very carefully made out. Every year the residents and visitors, and also the deaths, were put down, with full description of those who died. Therefore, he thought the figures were quite as good as figures got through traders or missionaries. Mr. Sprague raised an interesting medical question as to the age of going out to West Africa. That was a point that medical officers would be very thankful for the Institute to settle. To be able to say that when men reached a certain age they had a good chance of doing well, and that if they went out before that they had not a good chance, would be extremely helpful and useful to medical officers. He examined for two firms sending men out to India, and he thought medical officers would agree with him when he said they were rather less severe on people going out for trading purposes than they were on people for insurance. First, the time of life they were concerned with was much shorter as a rule; and, secondly, it often happened that the employers would say the man was very useful, and if he had a fair life they would send him out; so that there was a pressure on the medical officer to pass that man if he possibly could. But in insurance companies they had to be much more strict, and there was no pressure put upon them. With regard to acclimatization, it was becoming the received opinion that acclimatization in the ordinary sense of the word—namely, that a man could alter his organism so as to live as well in West Africa as in England—did not exist. Individual acclimatization did not occur at all, but what did occur in some cases was acclimatization by groups. As to after history, on making inquiries, he found that out of the officials that went out to West

Africa more than half came back within three years, and a large proportion of those were perfectly healthy. He did not think that would happen with traders, so that, as far as after history went, it might be that the officials would come out much better than the traders. Mr. Antrobus had pointed out to him that people who had been well brought up in the way of nourishment, people of the better classes, did decidedly better than those of other classes that were sent out, such as artisans. The latter did exceedingly badly. There was another point, in reference to geography. The West Coast was always likened to an inverted dish. First, there was the table land and then a quick slope down to the rim, and then came the sea. The rim represented the West Coast of Africa. The climate on the table-land was perfectly good, but on the rim it was bad. At Sierra Leone the rim was only about a third as broad as it was at Lagos and the Gold Coast, and as malaria and yellow fever and other tropical diseases lived in the lower levels, that might account for the fact that Sierra Leone appeared to be much more healthy than the Gold Coast or Lagos. He had been told that the Colonial Office in the future were going to make an enquiry into the whole matter under the care of Mr. Finlaison. Statistics were going to be obtained with great care in the following way: Returns of all European officers in the colonies; name, date of birth, date of first arrival in the colony, date of final departure from colony, total absences from the colony (with dates), occupation in the colony, expiration of service, subsequent life-history. Mr. Antrobus had said how surprised he was to find how well the subsequent life-history could be followed; and Mr. Antrobus felt sure that he would be able to get a perfectly complete account of the subsequent life-history of those who had been to the colonies.

Mr. J. R. HART said that Mr. Sprague had brought out a very high rate on the Congo—9·4 per-cent—which naturally led one to enquire what were the particular circumstances and risks attaching to that body of persons examined which might make the rate higher than that which would attach to any other body. The period of time over which Mr. Sprague's observations extended was not stated, but he understood it covered 15 years—from 1878 to 1892. Mr. Sprague's statistics would probably include, therefore, a large number of the pioneers that Stanley took out with him to found the Congo Free State between the years 1878-85. Mr. Stanley, in his book on the Congo, gave an analysis of the fearful mortality to which his force had been subject; and said that many of the deaths were the result of downright madness, and the majority of them could have been avoided. In addition to malaria, there was considerable risk in the cold winds and chilly nights, and variations in the temperature. Besides that, there was the moisture, which was very trying, and for that reason the English, well accustomed to the damp and mist of these islands, could stand the climate better than other Europeans. Mr. Stanley's experience seemed to have been useful as a means of guarding against malaria. In three ways it appeared that malaria could be minimized; a belt of trees round the station, a stream of running water, or the cultivation of the soil in the direct tract of the prevalent wind seemed to make a marked difference in the mortality. The experience of three expeditions might be taken as an interesting example of how

the pioneers in Africa really suffered so much more than those who now went out. In 1832, 49 men went up the Niger and 40 died. In 1843, 143 went up of whom 48 died; 11 years later, in another expedition, not a single death occurred, thanks to the lessons learned in the earlier expeditions; more attention being paid to diet and to the use of quinine. Mr. Sprague gave an example of how the rate of mortality had improved. He formerly brought out a rate of 7 per-cent for the West Coast, the maximum being 21 per-cent in the earlier years; he now brought out 4·7, with a maximum of 10 per-cent. With regard to the West Coast, Mr. Sprague compared 4·7 per-cent with the rate brought out by himself (Mr. Hart) with regard to Government officials of 5·2 per-cent. Mr. Sprague gave two reasons why he thought Government officials should be better lives; their leave of absence and medical examination. He (Mr. Hart) thought that the explanation of the higher rate he brought out was that amongst those lives he dealt with, there was a certain number of foremen of works, and Mr. Antrobus, whom he saw on the subject, stated that they were either dismissed very speedily or died from drink. Not only was drinking to excess very dangerous, but even drinking in moderation during the day, and for that reason he thought that the mortality of Government servants was likely to be higher than missionaries, as the latter were mostly teetotallers. Another reason why missionary mortality was rather lighter than that which might be taken as a guide for insurance purposes, was that the mission stations were generally on spots chosen for their salubrity. It might be of interest to mention that the African Trade Section of the Liverpool Chamber of Commerce had been taking up the subject of improving the conditions of health on the West Coast, and they had made representations to Mr. Chamberlain, who was doing a good deal in the matter. More medical officers were to be sent out, and the water supply of the Gold Coast was to be improved. Mr. Sprague, in his Congo experience of Government officials and missionaries, showed that the mortality among missionaries was rather higher than that among Government officials, but he (Mr. Hart) thought that was due to the fact that of the 128 missionaries 28 were females. By the kindness of Mr. Sprague he had had the opportunity of looking at fuller particulars than were given in the paper on the subject, and he found that those females were divided into three divisions; the first section did not give much information, but the other showed 18 per-cent mortality, compared to 8·9 among the males. If this 18 per-cent were a guide to the mortality among the 28 females, it would be a satisfactory explanation of the higher rate. He thought that probably the Government officials experienced a higher mortality than the missionaries.

Mr. ARCHIBALD HEWAT said that some years ago he had obtained from New York statistics relating to the mortality among white missionaries in Africa, which he passed on to the *Insurance Record*, where they were published on 30 January 1880, and if the members would permit him he would read them: "We can but remember that the mortality to white missionaries in Africa has been frightful. The Church Missionary Society lost thirty missionaries by death at Sierra Leone in twelve years. And the history of their Central African

Mission has been sad indeed. The London Missionary Society has also been sorely tried by the death of several of its missionaries within the last two years. . . . The Basle Society lost eleven out of seventeen missionaries on the Guinea Coast within one year after they entered on their work. The American Missionary Association lost on the West Coast five out of seven in a few weeks after they touched the Continent. The Wesleyan Society lost in West Africa thirteen out of the first eighteen missionaries in less than two years after they entered on their work. The same society sent seven others and three of them died within six weeks, and the other four within fifteen months." Lately he had looked into the question, and had obtained from official sources some statistics, from which he found that the rate of mortality did not come out nearly so badly as he thought it would have done. With regard to taking off the extras when a man came home, he quite agreed with Mr. Sprague, that a medical certificate of good health should be obtained. It was not known what was going to happen after the assured came home—possibly invalided—the extra risk was not thereby at an end. With regard to Central Africa, in Scotland they had a good deal of experience with regard to that class of life through the Scottish Presbyterian Churches, which had missions in Blantyre and Livingstonia. And as to the question of the Government officials being better lives than missionaries it should always be borne in mind that the missionary of the right type had a higher sense of duty than had a Government official, and he would risk a good deal more than a Government official would do, and that he thought was one reason why the missionaries died earlier. Another reason was that missionaries had not always the same stamina as military men, Government officials, traders, and such like. They had spent a great deal of time in hard study, immediately after which they went abroad enfeebled.

Mr. JAMES CHISHOLM said he had been interested in the subject of acclimatization. It seemed to him that Mr. Sprague made out two different kinds of acclimatization. He was surprised to find Dr. Lyon saying that the first kind of acclimatization—the acclimatization of the individual—was exploded. That would hardly seem to be so from the quotation that Mr. Sprague gave from Dr. Felkin, in which it was said that a lengthened residence gave strangers a certain immunity from certain diseases. It would almost seem that a group of individuals was acclimatized in much the same way as the body of an individual itself was acclimatized. The corpuscles that made up the bodies of each one of us were liable to attack in certain particular tracts by different kinds of diseases, and the attack by a particular disease killed out the particular corpuscles which were most susceptible. There was another fact on which he should further like to be enlightened, namely, as to whether a man lost the immunity he had secured by acclimatization on coming home on a lengthened furlough. The number of cases that came before him was very limited, but he had in his mind three cases—two on the West Coast of Africa and one in India. They all three came to insure their lives, and urged their residence abroad very strongly as a reason for some modification of terms being granted. They were insured, and in all three cases they

died within one year of their return to the places where they had previously resided and to which they were supposed to have become acclimatized. He could corroborate what had been said as to the teetotallers suffering more than others. At Mauritius there was a detachment of soldiers, and most of them were kept up on the High Level, but a number of them, some twelve or sixteen, were kept at the Fort at Port Louis, where the land was much lower and the water supply not so good. It was a fact, or was at the time he referred to, that the teetotallers among those soldiers who went down to the Fort were more subject to the fever, and many more succumbed than among those who were not teetotallers.

The PRESIDENT said that in all ventures into new tracts of enquiry men must obviously for a very considerable time be content with very limited data, such as that which had been available to Mr. Sprague, and the value of limited data had invariably consisted in the fact, not that they were able to assign any final results, but that they served the important function of indicating the direction in which men's practical judgments must prevail. He conveyed to Mr. Sprague the cordial acknowledgment from the members of his work.

Mr. SPRAGUE, in reply, thanked the members for the very cordial and hearty reception they had given him. Mr. Higham had wished to know about the occupations of the various classes of lives. Speaking from memory, he thought that nearly all those on the Congo were Government officials in the employ of the Belgian Government; nearly all those on the West Coast and in Central Africa were missionaries; and all those on the South West Coast were traders in the employ of a commercial company. As to the tables of premiums that he made, a not unusual way of getting at the extra premium was to say "here is a rate of mortality of 8 per-cent; according to the H^M Table there is a mortality of 1 per-cent; therefore the extra is, roughly speaking, 7 per-cent." He did not consider that very satisfactory, and he wanted to see if he could get some more scientific approximation of the minimum. He did not say that the premiums were what should be charged; he was only trying to find what was the lowest they could accept. Offices, as they all knew, differed in practice. For instance, with regard to India, his brother, in one office, was asked to pay an extra premium of 30s. per-cent. He promptly declined to pay that, and succeeded in getting insured in another office without any extra at all. It seemed to him that it was rather unfortunate there should be so much difference in the practice of offices, and he would be glad if his paper helped towards uniformity. He had been asked to define more specifically what Central Africa meant. He was very sorry he could not give any accurate definition. The returns came from a few missionary societies scattered about in various parts, probably, as Dr. Lyon assumed, parts of the Congo territory. From what he had heard, he should think there was some personal acclimatization as regards certain diseases. Malaria could be guarded against very considerably, as indicated by Mr. Hart in his remarks; and fatigue had a good deal to do with the risk of contracting it. Some of the crew of one of Her Majesty's ships on the West Coast were sent out in an open boat; half of them were kept awake nearly all the night

working, but the other half managed to get some sleep. The majority of those who were kept up all night were down with the fever next day, while those who got a proper night's rest for the most part escaped. It was, therefore, supposed that fatigue had a very great tendency to increase the liability to malaria. He was glad to see that Mr. Hewat agreed with him about not returning any extra premium without requiring some evidence of health. As far as he could make out, there seemed to be a considerable number of lives who returned to this country and died shortly afterwards.

On the Mortality among Government Officials on the West Coast of Africa. By J. R. HART, F.I.A.

A FEW statistics relating to this subject recently came into my hands, and although the results obtained from them cannot be considered to be of great weight, a short communication may be of interest. In the hope that useful information might be forthcoming, I made enquiry as to whether any record is kept of the dates of departure, death, or retirement of persons who go out in the employment of African merchants to the West Coast; and ascertained that it was unlikely that data could be supplied from that source. But I thought it worth while to make similar enquiry at the Colonial Office; and although the West African department could not officially furnish me with information, as they had none here of which they could vouch for the accuracy, I obtained, through the courtesy of one of the officials, the particulars given below. These were contained in a list of all the Europeans employed by the Governments of the four West African Colonies—the Gambia, Sierra Leone, the Gold Coast, and Lagos—during the ten years, 1 January 1881 to 31 December 1890, showing when their service began, and, if ended before 31 December 1890, when and why it ended. The list comprised 296 persons, of whom 138 served on the Gold Coast, 44 at Sierra Leone, 25 at Lagos, and 7 at Gambia; while, of the rest, 36 were employed both in Lagos and on the Gold Coast, and 46 in two or more of the other colonies, and in 16 cases the colony was not specified, the nature of the official's employment probably necessitating frequent changes from one place to another. The occupations of these persons were not stated; but I understand that most of them were of the better classes, including consular servants, secretaries, and constabulary, medical and customs' officers; but some held positions akin to that of foremen of works. A fair proportion of the last-mentioned become, I am told, addicted to drink, and are soon dismissed, their exit being

noted in the column headed "Other causes." If this be so, we may conjecture that the mortality, shown below, would be increased by the deaths, which occur subsequent to dismissal, brought about by this fatal indulgence.

Of the 296 lives, 77 had on 1st January 1881, already been for some time in the service, and 219 joined during the period 1881-90; so that the experience is short, the average being only $3\frac{1}{2}$ years. The causes of service ending were stated in four columns headed (1) death, (2) illness, (3) age of 55, (4) other causes, the totals of each being

Died	54
Retired on account of illness	37
Retired on reaching pensionable age	3
Retired from other causes (<i>i.e.</i> , promoted, resigned, or dismissed)	90
								184
Add, still surviving on 31 December 1890	112
								296
Total	296

No ages were given, but I understand that colonial servants are not sent out to West Africa under the age of 22, and, as stated above, the lives are pensioned at age 55. The years of entry and exit were given and not the exact dates. In the following table, the Exposed and Died are tabulated according to the period elapsed since first leaving this country. The total number of years of life are $1031\frac{1}{2}$, and the deaths 54, giving an average rate of mortality of 5.2 per-cent.

Year	Exposed to Risk	Died	Annual Rate of Mortality
$\frac{1}{2}$	$105\frac{1}{2}$	3	2.9
$1\frac{1}{2}$	184	14	7.6
$2\frac{1}{2}$	151	8	5.3
$3\frac{1}{2}$	113	5	4.4
$4\frac{1}{2}$	91	4	4.4
$5\frac{1}{2}$	73	6	5.3
$6\frac{1}{2}$	60	1	
$7\frac{1}{2}$	58	3	5.2
$8\frac{1}{2}$	52	6	5.1
$9\frac{1}{2}$	38	1	
$10\frac{1}{2}$	21	1	
$11\frac{1}{2}$	15	1	
$12\frac{1}{2}$	11	...	
$13\frac{1}{2}$	10	...	
$14\frac{1}{2}$	7	...	
$15\frac{1}{2}$	7	...	
$16\frac{1}{2}$	5	...	
$17\frac{1}{2}$	5	...	
$18\frac{1}{2}$	2	...	
$19\frac{1}{2}$	3	...	
$20\frac{1}{2}$	20	...	
Totals	<u><u>$1,031\frac{1}{2}$</u></u>	<u><u>54</u></u>	<u><u>5.2</u></u>

Among the 138 persons who were employed on the Gold Coast only, the mortality was 6·2 per-cent, while the 44 who served in Sierra Leone only experienced the average death of 5·2 per-cent; but to a comparison of these results little importance can be attached, as they must be disturbed to some extent by the selection which takes place on promotion, those who obtain higher posts on transfer from one colony to another being on the whole better lives than those who appear in the list as having served in one colony only.

The facts are too scanty to admit of our drawing any definite conclusions from a comparison of the rates at successive periods of duration of service; but the comparatively low annual rate of 2·9 per-cent in year $\frac{1}{2}$, and the sudden increase to 7·6 per-cent in the following year, may possibly be explicable by a comparison of the rates at which the lives retired on account of illness.

After adjusting the period of exposure of the "Died" and "Invalided", we get the following table, giving 1029 $\frac{1}{2}$ exposed and an average rate of 3·6 per-cent.

Year	Exposed	Retired from Illness	Annual Percentage
$\frac{1}{2}$	106 $\frac{1}{2}$	6	5·6
1 $\frac{1}{2}$	180	5	2·8
2 $\frac{1}{2}$	148	3	2·0
3 $\frac{1}{2}$	113	2	1·8
4 $\frac{1}{2}$	89	1	
5 $\frac{1}{2}$	71	2	
6 $\frac{1}{2}$	60	2	4·2
7 $\frac{1}{2}$	58	1	
8 $\frac{1}{2}$	51	4	
9 $\frac{1}{2}$	37	1	7·3
10 $\frac{1}{2}$	24	5	
11 $\frac{1}{2}$	15	1	
12 $\frac{1}{2}$	11	0	
13 $\frac{1}{2}$	11	1	
14 $\frac{1}{2}$	8	1	3·6
15 $\frac{1}{2}$	7	0	
16 $\frac{1}{2}$	6	1	
17 $\frac{1}{2}$	5	0	
18 $\frac{1}{2}$	4	0	
19 $\frac{1}{2}$	6	0	
20 $\frac{1}{2}$	19	1	
Totals . .	1,029 $\frac{1}{2}$	37	3·6

It will be observed that the annual rate at which these officers retired on account of illness was, during the first six months, double that of the following year. No information was furnished as to the subsequent experience of those who so retired; but, were they traced out, we may assume that the ratio of deaths to

exposures would be increased; and this assumption seems to be borne out by a comparison of the rates of mortality for years $\frac{1}{2}$ and 1.

Another point suggested by a consideration of Table I is that, with the exception of the first eighteen months after commencement of service, there is not at any time a great divergence from the average rate of 5·2 per-cent. This is probably the result of the special opportunities allowed to Government officials for recruiting their health, thus preventing a large proportion of the lives dying off in the early years to leave but a few exceptionally good lives at later periods. The rules as to leave, adopted by the Government, may be summarized as follows: Subject to the necessities of the service, officers, under the rank of governor, who were not themselves born in West Africa, and whose parents were neither of them born there, may be granted six months' leave of absence after they have completed (1) in Sierra Leone and Gambia fifteen, and (2) on the Gold Coast and in Lagos twelve, consecutive months of residential service. Notwithstanding the more extensive leave allowed in the case of the two latter colonies, the mortality among the officers resident there is shown by the present facts to be higher than the average.

It was impracticable to ascertain the years of exposure while resident in Africa apart from the time spent on leave, and the figures, therefore, relate to the experience of Government servants during their term of service, and cannot be taken as a guide to the risk attendant on continuous residence on the West Coast. While the precautions taken by the Government must tend to reduce the mortality among their servants, most persons coming before an insurance office will be somewhat similarly circumstanced in being able to recruit their health by some months' leave after a stay of a year or two in Africa. A fact that must not be overlooked in comparing the present results with others is, that nearly all these Government servants have to reside on the coast, and are not able to choose the more healthy spots on the higher ground farther inland.

On the Approximate Evaluation of the Integral for the Compound Survivorship Annuity. By RALPH TODHUNTER, F.I.A.

IN the various references in the *Journal* to the subject of the approximate evaluation of compound survivorship annuities and assurances, the possibility of using Mr. Emory McClintock's expression for the continuous annuity (*J.I.A.* xviii, 242) does not appear to have been noticed. By substituting Mr. McClintock's expression for the annuity under the integral sign, and integrating by parts, many compound survivorship benefits can be expressed more or less approximately in terms of simple survivorship benefits; but it may be admitted that the labour necessary to obtain a result as accurate as that obtainable by a summation formula would not in general recommend the method. There is, however, one case—that of the compound survivorship annuity involving three lives—in which the special interest that has always been taken in the function may be considered a sufficient apology for a result which is both complicated and limited in its range of practical applicability; that case it is proposed to investigate in this note.

As an introduction to the constants and other quantities employed in the investigation, it will be convenient to reproduce Mr. McClintock's work in a slightly different form, employing the notation of the *Text-Book*. The symbol λ is used throughout to denote the Napierian logarithm. In terms of Makeham's constants,

$$\bar{a}_x = \int_0^{\infty} (vs)^t g^{e^x c^t - 1} dt = g^{-c^x} \int_0^{\infty} (vs)^t e^{c^{x-t} \lambda g} dt$$

Let $\tau = -c^{x+t} \lambda g$, so that

$$d\tau = -c^{x+t} \lambda g \lambda c dt; \quad t \lambda c = \lambda \frac{\tau}{c^x \lambda \frac{1}{g}}; \quad \text{and} \quad \left(\frac{vs}{c}\right)^t = \tau^{\frac{\lambda \frac{vs}{c}}{\lambda c}} \left(c^x \lambda \frac{1}{g}\right)^{-\frac{\lambda \frac{vs}{c}}{\lambda c}}.$$

Also write β for $c^x \lambda \frac{1}{g}$, and ρ for $\frac{\lambda \frac{vs}{c}}{\lambda c}$ (for the rates of interest of practice the value of ρ lies between -1 and -2). Then, making the necessary alteration in the limits of integration,

$$\begin{aligned}
\bar{a}_x &= \frac{\beta^{-(\rho+1)} e^\beta}{\lambda c} \int_{\beta}^{\infty} \tau^{\rho} e^{-\tau} d\tau \\
&= -\frac{1}{\lambda c} \cdot \frac{1}{\rho+1} + \frac{\beta^{-(\rho+1)} e^\beta}{(\rho+1)\lambda c} \int_{\beta}^{\infty} \tau^{\rho+1} e^{-\tau} d\tau \\
&= -\frac{1}{\lambda c} \cdot \frac{1}{\rho+1} + \frac{\beta^{-(\rho+1)} e^\beta}{(\rho+1)\lambda c} \left[\int_0^{\infty} \tau^{\rho+1} e^{-\tau} d\tau - \int_0^{\beta} \tau^{\rho+1} e^{-\tau} d\tau \right] \\
&= \frac{e^\beta \beta^{-(\rho+1)}}{(\rho+1)\lambda c} \cdot \Gamma(\rho+2) - \frac{1}{\lambda c} \left[\frac{1}{\rho+1} + \frac{\beta}{(\rho+1)(\rho+2)} \right. \\
&\quad \left. + \frac{\beta^2}{(\rho+1)(\rho+2)(\rho+3) + \dots} \right]
\end{aligned}$$

or, since $(\rho+1)\lambda c = \lambda(vs) = \lambda s - \delta$,

$$\bar{a}_x = \frac{1}{\delta - \lambda s} \left[-e^\beta \beta^{-(\rho+1)} \Gamma(\rho+2) + 1 + \frac{\beta}{\rho+2} + \frac{\beta^2}{(\rho+2)(\rho+3)} + \dots \right] \quad (1)$$

If a is written for $-\lambda s$, r for $(\rho+2)$, v for β , v_2 for $\frac{\beta^2}{\rho+3}$, &c., this result will be found to agree with Mr. McClintock's.

A brief consideration of the numerical values of the quantities entering into the terms of the infinite series will suffice to show that a very close approximation to the value of \bar{a}_x can be obtained from (1) for the younger ages, but that the formula becomes quite unmanageable at the older ages on account of the number of terms to be summed. Taking, as a numerical example, $x=40$, working with the *Text-Book* graduation of the H^M Experience and 3 per-cent interest, and neglecting powers of β above the second, (1) gives $\bar{a}_{40} = 17.674$ —a fairly close approximation. The neglect of β^2 would, in this case, have involved an error of .047. At age 20 the error resulting from the neglect of the second power of β would be only .001, but at age 60 it will be found to be as much as 1.813, and at very advanced ages the series $\beta + \frac{\beta^2}{\rho+3} + \frac{\beta^3}{(\rho+3)(\rho+4)} + \dots$ will be temporarily divergent. This feature of the formula does not appear to have been noticed by Mr. McClintock, but it will be seen that it imposes a serious limitation upon the use of the formula in the calculation of compound survivorship annuities. The following table is given for the purpose of indicating the manner in which

the value of the infinite series $\frac{1}{\delta - \lambda s} \left[\frac{\beta}{\rho + 2} + \frac{\beta^2}{(\rho + 2)(\rho + 3)} + \dots \right]$ increases with the age:

Values of $\bar{a}_x + \frac{1}{\delta - \lambda s} \left[e^{\beta} \beta^{-(\rho+1)} \Gamma(\rho+2) - 1 \right]$; H^M (*Text-Book Graduation*) \pm per-cent.

x	$\bar{a}_x + \frac{1}{\delta - \lambda s} \left[e^{\beta} \beta^{-(\rho+1)} \Gamma(\rho+2) - 1 \right]$
20	·339
25	·457
30	·722
35	1·146
40	1·826
45	2·929
50	4·740
55	7·784
60	13·089

Proceeding to the main object of this note, the integral for $\bar{a}_{y:z|x}^1$ becomes, by substituting the expression in (1) for a_{x+t} ,

$$\frac{1}{\delta - \lambda s} \int_0^{\infty} (vs^3)^t g^{c^x + c^y + c^z} c^{t-1} \mu_{z+t} dt \left[1 + \frac{c^{x+t} \lambda \frac{1}{g}}{\rho + 2} + \frac{c^{2x+2t} \lambda^2 \frac{1}{g}}{(\rho + 2)(\rho + 3)} \right. \\ \left. + \dots - \Gamma(\rho + 2) \cdot g^{-c} \cdot c^{x+t} \left(c^{x+t} \lambda \frac{1}{g} \right)^{-\rho+1} \right]$$

The first term in this integral will be seen to be $\frac{1}{\delta - \lambda s} \bar{A}_{xy:z}^1$. The second term on integration by parts gives

$$\frac{c^x \lambda \frac{1}{g}}{(\rho + 2)(\delta - \lambda s)(c^x + c^y + c^z) \lambda c \lambda g} \left[\int_0^{\infty} (vs^3)^t g^{c^x + c^y + c^z} c^{t-1} \mu_{z+t} \right. \\ \left. - \int_0^{\infty} g^{c^x + c^y + c^z} c^{t-1} \left(\lambda (vs^3)^t (vs^3)^t \mu_{z+t} - (vs^3)^t \lambda g \lambda^2 c^{z+t} \right) dt \right]$$

or, since $-\lambda g \lambda c^{z+t} = \mu_{z+t} + \lambda s$,

$$\frac{c^x}{(\rho + 2)(\delta - \lambda s)(c^x + c^y + c^z) \lambda c} [\mu_z + \lambda (vs^3 c) \bar{A}_{xy:z}^1 + \lambda s \lambda c \bar{a}_{xy:z}].$$

The third and following terms may be evaluated in a similar way by successive integrations by parts. The last term becomes by obvious modifications

$$- \frac{\Gamma(\rho+2) \left(c^x \lambda \frac{1}{g} \right)^{-(\rho+1)} g^{-c^x}}{\delta - \lambda s} \int_0^\infty (s^2 \cdot vsc^{-(\rho+1)})^t g^{(c^x + c^y + c^z)(c^t - 1)} \mu_{z+t} dt$$

or, since $(\rho+1)\lambda c = \lambda(vs)$ whence $vsc^{-(\rho+1)} = 1$,

$$- \frac{\Gamma(\rho+2) \left(c^x \lambda \frac{1}{g} \right)^{-(\rho+1)}}{\delta - \lambda s} g^{-c^x} Q_{yz}^1$$

Collecting results, and expressing c^x , c^y , c^z in terms of the corresponding μ 's, and ρ in terms of v , s and c ,

$$\begin{aligned} \bar{a}_{yz|x} = & \frac{1}{\delta - \lambda s} \left[\bar{A}_{xyz}^1 - \Gamma \frac{\lambda(vs)}{\lambda c} \cdot \left(c^x \lambda \frac{1}{g} \right)^{-\frac{\lambda(vs)}{\lambda c}} g^{-c^x} Q_{yz}^1 \right. \\ & + \frac{\mu_x + \lambda s}{\lambda(vs)(\mu_x + \mu_y + \mu_z + 3\lambda s)} (\mu_z + \lambda(vs^3 c) \bar{A}_{xyz}^1 + \lambda s \lambda c \bar{a}_{xyz}) \\ & \left. + \int_0^\infty (vs^3)^t g^{(c^x + c^y + c^z)(c^t - 1)} \mu_{z+t} dt \left(\frac{c^{2x+2t} \lambda^2 \frac{1}{g} \lambda^2 c}{\lambda(vs) \lambda(vs c^2)} + \dots \right) \right] \quad (2) \end{aligned}$$

The evaluation of further terms of the integral would not add interest to the investigation, and would introduce co-efficients of too complicated a character for use in practical calculations. From what has been said above with reference to the infinite series in McClintock's formula for the ordinary annuity, it will be readily seen that the integral in (2) may be neglected without seriously affecting the result, provided the reversionary annuitant is young and the age of either of the counter-lives is such that the risk must determine one way or the other before the annuitant attains, say, age 60 or 65, for the errors in the annuity-values at the older ages will be multiplied by very small probabilities of joint-life survivorship.

The approximation obtained by neglecting the integral in (2) may be slightly improved by adding

$$\frac{1}{\delta - \lambda s} \int_0^\infty (vs^3)^t g^{(c^x + c^y + c^z)(c^t - 1)} \mu_{z+t} \cdot c^t \cdot dt \left(\frac{c^{2x} \lambda^2 \frac{1}{g} \lambda^2 c}{\lambda(vs) \lambda(vs c^2)} + \dots \right) \quad (3)$$

the value of which is obviously less than that of

$$\frac{1}{\delta - \lambda s} \int_0^{\infty} (vs^3)^t g^{c^x + c^y + c^z} c^{t-1} \mu_{z+t} dt \left(\frac{c^{2x+2t} \lambda^2 \frac{1}{g} \lambda^2 c}{\lambda(vsc) \lambda(vsc^2)} + \dots \right)$$

Substituting from (1)

$$\bar{a}_x + \frac{1}{\delta - \lambda s} \left[\Gamma \frac{\lambda(vsc)}{\lambda c} \left(c^x \lambda \frac{1}{g} \right)^{-\frac{\lambda(vsc)}{\lambda c}} g^{-c^x} - 1 - \frac{c^x \lambda \frac{1}{g} \lambda c}{\lambda(vsc)} \right]$$

$$\text{for } \frac{1}{\delta - \lambda s} \left[\frac{c^{2x} \lambda^2 \frac{1}{g} \lambda^2 c}{\lambda(vsc) \lambda(vsc^2)} + \frac{c^{3x} \lambda^3 \frac{1}{g} \lambda^3 c}{\lambda(vsc) \lambda(vsc^2) \lambda(vsc^3)} + \dots \right]$$

and evaluating $\int_0^{\infty} (vs^3)^t g^{c^x + c^y + c^z} c^{t-1} \mu_{z+t} c^t dt$ by integration by parts, (3) becomes

$$\left\{ \bar{a}_x + \frac{1}{\delta - \lambda s} \left[\Gamma \frac{\lambda(vsc)}{\lambda c} \left(c^x \lambda \frac{1}{g} \right)^{-\frac{\lambda(vsc)}{\lambda c}} g^{-c^x} - 1 - \frac{c^x \lambda \frac{1}{g} \lambda c}{\lambda(vsc)} \right] \right\} \\ \times \frac{\mu_z + \lambda(vsc) \bar{A}_{xyz}^1 + \lambda s \lambda c \bar{a}_{xyz}}{\mu_x + \mu_y + \mu_z + 3\lambda s}$$

The substitution of this result for the integral in (2) gives, as an inferior limit of value of $\bar{a}_{yz|x}^1$,

$$\frac{1}{\delta - \lambda s} \left[\bar{A}_{xyz}^1 - \Gamma \frac{\lambda(vsc)}{\lambda c} \left(c^x \lambda \frac{1}{g} \right)^{-\frac{\lambda(vsc)}{\lambda c}} g^{-c^x} Q_{yz}^1 \right] \\ + \left[\bar{a}_x + \frac{\Gamma \frac{\lambda(vsc)}{\lambda c} \left(c^x \lambda \frac{1}{g} \right)^{-\frac{\lambda(vsc)}{\lambda c}} g^{-c^x} - 1}{\delta - \lambda s} \right] \frac{\mu_z + \lambda(vsc) \bar{A}_{xyz}^1 + \lambda s \lambda c \bar{a}_{xyz}}{\mu_x + \mu_y + \mu_z + 3\lambda s}.$$

The same result may be obtained by another method which has the advantages of exhibiting a little more clearly the nature of the approximation just obtained, and of giving a superior limit of value. Reverting to the notation employed in the reproduction of Mr. McClintock's formula, and using β' to denote $c^{x+t} \lambda \frac{1}{g}$,

$$\bar{a}_{yz|x}^1 = \frac{1}{\delta - \lambda s} \int_0^{\infty} (vs^3)^t g^{c^x + c^y + c^z} c^{t-1} \mu_{z+t} dt \left[-e^{\beta' \beta'^{-\rho+1}} \Gamma(\rho+2) + 1 \right. \\ \left. + e^{\beta' \beta'^{-\rho+1}} \int_0^{\beta'} \tau^{\rho+1} e^{-\tau} d\tau \right]$$

$$\text{which} = \frac{1}{\delta - \lambda s} \left[\bar{A}_{xyz} - \Gamma \frac{\lambda(vsc)}{\lambda c} \left(c^x \lambda \frac{1}{g} \right)^{-\frac{\lambda(rs)}{\lambda c}} g^{-c^x} Q_{yz} \right. \\ \left. + \int_0^\infty (vs^3)^t g^{(c^x + c^y + c^z)(c^t - 1)} \mu_{z+t} dt \cdot g^{-c^{x+t}} \left(c^{x+t} \lambda \frac{1}{g} \right)^{-(\rho+1)} c^t \right. \\ \left. \times \int_0^{c^x \lambda \frac{1}{g}} (c^t \tau)^{\rho+1} e^{-c^t \tau} d\tau \right]$$

The double integral in this expression becomes, by obvious modifications, and by changing the *order of integration*,

$$g^{-c^x} \left(c^x \lambda \frac{1}{g} \right)^{-(\rho+1)} \int_0^{c^x \lambda \frac{1}{g}} \tau^{\rho+1} e^{-\tau} \int_0^\infty (vs^3)^t g^{(c^x + c^y + c^z)(c^t - 1)} \mu_{z+t} \cdot c^t \cdot dt d\tau,$$

where ξ is taken so that $\tau = c^\xi \lambda \frac{1}{g}$.

$$\text{Or, } g^{-c^x} \left(c^x \lambda \frac{1}{g} \right)^{-(\rho+1)} \int_0^{c^x \lambda \frac{1}{g}} \tau^{\rho+1} e^{-\tau} \cdot \frac{\mu_z + \lambda(vs^3c) \bar{A}_{\xi yz} + \lambda s \lambda c \bar{a}_{\xi yz}}{\mu_\xi + \mu_y + \mu_z + 3\lambda s} \cdot d\tau.$$

$$\text{Hence } \frac{1}{\delta - \lambda s} \int_0^\infty (vs^3)^t g^{(c^x + c^y + c^z)(c^t - 1)} \mu_{z+t} dt \cdot g^{-c^{x+t}} \left(c^{x+t} \lambda \frac{1}{g} \right)^{-(\rho+1)} c^t \\ \times \int_0^{c^x \lambda \frac{1}{g}} (c^t \tau)^{\rho+1} e^{-c^t \tau} d\tau$$

$$= \left[\frac{\bar{a}_x + \Gamma \frac{\lambda(vsc)}{\lambda c} \left(c^x \lambda \frac{1}{g} \right)^{-\frac{\lambda(rs)}{\lambda c}} g^{-c^x} - 1}{\delta - \lambda s} \right] \frac{\mu_z + \lambda(vs^3c) \bar{A}_{\xi yz} + \lambda s \lambda c \bar{a}_{\xi yz}}{\mu_{\xi'} + \mu_y + \mu_z + 3\lambda s} \quad \dots (4)$$

where ξ' is the average value of ξ for the range of values lying between the limits determined by $c^\xi = 0$ and $c^\xi = c^x$. An *inferior* limit of value is obviously obtained by putting $\xi' = x$; (4) then becomes

$$\left[\bar{a}_x + \frac{\Gamma \frac{\lambda(vsc)}{\lambda c} \left(c^x \lambda \frac{1}{g} \right)^{-\frac{\lambda(rs)}{\lambda c}} g^{-c^x} - 1}{\delta - \lambda s} \right] \frac{\mu_z + \lambda(rs^3c) \bar{A}_{xyz} + \lambda s \lambda c \bar{a}_{xyz}}{\mu_x + \mu_y + \mu_z + 3\lambda s}.$$

A *superior* limit of value is obtained by putting $c^\xi = 0$; (4) then becomes

$$\left[\bar{a}_x + \frac{\Gamma \frac{\lambda(vsc)}{\lambda c} \left(c^x \lambda \frac{1}{g} \right)^{-\frac{\lambda(rs)}{\lambda c}} g^{-c^x} - 1}{\delta - \lambda s} \right] \frac{\mu_z + \lambda(vs^3c) \bar{A}'_{yz} + \lambda s \lambda c \bar{a}'_{yz}}{\mu_y + \mu_z + 2\lambda s}$$

where \bar{A}' and \bar{a}' are calculated at a rate of interest i' given by $v' = vs$.

Hence the true value of $\bar{a}_{yz|x}^1$ lies between

$$\frac{1}{\delta - \lambda s} \left[\bar{A}_{xyz}^1 - \Gamma \frac{\lambda(vs c)}{\lambda c} \left(c^x \lambda \frac{1}{g} \right)^{-\frac{\lambda(vs)}{\lambda c}} g^{-c^x} Q_{yz}^1 \right] \\ + \left[\bar{a}_x + \frac{\Gamma \frac{\lambda(vs c)}{\lambda c} \left(c^x \lambda \frac{1}{g} \right)^{-\frac{\lambda(vs)}{\lambda c}} g^{-c^x} - 1}{\delta - \lambda s} \right] \frac{\mu_z + \lambda(rs^3 c) \bar{A}_{xyz}^1 + \lambda s \lambda c \bar{a}_{xyz}}{\mu_x + \mu_y + \mu_z + 3\lambda s}$$

and

$$\frac{1}{\delta - \lambda s} \left[\bar{A}_{xyz}^1 - \Gamma \frac{\lambda(vs c)}{\lambda c} \left(c^x \lambda \frac{1}{g} \right)^{-\frac{\lambda(vs)}{\lambda c}} g^{-c^x} Q_{yz}^1 \right] \\ + \left[\bar{a}_x + \frac{\Gamma \frac{\lambda(vs c)}{\lambda c} \left(c^x \lambda \frac{1}{g} \right)^{-\frac{\lambda(vs)}{\lambda c}} g^{-c^x} - 1}{\delta - \lambda s} \right] \frac{\mu_z + \lambda(rs^3 c) \bar{A}'_{yz}^1 + \lambda s \lambda c \bar{a}'_{yz}}{\mu_y + \mu_z + 2\lambda s}.$$

These results may, for convenience, be written in the following form:—

$$\text{Inferior limit} \quad \frac{\bar{A}_{xyz}^1 - a_x \cdot Q_{yz}^1}{\delta - \lambda s} + \beta_x \cdot \gamma_{xyz} \quad . \quad . \quad (5)$$

$$\text{Superior limit} \quad \frac{\bar{A}_{xyz}^1 - a_x \cdot Q_{yz}^1}{\delta - \lambda s} + \beta_x \cdot \gamma_{yz} \quad . \quad . \quad (6)$$

where γ_{yz} denotes the limiting value assumed by γ_{xyz} when $x = -\infty$.

From inspection of the quantities denoted by γ_{xyz} and γ_{yz} respectively, and of the 4 per-cent values of β_x given in the table on page 319, it will be apparent that, if x is young, and y and z are fairly advanced in age, there will be very little difference in the results obtained from (5) and (6), and, consequently, that either of these formulas will give a tolerably good approximation. Thus, in the *Text-book* case— $\bar{a}_{45.60.30}^1$ at 4 per-cent—(5) works out at 6.960 and (6) at 7.020, the value found by using formula 39(a) of chap. xxiv. of the *Text-book* being 6.996. If, however, the annuitant's age is taken as 60 instead of 30, (5) and (6) give results as widely different as -2.982 and 6.090, which are, of course, useless. With the object of enabling the degree of approximation given by (5) and (6) to be tested numerically, if not of rendering them of much practical use, the specimen values of a_x , β_x , Q_{yz}^1 and γ_{yz} , given in the appended table, have been calculated. The double interpolation (first, for the annuity

H^M (*Text-Book Graduation*) 3 per-cent
Values of α_x and β_x .

x	α	$\log \alpha$	β	$\log \beta$
20	·2064	$\bar{1}$ ·31471	·364	$\bar{1}$ ·56110
25	·2477	$\bar{1}$ ·39399	·480	$\bar{1}$ ·68124
30	·2980	$\bar{1}$ ·47420	·756	$\bar{1}$ ·87852
35	·3597	$\bar{1}$ ·55593	1·200	·07918
40	·4365	$\bar{1}$ ·64003	1·912	·28149

Values of Q_{yz}^1 and γ_{yz} .

y	z	Q_{yz}^1	$\log Q_{yz}^1$	γ_{yz}	$\log \gamma_{yz}$
45	60	·7540	$\bar{1}$ ·87736	1·5963	·20312
	65	·8179	$\bar{1}$ ·91269	1·5087	·17859
	70	·8685	$\bar{1}$ ·93876	1·4050	·14767
	75	·9069	$\bar{1}$ ·95757	1·3055	·11578
50	60	·6842	$\bar{1}$ ·83521	1·3861	·14181
	65	·7631	$\bar{1}$ ·88259	1·3697	·13662
	70	·8278	$\bar{1}$ ·91795	1·3182	·11997
	75	·8781	$\bar{1}$ ·94354	1·2529	·09790
55	60	·5978	$\bar{1}$ ·77653	1·1423	·05779
	65	·6904	$\bar{1}$ ·83908	1·1929	·07661
	70	·7710	$\bar{1}$ ·88705	1·1997	·07906
	75	·8362	$\bar{1}$ ·92229	1·1777	·07105
60	60	·5000	$\bar{1}$ ·69897	·8883	$\bar{1}$ ·94556
	65	·6010	$\bar{1}$ ·77891	·9879	$\bar{1}$ ·99470
	70	·6960	$\bar{1}$ ·84258	1·0493	·02059
	75	·7777	$\bar{1}$ ·89080	1·0753	·03153
65	65	·5000	$\bar{1}$ ·69897	·7731	$\bar{1}$ ·88825
	70	·6038	$\bar{1}$ ·78088	·8737	$\bar{1}$ ·94136
	75	·7006	$\bar{1}$ ·84545	·9446	$\bar{1}$ ·97523
70	70	·5000	$\bar{1}$ ·69897	·6886	$\bar{1}$ ·83797
	75	·6061	$\bar{1}$ ·78251	·7911	$\bar{1}$ ·89522
75	75	·5000	$\bar{1}$ ·69897	·6280	$\bar{1}$ ·79795

Constants.

	$\delta - \lambda s$	$\lambda (vs^2c)$	$-\lambda s, \lambda c$
Common Logarithm	$\bar{2}$ ·55329	$\bar{2}$ ·63526	$\bar{4}$ ·75236

On Governmental Regulation of Life Insurance in the United States of America, by SHEPPARD HOMANS, Consulting Actuary, New York.

[Read before the Institute, 22 February 1897.]

AS the members of the Institute are doubtless aware, government in the United States differs in at least one important particular from that in Great Britain. In the former country all legislation must conform to the requirements of a written Constitution, and such conformity or non-conformity is decided in each case as it arises, without appeal, by a Supreme Court. In the latter country, on the contrary, the validity or otherwise of any Act of Parliament is decided by unwritten traditions and evolutions developed by the growth and experience of centuries. It is not my object or province to discuss the relative advantages and disadvantages of the two systems; I merely state the fact which must be clearly kept in view in order to understand the unique position of life insurance in the United States as regards governmental intervention.

The written Constitution of the United States is the supreme law and the limit of the power and jurisdiction of the Federal government. No rights or powers can be acquired by the Federal government except those which are specified directly or by implication in that written Constitution. All rights and powers not so specified or implied are reserved to the individual States. The individual States are sovereign within their own boundaries in respect to all matters affecting the interests of their own citizens, subject alone, in matters affecting the people of the country as a whole, to the provisions in the written Constitution. In respect to the latter, the Federal government is supreme. All State legislation must yield, or must be made to conform, to the superior authority of the Federal government, within the powers given to it directly or by implication under the Constitution.

In other words, the theory upon which the government of the United States is based, is that the people of the individual States, as represented in their executive and legislative branches, are supreme as regards all local matters, while the people of the whole country, as represented in their national executive and legislative branches, are supreme as regards all matters affecting the interests of the people as a whole.

It may be mentioned, incidentally, that the States are still further divided into townships, or counties, or parishes, and that these latter are the political units having ample powers as regards local affairs, yet subject to the still higher powers of the State as regards matters affecting its citizens as a whole. But for my present purpose, it is only necessary to discuss the relative powers of the individual States and those of the Federal government, more particularly as affecting the business of life insurance.

It may be mentioned, incidentally also, that in almost all national elections, the contests have turned on the relative powers of the Federal and the State governments. The Democrats have generally insisted upon a strict construction of the Federal Constitution, and a strict limitation of Federal powers, while the Whigs, formerly, and their successors, the Republicans, have favoured a liberal construction of that instrument and a strong central government.

The Constitution of the United States has excited the admiration of the civilized world by reason of the wisdom, foresight and breadth of view evinced by its framers. Its strength is that of the giant oak, however, rather than that of a granite column. Its interpretations by the Supreme Court must be governed by enlightened but conservative wisdom, and in accordance with developments of modern life and the progressive demands of a higher civilization. Interpretations by the Supreme Court have, in several cases, been reversed by the same Court when, by reason of changed conditions, or by amendments, as prescribed in the instrument itself, it was deemed wise or necessary to do so. No less than fifteen amendments have been added to the Constitution since it was originally adopted.

The Supreme Court found occasion to change its constitutional construction in the matter of citizenship of corporations, mentioned in *Paul v. Virginia*. So, at first, it restricted admiralty jurisdiction to waters subject to the ebb and flow of the tides, as in England, but the inapplicability of any such theory to our vast country with its great lakes and rivers caused that restriction to be abandoned. A recent instance, is that of the income tax, held to be direct, after having twice been declared indirect. It is not unreasonable that its decision regarding insurance may, in the light of changed conditions and additional information, be also reversed.

It must be borne in mind that the Constitution was written and adopted more than one hundred years ago, and that times

have greatly changed in that interval. Railroads, steam navigation, electricity, the telegraph and the telephone were not dreamed of at the time of its adoption. Modern banking and life insurance were also then unknown. The Constitution was drawn without reference to either of these great economies which affect, more or less, every citizen and every family in the land. Hence the powers of the Federal government under the Constitution as regards these important interests, must be matters of implication rather than of expressed authorization.

These powers "keep pace with the progress of the country, " and adapt themselves to the new developments of times and " circumstances. They extend from the horse with its rider to the " stage coach, from the sailing vessel and the steamboat to the " railroad, and from the railroad to the telegraph, as these new " agencies are successively brought into use to meet the demands of " increasing population and wealth. They were intended for the " government of the business to which they relate, at all times and " under all circumstances. As they were intrusted to the general " government for the good of the nation, it is not only the right, " but the duty of Congress, to see to it that intercourse among the " States and the transmission of intelligence are not obstructed or " unnecessarily encumbered by State legislation."*

In the admirable treatise on "The Principles of Constitutional Law", by Hon. Thomas M. Cooley, LL.D., Ex-Chairman of the United States Inter-State Commission, it is stated, pp. 38, 39:

"The principles that, at one time, applied the power over " commerce to the regulation of navigation, at a latter day are " found equally applicable to traffic and travel by railroad and " communication by telegraph; and though these new applications " of principles do not in the least depart from or enlarge former " doctrines, they nevertheless strengthen the national power by " the immensity of the interests it is thus invited to take under " its control. The gradual energizing of Federal authority has " been accomplished quite as much by the course of public " events as by the new amendments to the Constitution. There " can be no question that the new interests, coming gradually " within the purview of Federal legislation, and the increase in " magnitude and importance of those already under Federal " control, must have a still further tendency in the direction " indicated."

* *Pensacola Tel. Co. v. Western Union Tel. Co.*, U.S. 1, 9.

Article 1, section 8, of the Constitution of the United States, confers upon Congress the power "to regulate commerce with "foreign nations, and among the several states, and with the "Indian tribes."

The question at once arises, what is commerce within the meaning of the clause above quoted of the Federal Constitution?

"The word commerce is not limited to traffic; to buying and "selling and the exchange of commodities; but it comprehends "navigation also, and all that is included in commercial inter- "course between nations and parts of nations in all its branches, "and is regulated by prescribing rules for carrying on that "intercourse."*

Said Chief Justice Marshall, in *Gibbons v. Ogden*, Op. 9 Wh., p. 189: "Commerce is undoubtedly traffic, but it is something more; it is intercourse." The learned Chief Justice also said, "to confine commerce to the interchange of commodities would "restrict a general term, applicable to many objects, to one of its "significations."

Commerce, especially inter-State commerce, is usually carried on by corporations. But corporations are, with very rare exceptions, the creations of State legislatures. A State corporation, deriving its existence and powers from the legislature of its own State, can be admitted to do business in any other State only by consent of the latter, which may, in the absence of Federal legislation, impose such restrictions, taxes, and conditions upon all corporations of other States seeking to do business within its borders, as its legislators shall see fit to impose, even to the extent of absolute prohibition.

A corporation organized in one State can exercise its corporate power in another only by the comity of the latter.†

There are, however, two exceptions to this statement:

First, where the corporation has been vested by the Federal government with the execution of one of its express or implied powers.‡

* *Henderson v. New York*, 92 U.S., 259; *Pensacola Tel. Co. v. West, &c.*, Tel. Co., 96 U.S., 1, 9; *Gibbons v. Ogden*, 9 Wheat, 1, 189.

† *McCulloch v. Maryland*, 4 Wh., 316.

‡ Op. of Ct. *Pembina Mining Co. v. Pa.*, 125 U.S., p. 186, adopting an opinion of Justice Bradley in *Stoeton v. Balt. and N.Y.R.R. Co.*, 32 Fed. Rep., 9.

See also

Pensacola Tel. Co. v. W. U. Tel. Co., 96 U.S., 1; *McCall v. Cal.*, 136 U.S., 104; *Norfolk, &c., Ry. Co. v. Pa.*, 136 U.S., 114.

Second, where the corporation is engaged in inter-State business.*

“Congress may make any law, not by the Constitution expressly or impliedly prohibited, which it shall deem conducive to the execution of any express power.”†

“It may therefore charter a national bank as a necessary and useful instrument in the fiscal operations of the government.”‡

“It may give a preference to the demands of the United States in case of insolvent estates.”§

Banking, inter-State steam navigation, railroad transportation and telegraphy were not commerce within the meaning of the Constitution until Congress made them so by appropriate legislation. In the absence of such national legislation, each State is sovereign, without appeal, as to the conditions upon which banks, railroads, telegraphs, and all other corporations of any other State, or of foreign countries, shall be allowed to transact business within its borders. When, by appropriate legislation, Congress saw fit to regulate these important economics, as, *in its opinion*, branches, adjuncts, or incidents of inter-State commerce or commercial intercourse, there was no question as to its power and right to do so in the Supreme Court or elsewhere.

We are now prepared to consider the relative power and authority of the individual States, and of the Federal government, in relation to the business of life assurance as transacted by life insurance companies which, with one exception, derive their existence, powers and functions from acts of individual States.

There are now forty-five States in the American Union, and five Territories which, sooner or later, may be added to the list. In addition, there is the District of Columbia, which is neither a State nor a Territory, but which has its Insurance Department, and a code of Insurance Laws, enacted directly by the Federal Congress.

In the absence of legislation by Congress regulating insurance, each State is sovereign, without appeal, as regards restrictions, taxes, and other conditions which its legislature may see fit to

* *Paul v. Virginia*, 8 Wall, p. 183.

† *Legal Tender Cases*, 12 Wall, 457, 539; *Martin v. Hunter*, 1 Wheat, 304.

‡ *McCulloch v. Maryland*, 4 Wheat., 316, 413.

§ *United States v. Fisher*, 2 Craneh, 358.

impose upon insurance corporations of other States, or of foreign countries which seek permission to transact business within its borders. As a consequence, legislation in the different States may not only be incongruous, burdensome, and conflicting, but it is subject to sweeping and frequent changes. Insurance corporations, or more correctly, their policyholders, are subjected to heavy expenses in efforts, necessary each year, to prevent injurious and even hostile legislation in the several States. The business is thus greatly hampered and even endangered. No branch of business requires and deserves wise, friendly, and stable legislation more than that of insurance.

It has been generally assumed that by the doctrine enunciated in the opinion of the United States Supreme Court, in *Paul v. Virginia*, 8 Wall, 168, any Federal insurance legislation would be unconstitutional; but a careful examination of the points involved would show, perhaps, that such assumption is not wholly warranted, at least so far as regards the life insurance branch of the business.

In the words of Hon. Edwin B. Smith, for many years Deputy Attorney-General of the United States:—"Mr. Paul was indicted, convicted, and fined, for refusing to comply with Virginia laws, requiring from agents of all foreign insurance companies (*i.e.*, those chartered outside that State, whatever was the subject of their insurance) a license fee and deposit of bonds, not exacted from similar corporations existing under a Virginia charter. The proceedings against Mr. Paul—for acting within the State of Virginia as agent of several New York fire insurance companies, without having made the required deposit and obtained the license—being in a State Court, were taken by writ of error to the Supreme Court of the United States, solely upon the Federal question presented upon the record, whether the Virginia statutes violated either the 8th section of Article I of the Constitution of the United States, relating to the Congressional power to regulate commerce, or the 2nd section of Article IV, entitling the citizens of each State to all the privileges and immunities granted citizens in any other State.

"The Court held a corporation not to be a citizen, within the meaning of Article IV, sec. 2; hence, entitled, as of individual right, to no privilege beyond the lines of the State granting its charter. It might be excluded altogether from every other State, or admitted upon just such conditions as each State might

“ choose to prescribe, however onerous or stringent. This
 “ conclusion being reached, it necessarily followed that the
 “ prescription of whatever condition Virginia chose to impose
 “ was *not* a regulation of commerce, but the price of being
 “ permitted to do *any* corporate business in that State.”

Mr. Justice Field, in delivering the opinion of the Court in the case of *Paul v. Virginia*, states:—“Corporations are not
 “ citizens within the meaning of Article IV, sec. 2, of the Con-
 “ stitution, which declares that ‘the citizens of each State shall
 “ ‘be entitled to all the privileges and immunities of citizens in
 “ ‘the several States.’

“ Issuing a policy of insurance is not a transaction of
 “ commerce. The policies are simple contracts of indemnity
 “ against loss by fire, entered into between the corporations and
 “ the assured for a consideration paid by the latter. These
 “ contracts are not articles of commerce in any proper meaning of
 “ the word; they are not subjects of trade and barter, offered in
 “ the market as something having an existence and value
 “ independent of the parties to them; they are not commodities,
 “ to be shipped or forwarded from one State to another, and thus
 “ put up for sale. They are like other personal contracts between
 “ parties which are completed by their signature and the transfer
 “ of a consideration. Such contracts are not inter-State trans-
 “ actions, though the parties may be domiciled in different States.
 “ Policies do not take effect—are not executed contracts—until
 “ delivered by the agent in Virginia. They are, then, local
 “ transactions, and are governed by the local law. They do not
 “ constitute a part of the commerce between the States, any more
 “ than a contract for the purchase and sale of goods in Virginia
 “ by a citizen of New York whilst in Virginia would constitute a
 “ portion of such commerce.”

The same Court subsequently decided, in the case of *Santa Clara County v. South Pacific Railroad Company*, 118 U.S., 396, that, under Amendment XIV, sec. 1, corporations *are* persons, and, as such, *are* entitled to the equal protection of the law.

It will be noticed that the above opinion refers specifically to fire insurance. The same Court, in the subsequent case of the *Conn. Mutual Life Insurance Company v. Schaefer*, 94 U.S., 457, clearly sets forth one essential and vital distinction between fire and life insurance contracts.

In that opinion, Bradley, J., states:—

“ In marine and fire insurance, the . . . insurance *is*

“considered as strictly an indemnity. But in life insurance the loss can seldom be measured by pecuniary values.”

In that opinion is cited the case of *Dalby v. Life Insurance Company*, 15 C.B., 363, over-ruling the previous case of *Goodsel v. Boldero*, 9 East., 72, decided by Lord Ellenborough. This latter case proceeded “upon the idea that life insurance is a mere contract of indemnity.”

Baron Parke, in commenting upon this case, very justly says:—
“Upon considering this case, it is certain that Lord Ellenborough decided it upon the assumption that a life policy was, in its nature, a mere contract of indemnity, as policies on marine risks and against fire undoubtedly are . . . and his Lordship relied upon the decision of Lord Mansfield in *Hamilton v. Burr*, 1270, that the plaintiff’s demand was for an indemnity only. Lord Mansfield was speaking of a policy against marine risks, which is, in its terms, a contract for indemnity only. *But that is not the nature of what is termed an insurance for life; it really is what it is on the face of it—a contract to pay a certain sum in the event of death.*”

Mr. Justice Bradley then continues:—“As thus interpreted, we might almost regard the English statute as declaratory of the original common law, and as indicating the proper rule to be observed in this country when that law furnishes the only rule of decision.”

“Life insurance is a contract whereby one undertakes to pay another a sum of money upon the happening of an unknown or contingent event dependent upon the existence of life. It is not a mere contract of indemnity.”*

“And it may also be asserted, with the same universality, that the Courts have decided that a life policy is not a contract of indemnity.”†

In many other important respects the differences between fire insurance policies, as interpreted by Mr. Justice Field, in his opinion quoted above, and life insurance policies are clearly

* *Alexander on Life Insurance*, page 1; *Rawls v. Mut. Life Ins. Co.*, 27 N.Y., 282.

† *Biddle on Insurance*, page 186; *Dalby v. India and Lond. L. Assurance Co.*, 15 Q.B., 365; *Trenton Mut. L. and F. Ins. Co. v. Johnson*, 4 Zab. (N.Y.), 576; *De Rouge v. Elliott*, 8 Green (N.J.), 486; *Rawls v. Am. Mut. L. Ins. Co.*, 27 N.Y., 282; *Mut. Life Ins. Co. v. Allen*, 13 Ins. L.J., 897 (Mass.); *Scott v. Dickson*, 108 Pa. St., 6; *Carson’s Ap.*, 13 Pa. St., 438; *Mowry v. Home L. Ins. Co.*, 9 R.I., 346; *Conn. Mut. L. Ins. Co. v. Schaefer*, 94 U.S., 457; *Warnock v. Davis*, 104 U.S., 775.

defined and vital. Unlike the former, life insurance policies frequently *are* "subjects of trade and barter." They frequently *are* "offered in the market as something having an existence and value independent of the parties to them." This value (reserve) is recognized by the statutes of all the States. Life insurance policies *have been* "put up for sale", public as well as private, especially in Great Britain. They constitute a valid collateral for loans or other business transactions. They are *not* "completed by their signature and the transfer of their consideration." They involve yearly payments of premiums, in which are included yearly deposits to be gradually accumulated as trust funds (reserves) necessary to meet claims at their maturity in the distant future. Such policies *do* "take effect", *are* "executed contracts", when signed by the Home Office of the Company and before they are "delivered by the agent." They are *not* "then local transactions", and are *not* "governed by the local law." On the contrary, by reason of various State enactments regulating the business of companies admitted from other States, which enactments are, in some cases, discriminating, retaliatory and conflicting, life insurance contracts are essentially inter-State, if not of commerce, certainly of commercial intercourse.

The business of life insurance in the United States is usually conducted on the mutual plan. The members of a mutual life insurance company constitute a society, and each member is at once insurer and insured, entitled to participate in the management and share in the profits in proportion to his interest. The managers of such institutions are simply the collecting and disbursing agents of the policyholders who select them, and the custodians of the trust funds created by them for the benefit of their future widows and orphans.

A life insurance company essentially consists of a trust fund, and the proper administration of such a fund demands *equality* between the beneficiaries. As the policyholders, the sole beneficiaries of such a trust fund, reside in different States, the necessity of uniform laws for the administration of the trust inheres in its very nature. The rights of a beneficiary residing in one State in the administration of such a trust should be determined by the same law which governs those of his fellow beneficiaries residing in other States. Any discrimination in favour of a beneficiary residing in one State would impair the vested rights of beneficiaries residing in other States. Uniform law for life insurance is demanded, precisely as in the case of the administration of the

Peabody Trust Fund, where the statutes in one State govern its transactions in all other States. Uniform, stable, and just law is an absolute essential to safety and success in the administration of the business of life insurance.

The administration of the trust funds in life insurance companies is more nearly akin to the administration of trust funds in banks, trust companies or savings banks, except that the trust in the former case is more delicate and sacred, because the settlements may not fall due for many years, or until after the deaths of those whose payments create the fund. Such contracts are based upon scientific computations in regard to the probabilities of living and dying, and the improvement of money by compound interest extending over a long series of future years. It is necessary to "preserve a fixed relation between the premiums and the amount insured, as required by the principles of life insurance." Opinion of Mr. Justice Bradley, *Supra*. The determination of the contingent liabilities of a life insurance company, and of the considerations to be paid therefor, involve the careful weighing of measurable chances. Discriminating and, perhaps, prohibitory legislation are not measurable chances, and they might absolutely prevent the fulfilment of life insurance obligations which, in themselves, are not only legitimate and meritorious, but involve the happiness and well-being of millions of our citizens.

It will be noted that the case of *Paul v. Virginia* did not touch the business of insurance *per se*, but involved the right of the State of Virginia to regulate the admission of foreign corporations to do *any* business within her limits. As the opinion states, commerce can be carried on between the States, and this *commerce* Congress can regulate. That case concerned only the domiciliation of foreign corporations in Virginia, for the purpose of transacting any business within the State. No case has yet held that Congress cannot regulate the business of life insurance, and incidentally protect the agencies by which that business can be transacted. Nor can this question ever be raised—still less be judicially decided, until Congress shall, by appropriate legislation, enact laws regulating the business of insurance. Even then, the question can be brought before the Supreme Court only by a suit in which its constitutionality is questioned. Some of the uncalled-for reasoning of the opinion in *Paul v. Virginia*, militates against the right of Congress to so legislate, but it is evident that such reasoning is *obiter dicta*—not necessary to the decision of the points at issue. The essential points at issue can

only be presented to the Supreme Court when Congress shall have enacted a law involving Federal regulation of life insurance, and the constitutionality of that law shall have been called in question.

This is evidenced by the following citations from decisions of the Supreme Court itself, and from the opinions of individual justices, and also from extracts from Judge Cooley's *Principles of Constitutional Law*.

"The power that controls commerce must, from the very nature of things, include the power to restrict and limit—to prohibit as to certain things, and to suspend altogether when, for the time, it seems wise. *It is a sovereign power, and therefore knows no limit.*" *

"This is a power which Congress may or may not exercise, and, when it abstains from doing so, the States are at liberty to legislate on the subject. Nevertheless, their legislation must yield to uniform laws, whenever Congress shall think fit to pass them." †

"The mere existence of this power in Congress does not necessarily exclude the States from all authority whatever which might affect the commerce falling within the control of the Congress, *provided no actual legislation of Congress is interfered with.*" ‡

"But in respect to the commerce that properly falls within the control of Congress, its authority is necessarily exclusive so far as it is exercised, *and it is competent for Congress to extend its regulations to the most minute particulars.*" §

"And Congress is of necessity the exclusive judge of what is needful and proper when the means chosen conduce to the end and are not forbidden." ||

"The question of the validity of a statute must always be one of legislative competency to enact it; not one of policy, propriety, or of strict justice." ¶

* Const. Law, Cooley, p. 68.

† *Sturges v. Crowninshield*, 4 Wheat., 122; *Ogden v. Saunders*, 12 Wheat., 213; *Baldwin v. Hale*, 1 Wall, 223; *Ex parte Eames*, 2 Sory, 322.

‡ Const. Law, Cooley, p. 69.

§ Const. Law, Cooley, p. 69.

|| *McCulloch v. Maryland*, 4 Wheat., 316, 413.

¶ Const. Law, Cooley, p. 149.

“ When State legislation is in its essence, and of necessity a
“ regulation or foreign or inter-State commerce, and therefore of
“ national importance, it is an encroachment upon the power of
“ Congress over the subject, even though Congress may never
“ have legislated on the subject.” *

“ By refraining from action, Congress in effect adopts as its
“ own regulations those which the common law, or the civil law
“ when that prevails, has provided for the government of such
“ business, and those which the States, in the regulation of their
“ domestic concerns, have established affecting commerce, but not
“ regulating it within the manner of the Constitution. In fact,
“ Congressional legislation is only necessary to correct defects in
“ existing laws, as they are discovered, and to adapt such laws to
“ new developments of trade.” †

“ Inaction by Congress is equivalent to the declaration that
“ the commerce under its control shall remain free and un-
“ trammelled.” ‡

“ No legislative body can delegate to another department
“ of the government, or to any other authority, the power, either
“ generally or specially, to enact laws. The reason is found in
“ the very existence of its own powers. This high prerogative has
“ been intrusted to its own wisdom, judgment, and patriotism,
“ and not to those of other persons, and it will act *ultra vires* if
“ it undertakes to delegate the trust instead of executing it.” §

“ The judiciary, though the final judgment of what the law is,
“ is not the judge of what the law should be.” ||

“ The propriety, or justice, or policy of legislation, within the
“ limits of the Constitution, is exclusively for the legislative
“ department to determine; and the moment a Court ventures to
“ substitute its own judgment for that of the legislature, it
“ passes beyond its legitimate authority, and enters a field where
“ it would be impossible to set limits to its interference, except as
“ should be prescribed in its own discretion. The protection
“ against unwise or oppressive legislation, within constitutional

* *Welton v. Missouri*, 91 U.S., 275.

† *Hall v. De Cuir*, 95 U.S., 485, 490.

‡ *Welton v. Missouri*, 91 U.S., 275, 282.

§ *Locke on Civil Government*, 142; *Barto v. Himrod*, 8 N.Y., 483; *Rice v. Foster*, 4 Harr., 479; *Cooley, Const. Lim.*, 4th ed., 141-152.

|| *Const. Law*, *Cooley*, p. 139.

“ bounds, is by an appeal to the justice and patriotism of the
“ representatives of the people. If this fail, the people in their
“ sovereign capacity can correct the evil, but the Courts cannot
“ assume their rights.” *

“ The judiciary can only arrest the execution of a statute
“ when it conflicts with the Constitution. It cannot run a race
“ of opinions upon points of right, reason, and expediency with the
“ law-making power.” †

Freedom of inter-State trade and commercial intercourse, if not the sole, was at least the chief, object of the grant to Congress of the power to regulate commerce.

From the foregoing, it would seem to be clear that Congress has the power, *in its discretion*, to regulate the business of insurance, or at least the life insurance branch of it, as an adjunct or incident of commerce, or of commercial intercourse. It remains, then, to consider whether the national regulation is desirable and for the best interests of the policyholders in life insurance companies.

The life insurance interest in the United States, present and contingent, exceeds in amount the combined capital of banks, railroads, steamboats and telegraph companies. The insurance in force, January 1 1896, in the regular companies, was \$6,617,508,355, and in assessment associations about \$7,500,000,000, or a total of more than fourteen billion dollars, which is equal to nearly one-fifth of the total accumulated wealth of the country. Their assets, representing the savings of policyholders, are about \$1,300,000,000. These figures are exclusive of those of fraternal societies, such as Oddfellows, Masonic, and in other societies of that nature. Uniform, wise and stable legislation is imperatively demanded for the safety of these important interests. Such legislation cannot be secured under forty-five or fifty separate and independent sovereignties, each with its own legislature and insurance department. National legislation and regulation alone can secure it.

As examples of conflicting or discriminating State legislation, the non-forfeiture statutes of different States may be cited. The

* Const. Law, Cooley, p. 148; *Bennett v. Bull*, *Baldw.*, 74; *Pennsylvania R. R. Co. v. Riblett*, 66 *Penn. St.*, 164.

† *Madison, &c.*, *R. R. Co. v. Whiteneck*, 8 *Ind.*, 217; *Bull v. Read*, 13 *Grat. (Va.)*, 98.

statutes of Missouri, for instance, provide that after the payment of *two* annual premiums, 75 per-cent of the net reserve (American Experience $4\frac{1}{2}$ per-cent) on any policy *shall* be applied to the purchase of extended insurance for the full face value of the policy. The statutes of the State of New York provide that after the payment of *three* annual premiums, two-thirds of the net reserve by the same standard may be applied, as shall be expressed in the contract, either to the purchase of paid-up or extended insurance. Again, the statutes of Massachusetts provide that after the payment of *two* annual premiums, the full reserve (Actuaries' 4 per-cent), less a surrender charge of 8 per-cent of the "insurance value", shall be applied to the purchase of paid-up or extended insurance or may be recovered by the insured in cash.

Commissioner Merrill points out, on page 12 of his last report, that in exempting life insurance companies of other States admitted to do business in Massachusetts from the stringent requirements of its non-forfeiture statute, there has resulted "a serious *discrimination* and handicap upon the Massachusetts companies in the "competition for business."

These different State statutes are still in force, and they have been upheld by the decisions of State Courts in numerous cases, as they are by the sentiments of the large number of our citizens who believe in State rights. On the other hand, the Supreme Court of the United States has ruled: "A discriminating law by "any State as against other States is, in itself, a regulation of "inter-State commerce, and is, therefore, an infringement upon the "power delegated solely to Congress."*

Again, in the case of *Hoadley v. Board of Insurance Commissioners of the State of Florida*, it was decided in the Supreme Court that the law requiring corporations and individuals, not citizens of that State, to invest \$150,000 as a condition precedent to transacting insurance business in that State, while domestic corporations and individuals were required to have a capital of \$25,000 only, in order to obtain the same privilege, was *invalid*.

Mahey, J., held: "As shown by the legislation of this State "referred to, both resident and non-resident citizens are authorized "to do insurance business in this State, and it is too clear to "require any argument that a discrimination has been made as to "the conditions upon which the business shall be conducted.

* *Tiernan v. Rinker*, 102 U.S., 123.

“The extent of the discrimination is not important; the only question is—Does it exist?”

“The State has expressly authorized both classes of citizens to engage in such business here, but has imposed conditions on the one, and none, or, if any, clearly discriminative ones, on the other. It is also apparent that insurance commissioners revoked the certificate of authority granted to relators, solely upon the ground that they had not complied with the requirements of the statute imposing discriminations upon non-resident citizens, and, for the reasons given, we think there is no reasonable doubt about the invalidity of the questioned provision in the statute.”
—*Filed May 12, 1896.*

Article XIV, sec. 1, “Amendments of the Federal Constitution”, provides, “nor shall any State deprive any person of life, liberty, or property, without due process of law, nor deny to any person within its jurisdiction the equal protection of the laws.”

Corporations are “persons” included within the above amendment, and are there protected against the deprivation of liberty or property without due process of law, and are protected against the denial of the equal protection of the law.*

The liberty to make lawful contracts, and to be protected thereunder, would seem to be the right guaranteed by the Federal Constitution to every individual and to every corporation in the United States.

In order to afford sufficient scope for the proper working of the law of average, the operations of a life insurance company cannot well be confined to the limits of a single State. . . . No company is so restricted. Large numbers of members, scattered over wide areas, are desirable, if not essential, to the success of the business. The policyholders, as citizens of the whole United States, are entitled to the benefit of wise, stable, and uniform laws. Such are absolutely essential to the full protection and preservation of their interests and the interests of their future widows and orphans.

American companies have been managed, generally, with consummate ability, and their managers are quite able to take care of the interests committed to them. They are the trustees of the funds committed to their care, but they are at the same

* *Santa Clara County v. So. Pac. Ry. Co.*, 118 U.S., 396; *Pembina Co. v. Pennsylvania*, 125 U.S., 181.

time amenable to the laws of the different States in which they are permitted to do business, and must conform to the same, however onerous and oppressive these laws may be. Dangers, expenses and mischiefs, due to oppressive, discriminating, or conflicting laws, fall, not upon the management of these companies, but upon individual policyholders and their beneficiaries.

From the foregoing, it will be seen that life insurance in the United States is "beset behind and before." On the one hand, we have State supervision and Insurance Departments in nearly all the States. The *status quo* will be stoutly urged by tradition, by State pride, and by the influence of the present office-holders, and by those who hope to become so. On the other hand, we have the stupendous interests of the life insurance companies—that is to say, of policyholders and their future widows and orphans—which are hampered, perhaps even endangered, by numerous conflicting, discriminating, and unstable laws, which are unavoidable under the independent legislation and jurisdiction of forty-five or fifty sovereign and independent States.

The settlement of this important matter of Federal or separate State regulation of life insurance is one that cannot be deferred for any great length of time. We have great confidence that, ultimately, the good sense and practical wisdom of the American people will compel a right settlement, despite the opposition which must be expected from those who are influenced by prejudice or by self-interest. I am quite sure that, in our efforts to secure a right settlement of this question, we may count upon the sympathy and good wishes of our insurance brethren in Britain, and especially of the members of the London Institute of Actuaries.

DISCUSSION.

The PRESIDENT (Mr. T. E. Young) said it had always been held to be a striking sign of the vitality and authority of a scientific body when the contributors to its knowledge in its earlier years had reverted to it in the maturity of their powers with fresh contributions. The present paper, accordingly, not merely afforded an indication of that character, but also gave the members an opportunity of very cordially welcoming Mr. Homans once more, after the lapse of 33 years. For it was in October 1863 that Mr. Homans contributed to their *Journal* his important paper upon the "Equitable Distribution of Surplus", which from the ingenuity and grasp of principle which it displayed, had been a source of much useful thought and research since that date. The present occasion was, he believed, the first on

which they had had submitted for their consideration, by one whom he might almost term a legal expert, the commercial constitution of the United States in its relation to insurance, with its essential distinction of Federal and State Legislation. Those who had studied the history of American legislation must thoroughly concur with Mr. Homans in the aptness of the illustration which he employed when he likened the general constitution of the United States, as a piece of legislative mechanism, with an oak, capable of progressive adaptation and growth. For, unfortunately, it had usually been the malign fortune of national constitutions, framed of course in more primitive times, that they had been formed of so rigid and doctrinaire a character as to exclude natural and unforced expansion adapted to the changing conditions of social and economic life. But when they passed from this appreciation of the general constitution of the United States, they must join equally heartily with Mr. Homans in his adverse strictures upon separate State Supervision and Insurance Departments, with their crippling and retarding effect upon the natural and normal growth of insurance associations; and Mr. Homans very justly concluded that, in his efforts to reform and re-vitalize those oppressive institutions, he possessed the warm sympathy of British actuaries.

Mr. CHISHOLM said that Mr. Homans was a very old friend, and needed no introduction. He was Corresponding Member of the Institute for the United States, and he thought that he must be known personally to at least two generations of British actuaries. They were doubly indebted to Mr. Homans, because he had given them a paper which took them out of their accustomed groove, and which introduced a fresh subject. The Institute would equally gladly welcome papers of a similar character from its now numerous members who were growing up on the other side of the sea, in Canada and Australia. He (Mr. Chisholm) had thought it would be an interesting thing if he were to recapitulate the separate laws in the States, and for that purpose he had studied the "Insurance Year Book", which gave a synopsis of them. Mr. Homans stated that there were 50 States. In the book to which he had referred, dated 1896, there were only 49, so that there must have been one born since the book was written. On classifying the different provisions, he found they might be arranged roughly into three groups. I, in the first, were those intended to safeguard the supremacy of the State; II, in the second, those intended to ensure the *bonâ fides* and solvency of the companies, and to watch over their proper working; III, in the third, miscellaneous regulations dealing more or less with matters of detail, or at any rate of relatively minor importance and intended for the benefit sometimes of the assured and sometimes of the company.

I.—In the first group would come

(1) The conditions on which the authorization to transact business in the State could be obtained. Those consisted generally in filing a copy of the charter and furnishing particulars of the constitution of the company. All the States except New Mexico required that condition to be observed. It was practically universal.

(2) All the States except New Jersey required an attorney or attorneys to be appointed, upon whom notice of legal process to be

taken within the State could be served. There was considerable variety in the persons who were allowed to be appointed attorneys. In six cases, all agents within the State must be qualified to accept service. In one case there must be an attorney in each county. In 13 cases the attorney must be the Insurance Commissioner, and in one case, Pennsylvania, he must *not* be the Insurance Commissioner. But the chief thing to notice was that all the States were most jealous in requiring actions to be tried within their own borders. That was shown particularly in the case of Illinois. There was a stipulation in the laws of this State "that the authority of the company to do business shall cease and determine whenever it shall make application to remove into any United States' Court any action commenced against it in any of the State Courts." There was a provision in Kansas somewhat to the same effect.

(3) The third condition coming under the heading of the first group related to the taxation. There were fees and licenses in addition to the taxation, but they were generally of small account, and might be passed over for the present purpose. The taxation, however, was important, and obtained in all but 8 of the States. It was most generally (in 24 cases) in the shape of a percentage varying from 1 to $2\frac{1}{2}$ per-cent of the gross premiums collected in the State within the year. In 8 cases it was a percentage on the net premiums after deducting losses, and in 4 of those cases the rate was the same as was charged in taxing personal property. They did not state any definite rate, but it was fixed at the same rate as for personal property. In two cases the tax was a graduated license duty according to the amount of premiums received. In one case it was one quarter of 1 per-cent annually on the net value of the policies. In two cases (North Carolina and Wisconsin) the tax was reducible by one-half if certain portions of the premium receipts were invested within the State, and in Vermont there was a very curious provision to the effect that the State's own companies must pay an *additional* tax of 1 per-cent annually on the surplus above the necessary reserve computed at 4 per-cent. That was actually a condition operating against their own companies, and protective in favour of the outside companies. That completed the group of provisions relating to the supremacy of the State.

II.—The second group of provisions of the State law comprised—

(1) The possession by the companies of a certain amount of funds. In 37 cases it was a fixed sum, generally £100,000 or £200,000. In four cases it was simply stated that the assets must equal or exceed the liabilities, and in eight cases there was no requirement. Frequently a deposit with the State was also required, but the conditions were not in any way onerous. Foreign companies were equally well treated, the only additional proviso in a number of cases being that they must have lodged with the proper authorities in some one State of the Union a deposit generally of £100,000 or £200,000.

(2) The second proviso was that the annual accounts must be filed every year within a definite time. This was the most universally adopted of all the conditions, there being no exception. All the States made that requirement, and in some cases half-yearly statements had to be furnished, and frequently arrangements had to be

made for publishing and advertising them. Bearing on that, there was one very curiously minute provision which would be interesting. It occurred in the laws of South Dakota, and ran as follows—"Annual statements must be filed within one month from 1 January and must be published at least three times in one of the three papers designated by the auditor and printed and published in each judicial circuit of the State in which the company shall have an agency within 90 days from the filing of the statement. Advertisements must be paid for at the rate of 75 cents per square of 12 lines of nonpareil type or its equivalent for the first insertion, and 50 cents per square for each subsequent insertion." If that was the letter of the law, in what spirit could they expect it to be administered by the officials? These had certainly every encouragement to be particular, minute and exacting.

(3) The third of the provisos related to the valuation of the company's policies. In 20 cases there was no provision of this sort, but in 29 States the basis was laid down, and it was generally either Combined Offices' Experience 4 per-cent or American Experience $4\frac{1}{2}$ per-cent. In New York State, however, the table was to be that adopted by the company, and the Insurance Commissioner was empowered to vary the standard both of interest and mortality in the case of foreign companies and of invalid lives. The commissioner of a particular State was often allowed to accept the certificate of valuation by the Commissioner of the State in which the company had its origin, but he was unable to say whether that was permissible in all cases. Neither had he been able to ascertain the extent to which a valuation of the assets was necessary. He had always understood that in valuing the affairs of a company not only the liabilities, but also the assets, were valued; indeed, a valuation would be of very little use otherwise. But the particular provision in the law did not seem to give effect to this, or, at least, the record he had of what the provision was did not show it.

III. The third group comprised a number of miscellaneous regulations of which he would mention a few.

(1) The first was that all agents must procure an annual certificate or license from the State authority, and pay a moderate fee for it. He was not sure whether that rule should be considered as most for the benefit of the assured or of the company, perhaps both, but it was a good regulation and he would like to see it adopted here. In view of that time coming, he might inform their home agents if they wished then to practice unlicensed in America, that the only places in which they would be able to do so were Columbia, Michigan, North Carolina, Virginia and Washington—five States. In all the other 44 States a license was compulsory.

(2) The second regulation of the miscellaneous group that he would call attention to was "discrimination." That would hardly be understood until the explanation was read out in full. In the State of Connecticut the paragraph relating to discrimination ran as follows:—"Companies are prohibited from making any discrimination in favour of individuals between insureds of the same class and expectation of life in the amount of premiums or rates charged, or any dividends or other benefits payable, or any other terms and

conditions of the contract. All conditions must be plainly expressed in the policy. Rebate on premium, directly or indirectly, prohibited." It would be seen that it was thus made impossible to exact from the insured any payment to meet the percentage of premium imposed as a tax by the State. He had not been able to ascertain whether that condition was considered as preventing the rating-up of invalid lives. In all, 25 States had adopted that regulation, and its infringement generally involved the infliction of a considerable fine, \$500 to \$1,000, and the loss of the license. In three States (Massachusetts, Vermont, and Wisconsin), discrimination against coloured persons, wholly or partially of African descent, was also prohibited.

(3) Third,—fraudulent statements. That was a provision entirely for the benefit of the companies. In Pennsylvania the provision ran as follows:—"Any agent, physician, or other person making false representations for the purpose of procuring insurance is deemed guilty of misdemeanour, and may be fined not more than \$1,000 or be imprisoned for a term not exceeding one year." A similar regulation was adopted in nine other States. That was a very drastic regulation. He did not see why something similar should not be applied here; they would all be glad to see it, at any rate in insurance offices.

(4) Fourth, paid-up insurance. In six States there was a proviso that after two, or in most cases three, annual premiums had been paid, a paid-up policy must be issued for such an amount as the reserve—Mr. Homans stated that in two cases it was a portion of the reserve only—taken as a single premium, would purchase. The States which were thus distinguished were California, Colorado, Massachusetts, Montana, New Jersey, and New York. There were fixed regulations in two instances specifying what was to be done in case of mis-statement of age, and, as if to show their independence of one another, in the case of one State it was laid down that the company should be entitled to recover from the assured the difference of premium, and in the case of the other State it was laid down that the sum payable was to be that smaller sum only which the premium actually paid would have assured. In Maine there was a proviso that proposals drawn by the agent were binding on the company, but not upon the insured, although signed by him, and in Georgia there was a curious proviso that it was unlawful for companies or agents to enter into any compact for preventing or lessening competition. The penalty for violation was revocation of the license.

They could well understand that the necessity for paying close attention to the minute provisions of so many different laws of a restrictive character affecting insurance must be exceedingly irksome, and place very great difficulties in the way of companies. They would have to keep books of account for each State, so as to show correctly the amount of tax payable. But the proviso giving the greatest amount of trouble was probably that relating to the valuation, for the companies were exposed to the possibility of each separate Commissioner making, and charging them for, a valuation of his own, although this might not often be actually carried out in practice. They might well ask themselves how it was that a common-sense and practical people like the Americans had endured such restrictions so

long? There were two answers that might be given. The first was that America was a country which, as Lord Salisbury advised them, was pre-eminently one to be studied with large maps before them. It was a country of vast distances. The distance from New York to New Orleans was very nearly as far as from London to St. Petersburg or Bucharest. That was only down one side of the American Continent. If they took a straight line from New York right across the country to San Francisco, then a similar line, of equal length, from London right across the Continent of Europe would land them in Asia somewhere north of the Caspian. Of course, the conditions in America were different. The towns and cities were connected by railroads, telegraphs, and telephones, but even allowing for all that, it was a large stretch of imagination for a person who was as far off as the Caspian Sea to insure his life in a London office. He did not think they could be surprised that the separate Legislatures should try to make some provision in the interests of their own citizens, and see that they were insuring in some good and solvent company. But that was a condition which could be equally well carried out under a Federal law. The real obstacle lay in the constitution of the States themselves, as Mr. Homans had pointed out. Mr. Homans stated that in almost all national elections the contests had turned on the relative powers of the Federal and the State Governments. To understand the position, it must be remembered how very different their history was from ours. In Britain our liberties had broadened slowly down from precedent to precedent. We had won each step with difficulty from an autocratic central power. The main contests had long since passed, and as far as personal liberty and rights before the law were concerned, we were one of the freest countries of the world. But whenever a proposal was brought forward to deprive the central government of some of its power, even if it should relieve it from the plethora of work from which it suffered, it was sure to be received with the greatest doubt and hesitation. That could be seen in the long controversies which had arisen over the proposal to give some kind of local parliament to Ireland, and in the proposal last year to devolve the care of English education on the counties. We wanted to be quite sure of our ground before we made a change for the sake of some fancied improvement. In the United States, the position was exactly the reverse of ours. The various States might exclaim with St. Paul, "But we were free born." They sprang fully equipped from the womb of independence, and they were very jealous of their origin and their rights. It had cost them a civil war to learn that, if they were to be a nation at all, the Federal power must be supreme. But the States were very careful about depriving themselves of any of their rights and privileges, and they looked, no doubt, on any proposal to do so, and to confer any portion of them on the central power, much as we looked on the proposal, for instance, to create a local parliament in Ireland. He was afraid he had wandered somewhat from the usual tenour of their discussions, but he had felt that, if we were to appreciate the mode of feeling in the States, we should have to realize their position and their history, and contrast it with our own. We could now send a message of cordial goodwill across the Atlantic, and with every feeling of sympathy for

the retention of full State independence, we might express the hope that at some early date it might be held in the supreme Federal Courts that insurance was a branch of commerce. We might tell them that we derived the greatest benefit from having one set of regulations governing the various companies, whether in England, Scotland, or Ireland; and that in cases where we had some slight difference in the law, as in the case of the Married Women's Property Acts, we had experienced great trouble and annoyance, and the emergence of questions which were very difficult to settle. He hoped the time would come when, in the evolution of their own affairs, the people of the United States might find it possible to release their life companies from some of the trammels which now bound them. We might assure them that, in our own case, we had found sufficient safeguards for the assured by ensuring publicity on the part of the companies, and leaving to each company the fullest possible measure of freedom.

We ought always to enquire, however, whether it was possible to extract any modicum of good out of everything evil, and in the restrictive State legislation of the United States he saw counterbalancing advantages. He had a strong conviction that the United States' insurance companies would not have been educated up to the mark to face all the requirements of Continental countries if they had not already been trained by bitter experience of the regulations of their own States. Putting himself in the position of an insurance manager in the States, he thought that this was something they might count to the good. In Britain, however, they had no difficult regulations to face. The field was as open to them as to our own companies, and therein we too had reaped an advantage. We all knew the difficulty involved in finding avenues for placing the large increase of savings which had taken place every year in this country. There was a margin of about eight millions a year for which investments must be found from life assurance savings only. We could not go all over the world to find places where money could be invested; To do so would involve the establishment of offices and officials and proper experts to advise on securities in distant countries. Some offices had done that, but it would be almost impossible for every company to do so, therefore, it was a distinct benefit to us that some of the money, which would otherwise have been an addition to that eight millions, had been collected by American companies and taken over to America to be invested there, where they were on the spot and could judge the securities better than we could. This must have relieved to an appreciable extent the pressure of the investment question here, and it would tend to equalize the rate of interest in this country and in America. He begged to thank Mr. Homans very cordially for the interesting paper that he had sent them.

Mr. COLENSO said the paper was not one which lent itself to much discussion, and certainly not to controversy. It was rather strange at first to read that life assurance in the United States—of all commercial growths the most redundant—was "beset behind and before." They had been accustomed to regard the American life offices, with their feelers-out in all the countries of the globe, as peculiarly exposed to the inconvenience of State interference. The

paper made it clear that in the States themselves there was a liability to discriminating legislation. Mr. Homans had given two or three illustrations of the practical meaning of that. They would all sympathise with Mr. Homans in his feeling that there ought to be one comprehensive Federal enactment on the subject. He had been struck with the lawyer-like manner in which Mr. Homans had covered his ground. His arguments brought one to the conclusion that in point of logic and law there was a clear avenue open for a comprehensive enactment by Congress, which would not only meet his requirements, but also stand the test of an appeal to the Supreme Court.

Mr. H. R. HARDING said that to their expressions of welcome to-night he was sure they might fitly add an emphatic expression of sympathy with their friends at the embarrassing state of the laws at present in force in the United States in regard to life assurance. When they considered, as the author had pointed out, that there are some fifty separate States, with separate Legislatures, all ready and willing to legislate on life assurance matters, and each, as it were, endeavouring to improve on the legislation of its neighbours, he thought that they could only come to the conclusion that the life of an assurance manager in the United States was scarcely worth living, and that those managers must regard with envious feelings the more perfect unity which exists, and the compact legislation which governs us, in this country. Those of the English managers who were brought into contact with the legislation of our own Colonies would feel the most sympathy with Mr. Homans' difficulties. He (Mr. Harding) was very much of the opinion that it was the example set in the United States which had led their own Colonies to pass enactments which more or less hampered companies working there. The remedy in their own case, in connection with their own Colonies, was not, of course, to be found in the same direction as that indicated by the author in connection with conflicting enactments passed by the various States, and they could only hope that in the near future their own Colonies would show a little more broad-mindedness in the legislation which they adopted. It was probably known to Mr. Homans, and to many of those present, that the same difficulty to which the author alluded existed in the Dominion of Canada. There it was still a moot point as to whether the Dominion Legislature or the separate Provincial Legislatures were authorized by the constitution to legislate on life assurance matters. As in the United States, the Provinces claimed that power. The analogy must not, however, be carried further, as the Federal Government in the United States had not legislated on the subject, whilst the Dominion Legislature had done so. Although the various Acts passed by certain individual Provinces were in themselves a source of inconvenience, the inconvenience was reduced to a minimum by the fact that the Dominion Legislature did not interfere with Provincial companies so long as they confined their operations to their own respective Provinces; and the Provinces themselves, as they had not, so far, with the exception of Ontario, passed any very comprehensive legislation on the subject, did not conflict with the Dominion legislation. He would be a very bold man who thought

he could usefully give their American cousins any advice on the subject of life assurance; but he (Mr. Harding) quite agreed that Mr. Homans' suggestion was a good one, and possibly, if the Federal Legislature could be prevailed upon to pass a comprehensive Act, it would be found that the individual States would be willing to bring themselves into line with it without the difficulty anticipated. No doubt the necessity of taking the course suggested by the author would be recognized in the near future. Their friends were not unversed in the necessary steps to bring about the desirable and desired change.

Mr. M. N. ADLER said the last speaker seemed rather sanguine as to the probability of the Federal law governing life assurance institutions in the States being passed. He himself was not so sanguine. A similar attempt was made with regard to the law governing bankruptcy. In fact, on three occasions, Congress had passed such laws which would over-ride the regulations of the several States. Each of these Federal laws was subsequently abrogated. He mentioned the case as of importance to English creditors in the event of an American company suspending payment. As a matter of fact, twenty years ago an American life company, the "Continental", opened business in England, having its offices in Palmerston Buildings, and advertised its assets as something like a million-and-a-half. One fine day it closed its doors, and the assured and the public here never heard anything more about it. Those who wished to know something of the laws as to bankruptcy in the United States, he would refer to a very useful publication which was issued as an appendix to the 9th edition of the "Encyclopædia Britannica." It was called the "Encyclopædia Americana", published in Boston, but was readily accessible in the public libraries here. The date of the work was 1883, but no material alteration had, he understood, since taken place in the law. In the article on Bankruptcy in that work, it was stated that under the power which the constitution gave to Congress, a general bankruptcy law for the States was passed in 1800, and remained in operation for two years; again, in 1840, when it remained in operation eleven months. Those bankruptcy laws were passed at the urgent request of the debtors of the country, who had suffered in the commercial crisis, but they were repealed when they had done their work. In 1867 there was a similar pressure, and Mr. J. A. Jenkes, of Rhode Island, got a law passed by Congress to meet both the views of creditors and debtors, the former by providing against preferences to favoured creditors, the latter by giving a discharge to honest debtors on conditions which were not onerous. In 1874, under stress of hard times, Congress made changes in the law, which rendered it very difficult for creditors to compel an adjudication of bankruptcy, and introduced a highly objectionable mode of discharge by composition with creditors. That did not work well. In England, the system in force less than thirty years ago of compounding with creditors had not worked well, and subsequent Acts were passed, one after the other, which, he thought, had put the law of bankruptcy on a more satisfactory footing in this country. But, as far as America was concerned, the changes made in 1874 displeased the creditors of the

country, and, debtors having been generally discharged, the law was repealed in 1878. In spite of all outside efforts, Congress had not passed a fresh law. The article went on to state that, since the abrogation of the Act of 1874, each State was governed by its own laws as to bankruptcy. The laws of nearly all the States recognized the virtue of equality of creditors; but, as to practically carrying it out, they provided that, where an insolvent debtor made a general assignment for the benefit of his creditors, then they shared all alike. These laws were inadequate to enforce equality in many of the most important cases, because it remained optional with the debtor whether he should make a general assignment. If he did not, but conveyed away his property to a few favoured creditors, one by one, the laws were evaded and without remedy. The New England States had mostly complete systems of bankruptcy laws. But there were many States which had not even provided for equality in general assignments, such as New York, Maryland, Virginia, the district of Columbia, and about ten other States. In New York State there was no full bankruptcy law; there was a voluntary bankruptcy law, which required that two-thirds of the creditors in the United States should join with the debtor in his petition. This was found so cumbrous that it was practically useless in case of fraud, and difficult in many other cases. Insolvent estates were, therefore, generally managed by assignments, in which preferences, though discouraged, were not actually forbidden. The statutes of the State of New York and of other States, as to insurance companies, were, no doubt, very complete, and possibly the bankruptcy laws might have been placed on a more satisfactory footing within recent years, yet it was not very comforting for those who had contracts in force in the United States, especially if they were foreigners, to know that, in the event of insolvency, A and B might be paid in full, and C be left out in the cold.

The PRESIDENT said it had struck him in reading the paper that there was a very interesting historical problem involved. Mr. Homans had reminded them that life assurance institutions in the United States were essentially of a mutual character. Now, the historical development of insurance bodies in this country had been a very distinct one; mutual offices at the outset, succeeded by purely proprietary companies, and they, in their turn, followed by companies of a mixed character where a share capital existed, with participation in profits by the assured, and the interesting question arose whether the United States were still in the early stages of life assurance history, and were likely to show a similar evolution to that which had existed in this country, or whether the present long-continued character of their institutions indicated an arrest and completion. He was sure the members would by acclamation accord their very grateful thanks to Mr. Homans, not merely for remembering his old friends at the Institute of Actuaries, but also for the very interesting and practical paper which he had submitted.

The vote of thanks was carried by acclamation.

AS REGARDS LIFE ASSURANCE COMPANIES.

the disposal of the Institute.

ts	Fees, Licenses and Taxes	Discrimination	Miscellaneous
rtificate	License, \$100 per ann. Tax, 1 per-cent on gross premiums re- ceived	Forbidden under penalties	No Claim to be contested after Policy in force 3 years
	Sundry Fees; no Li- cense; Tax, 1½ per- cent on gross pre- miums received	Nil	...
	Sundry Fees; no Li- cense; Tax, 2½ per- cent on prems. less losses and comms.	...	Officers or Agents making false statements guilty of felony—3-10 years' peni- tentiary
	Sundry Fees; Commr.'s expenses not covered by Fees assessed on Companies	Nil	Reciprocal legislation; paid- up Policies after 3 years; full reserve applied; re- moval of suit, revocation
	Sundry Fees; Tax, 2 per-cent on gross premiums	Prohibited	Paid-up insce. after 3 years; Sols., Agents, or Med. Exrs., making false statements guilty of misdemeanour
	Sundry Fees; Taxes governed by recip- rocal legislation	Do.	Copy of Proposal to be fur- nished on demand
	Sundry Fees; Tax, 1½ per-cent on gross premiums	Anti-rebate law in force	Has reciprocal law
Agent's e, \$15, others	Tax, 1 per-cent of gross receipts	...	Copy of Proposal must be attached to Policy
ense	Tax, 2 per-cent on gross premiums
rtificate	Tax, 1 per-cent on gross premiums	...	Compacts to lessen competi- tion unlawful
	License, \$50; sundry Fees
	Taxes governed by re- ciprocal legislation	Prohibited	...
	Tax, 3% of prems. re- ceived less losses; also reciprocal if Indiana Co.'s pay more	...	Fraudulent statements, or effecting an insurance without knowledge of life, as in (5)
	Tax, 2½ per-cent on gross premiums	Forbidden	Med. Examnrs.' Certif. binds Company as to responsibility
	School Fund, \$50; Fees same as (4); Taxes, reciprocal provisions

SCHEDULE OF THE PRINCIPAL REQUIREMENTS OF THE VARIOUS STATES OF THE AMERICAN UNION AS REGARDS LIFE ASSURANCE COMPANIES.

N.B.—This Schedule, prepared by Mr. CROSBY, and revised by Mr. SULLIVAN, has been kindly placed at the disposal of the Institute.

	States	Preliminary Documents	Attorney	Funds on Deposit	Computation of Foreign Countries	Valuation	Age of State Agents	Age of	License	Prohibition	Modifications
1	ALABAMA	Copy of Charter and last Annual Statement	Must appoint Insurance Commissioner	\$100,000	Statement to cover only business in U.S., \$200,000 in some State	Am. Exp., 1 per cent; Commission may accept Valuation when made by any other State	Annual Account, 1 March	Annual Certificate required	License, \$100 per ann. Tax, 1 per cent on gross premiums received	Forbidden under penalties	No Class 1 to be contested after Policy in force 3 years
2	ARIZONA	Statement of particulars regarding company, and Bond for \$15,000	All Agents	No requirement	\$15,000 Bond	No provision	Annual Account, March	Do.	Sundry Fees, no License, Tax, 1 per cent on gross premiums received	Nil	
3	ARKANSAS	As in (1) and (2), Bond for \$20,000	Must designate an Attorney	Assets must equal value of policies, Capital, \$100,000	Statement of business done outside U.S., Capital \$100,000	Am. Exp., 1 per cent; Certified Valuation accepted from Ins. Com. of State of origin of Co.	Annual Account, 1 March, Foreign Co's 1 July, of other than U.S. has	Do.	Sundry Fees, no License, Tax, 2 1/2 per cent on gross premiums less losses and claims		Officers or Agents making false statements guilty of felony, 3-10 years' imprisonment
4	CALIFORNIA	Similar, no Bond	Do.	\$200,000 in excess of liabilities	Same as (2), General Agent must give \$2,000 Bond	C.O.E., 1 per cent	Annual Account, 10 March, also statement of business done in State, 10 Jan.	Do.	Sundry Fees, Company's expenses not covered by Fee assessed on Companies	Nil	Reciprocal legislation required up to 10 years after 4 years, full review applied, to avoid 4 out, reciprocal
5	COLORADO	Do.	Attorney must be the Superintendent of Insurance	\$100,000 Cash Capital	\$100,000, some State	C.O.E., 1 per cent	Acts, 1 March	Do.	Sundry Fees, Tax, 2 per cent on gross premiums	Prohibited	Reciprocal legislation required up to 10 years after 4 years, full review applied, to avoid 4 out, reciprocal
6	CONNECTICUT	Do.	Do.	Assets must exceed liabilities	Must furnish Certificate of Home Government, and if Ins. Com. satisfied may be excused	C.O.E., 1 per cent, condition as in (3)	Acts, 1 March	Do.	Sundry Fees, Taxes governed by reciprocal legislation	Do.	Copy of Proposal to be furnished on demand
7	DELAWARE	Do.	Must appoint resident Attorney	No requirement	Manager of U.S. Branch report on behalf of his Company in such form as Ins. Com. designates	On basis of home State	Acts, 1 February	Do.	Sundry Fees, Tax, 1 per cent on gross premiums	Anticipate law in force	Has reciprocal law
8	DISTRICT OF COLUMBIA	Do.	Do.	Same as (6)	Nil	Am. Exp., 1 per cent	Acts, 1 March, also half-yearly status required	General Agents Certificate, \$15, covers all other	Tax, 1 per cent of gross receipts		Copy of Proposal must be attached to Policy
9	FLORIDA	Do.	Do.	\$100,000 in special securities	Nil	Nil	Acts, January	\$10 License	Tax, 2 per cent on gross premiums		
10	GEORGIA	Do.	Do.	\$100,000	Must make reports same as domestic	C.O.E., 1 per cent	1 Jan. or 60 days, 1/2 yearly status April & August	Annual Certificate required	Tax, 1 per cent on gross premiums		Companies to be lessor companies, non-indemnity
11	IDAHO	Do.	Do.	\$100,000	Capital \$100,000	Nil	Accounts, April	Do.	License, \$50, sundry Fees		
12	ILLINOIS	Do, authority ceases as soon as Company attempts to remove action in State Court into U.S. Court	Do.	\$100,000	Must conform to law regulating domestic companies, \$100,000	C.O.E., 1 per cent	Acts, 1 March, Foreign Co's, April 1	Do.	Taxes governed by reciprocal legislation	Prohibited	
13	INDIANA	Same as (1)	All Agents	Capital & Stock, \$100,000	\$100,000 in some State	Nil	Half-yearly statements Jan. and July	Do.	Tax 3% of premiums received less losses, also reciprocal of Indiana Co's pay none		Prudent statements of effecting an insurance without knowledge of life as in (5)
14	IOWA	Do.	Do.	\$100,000 in excess of liabilities	Do.	Am. Exp., 4 1/2 per cent, on C.O.E., 4 per cent	Accounts, 1 April	Do.	Tax, 2 1/2 per cent on gross premiums	Forbidden	Med. Examiners' Certificate Company as to speciality
15	KANSAS	Do.	Attorney must be Supt. of Ins., all actions must be tried in State	\$100,000	Do.	Am. Exp., 1/2 per cent	Acts, 1 March	Do.	School Fund \$50, Fees same as (1), Taxes, reciprocal provisions		

* In Session, changes may be made

	State	Preliminary Documents	Attorney	Paid-up Deposit	Companies of Foreign Countries	Valuations	Annual Statements	Agent	Fees, Licenses and Taxes	Discrimination	Miscellaneous
16	KENTUCKY	Same as (4)	All Agents	Deposit \$100,000	\$100,000 in some State	C.O.E., 4 per cent; condition as in (3)	Accts., 10 March	Annual Certificate required	Fees, same as (1). Tax, 2 per cent on premiums; Louisville 2½% addl.	Prohibited	Has recip. law, suit not to be removed from State Court without consent both parties
17	LOUISIANA	Do	Same as (7)	Nil	\$25,000 in some State	Nil	Accts. in 90 days	Do.	Licenses gratis, and immunities enttd. to exact same amounts as State	Do.	
18	MAINE	Do.; Removal of suit from State Court revokes	Insurance Commr.	\$100,000	\$100,000 in some State	Nil	Accts. on 31 Jan.	Do.	Tax, 1½ per cent gross License \$300	Do.	Props. drawn by Agt at binding on Comp., but not on cash although signed by him
19	MARYLAND	Do.	Do.	\$100,000	Annual Statements to be filed	Am. Exp., 4½ per cent	Accts. in 60 days; For. Co.'s of Ins. other than U.S. prior 1 July	Do.	Sundry Fees, Tax, 1½ per cent gross premiums	Do.	False statements as in (5). Retaliatory law, warranty no effect unless matured
20	MASSACHUSETTS	Do	Same as (5)	\$100,000	Funds in U.S. not less than the net value of its U.S. Pds.; such Funds to be not less than \$200,000	C.O.E., 4 per cent	Accts., 15 Jan.	Do.	Tax, ¼ of 1 per cent on net value of pds.	Do. (also against Africans)	False statements as in (5). Paid-up insurances after 3 years (applies Mass. Companies only)
21	MICHIGAN	Do	Same as (3)	\$100,000	\$100,000 in some State	Am. Exp., 4 per cent	Accts., January	No requirement	Tax, 2 per cent on premiums	Prohibited	False stat. Mod. Exams. in person, 3 mos., corp. law
22	MINNESOTA	Do	Same as (7)	\$100,000	Must have in funds of U.S. citizens funds equal to its liabilities on U.S. Pds., not less than \$100,000	Am. Exp., 4½ per cent	Accts. in 10 days	Annual Certificate required	Tax, 2 per cent on premiums	Do.	Fraudulent statements as in (5)
23	MISSISSIPPI	Do	Same as (7)	\$150,000	\$150,000 in some State	No provision	Accts., 1 March	Do.	License \$1,000		
24	MISSOURI	Do.	Same as (5)	Deposit \$100,000	\$100,000 in some State	C.O.E., 1 per cent; annual valuations	Accts. in 60 days	Do	Tax, 2 per cent on premiums received		Paid-up insurance after 3 years
25	MONTANA	Do.	Same as (7)	Do	Do.	No provision	Accts. in 90 days		Sundry Fees, License \$125 for first \$5,000 of gross and \$20 per \$1,000 after the first \$5,000, or 30.2 grs.		
26	NEBRASKA	Do	Same as (3)	\$100,000	Must invest \$100,000 in U.S. securities, such to be held by citizens	No provision	Accts. in January	Do	License taxed same as prop. prop., License \$100		
27	NEVADA	Do	Same as (7)	\$200,000	Must have in some State \$200,000 in excess of liabilities in U.S.	Valuation of other State shall be accepted	Accts., 1 March	Do	Sundry Fees		
28	NEW HAMPSHIRE	Do	Same as (5)	\$200,000	Assets of \$200,000	C.O.E., 1 per cent	Accts., 1 Feb.	Do	Tax, 1 per cent on gross premiums	Forbidden	
29	NEW JERSEY	Do	Any Agent, Clerk, or Officer	\$100,000	Must have Cap. Stock \$150,000	C.O.E., 1 per cent	Accts., January	Do	Tax, 1 per cent	Do	Home Office paid-up out of 3 yrs. and approval after 3 years, deniable within 4 mos. Required law
30	NEW MEXICO	Do	Must appoint Attorney in equity Co. does business in	\$200,000 Capital Stock, 50 per cent paid up	\$100,000 in some State	Nil	Accts. 30 Jan. & Synopsis published	Do	Gratd. for Agents Tax on net premiums as in prop.		
31	NEW YORK	Do	Same as (5)	\$100,000 in special securities deposit \$100,000	\$200,000 in some State	Annual Values by State Super., not premium business Table that adopted by Co.	Accts., 1 March	Do	1. License Co. only 2. companies, other than Insurers not allowed	Prohibited	Paid-up after 3 years policy cannot be forfeited within 60 days and of premiums 2 mos.
32	NORTH CAROLINA	Do	Same as (7)	If \$200,000 to be paid deposit with State Co. shall be declared solvent, if not Co. must satisfy Sec. of State or solvent		Nil	Accts., 1 March	Do	1. Tax \$250, Tax 2 per cent on gross premiums collectible no 1 per cent discount 2. Receipts recorded in State		No provision that suit must be brought within less than one year after withholding fund. Policy may not be deemed contracts made in State if app. taken there in

ts	Fees, Licenses and Taxes	Discrimination	Miscellaneous
rtificate	Fees, same as (4); Tax, 2 per-cent on prems.; Louisville 2½%, addl.	Prohibited	Has recip. law; suit not to be removed from State Court without consent both parties
	Licenses gradtd. and municipalities entd. to exact same amnts. as State	Do.	...
	Tax, 1½ per-cent gross License \$300	Do.	Props. drawn by Agent binding on Compy. but not on insd. although signed by him
	Sundry Fees; Tax, 1½ per-cent gross prems.	Do.	False statements as in (5); Retaliatory law, warranty no effect unless matured
	Tax, ¼ of 1 per-cent on net value of polys.	Do. (also against Africans)	False statements as in (5); paid-up insurance after 3 years (applies Mass. Companies only)
ement	Tax, 2 per-cent on premiums	Prohibited	False stat. Med. Exams. imprisonment. 3 mos., recip. law
rtificate	Tax, 2 per-cent on premiums	Do.	Fraudulent statements as in (5)
	License \$1,000
	Tax, 2 per-cent on premiums received	...	Paid-up insurance after 3 years
	Sundry Fees; License \$125 for first \$5,000 of prems. and \$20 per \$1,000 after the first \$5,000, or say 2% grs.
	Prem. taxed same as pers. prop.; License \$100
	Sundry Fees
	Tax, 1 per-cent on gross premiums	Forbidden	...
	Taxes, reciprocal	Do.	Home Co's. polys. incontest. aft. 3 yrs., paid-up polys. aft. 3 yrs. demandable within 3 mos.
	Gradtd. Lic. for Agents; Tax on net prems. same as pers. prop.	...	Reciprocal law
	Tax, Foreign Cos. only 2% on prems., other Co's. Taxes reciprocal	Prohibited	Paid-up ins. after 3 years, polys. cannot be forf'd. witht. 15 days not. of prems. becg. due
	License \$250, Tax 2 per-cent on gross prems. reducible to 1 per-cent if one qtr. of receipts is invested in State	...	No proviso that suit must be brought within less than one year; agts. withholding funds, felony. Policies must be deemed contracts made in State if app. taken therein

	State	Preliminary Demands	Attorney	Funds and Deposit	Companies of Foreign Origin	Valuation	Amount, State agents	Agents	Excess, License and Tax	Discrimination	Miscellaneous
33	NORTH DAKOTA	Same as (1)	Same as (5)	\$100,000	\$100,000 in some State	C.O.E., 1 per cent	Assets, 1 March	Annual Certificate required	Tax, 2½ per cent on gross premiums		
34	OHIO	Do	All Agents	\$100,000 in specified securities	\$100,000 to be deposited in Ohio	Am. Exp., 1 per cent	Assets, in 60 days	Do	Tax, 2½ per cent on gross premiums	Prohibited	False statements as in (21), copies of application on demand
35	OKLAHOMA	Do	Same as (7)	\$100,000	Assets must equal liabilities	C.O.E., 1 per cent	Assets, 1 March	Do	Sundry Fees, License \$25		Must not remove suits, record books
36	OREGON	Do	Do	\$100,000 cash Capital	\$200,000 in some State and \$50,000 in State	Accept Valuation of other State	Assets, 1 March	Do	Sundry Fees, Tax 2½ less Losses and Dividends, License \$100		
37	PENNSYLVANIA	Do	Do (Must not be Insurance Commissioner)	Same as (6)		C.O.E., 1 per cent	Assets, 1 March Foreign Co's., 1 April	Do	Tax, 2 per cent on premiums	Prohibited	Fraudulent statements as in (5)
38	RHODE ISLAND	Do	Same as (5)	\$100,000	\$100,000 in some State	C.O.E., 1 per cent	Assets, in 30 days	Do	Tax, ½ per cent gross profits to State, License \$100, local authorities satisfy same profits at persons info		General Agent must give bond to State
39	SOUTH CAROLINA	Do	Resident Agent must be appointed	Nil		Nil	Assets, 31 March	Copy application to be filed	Tax, 2½ per cent on gross premiums		Acting by Comptroller limited to within 2 yrs. from date of policy of solvency from home State accepted
40	SOUTH DAKOTA	Do	Same as (7)	Deposit \$100,000	\$100,000 in some State	Am. Exp., ½ per cent, or C.O.E., 1 per cent	Assets in 1 month	Annual Certificate required	Tax, 2½ per cent on premiums received	Prohibited	Recap law, fraudulent statements as in (5)
41	TENNESSEE	Do	Same as (5)	\$100,000	Do	C.O.E. or Am. Exp. 1 per cent	Assets, in 30 days	Do (Name of Agent to be reported)	Tax, 2 per cent on gross premiums, General Agent at \$50		
42	TEXAS	Do	All Agents	\$100,000 Deposit \$100,000	Do	1½ per cent	Assets, in 60 days	Do	Sundry Fees, 1½ per cent gross premiums		
43	UTAH	Do (Foreign Companies only)	Same as (7)	\$100,000		Nil	Assets, 30 June	Do	Tax, 2 per cent on premiums, 1 per cent additional tax on surplus above 1 per cent	Prohibited (colon, age, benefits and rates)	
44	VERMONT	Do	Must appoint Secretary of State	\$100,000	Must have paid up Capital \$100,000	C.O.E., 1 per cent	Assets, 1 March	Do	Tax, 1 per cent on premiums, License \$200		
45	VIRGINIA	Do	Same as (7)	Deposit \$100,000 to \$50,000		Am. Exp., 1½ per cent	Assets, 31 March	Nil	Tax, 2 per cent on premiums, less losses		
46	WASHINGTON	Do	Do	\$100,000	\$200,000 in some State	Nil	Assets, of Wash. Insurers, 15 Feb. General State account, 1 March	Annual Certificate required		Prohibited	
47	WEST VIRGINIA	Do	Do	\$100,000	\$100,000 in some State		Assets in January	Do	Tax, 2 per cent on gross premiums, reducible to 1 per cent if net receipts are invested in State		Reciprocal Law
48	WISCONSIN	Do	Same as (5)	\$100,000		Am. Exp., 1½ per cent	Assets, 1 March	Do	Sundry Fees & License, License \$400	Prohibited as in (20)	Reciprocal Law
49	WYOMING	Do	Same as (40)	\$100,000	Deposit \$100,000 some State	Nil	Assets, in 60 days	Do	Tax, 2½ per cent gross premiums	Prohibited	Reciprocal Law

* In Session, changes to be made

NOTE: Fuller particulars regarding the insurance law in the various States may be obtained from the *Insurance Year Book*, published annually by the Spectator Co. in New York and from the *Annual Cyclopaedia of Insurance in the United States*, edited by H. R. Hayden, and published by the Underwriter Printing and Publishing Co., New York.

ats	Fees, Licenses and Taxes	Discrimination	Miscellaneous
ertificate	Tax $2\frac{1}{2}$ per-cent on gross premiums
o.	Tax, $2\frac{1}{2}$ per-cent on gross premiums	Prohibited	False statements as in (21), copies of application on demand
o.	Sundry Fees; License \$25	...	Must not remove suits; reciprocal law
o.	Sundry Fees; Tax $2\frac{1}{2}$ less Losses and Dividends; License \$100
o.	Tax, 2 per-cent on premiums	Prohibited	Fraudulent statements as in (5)
o.	Tax, $\frac{1}{2}$ per-cent gross prems. to State; License \$100; Local authorities also tax same prems. at pers. rate	...	General Agent must give bond to State
oint. to	Tax, $2\frac{1}{2}$ per-cent on gross premiums	...	Action by Comp'y. <i>re</i> misrep'n. limited to within 2 yrs. from date of pol.; cert. of solvency from home State accepted
ertificate	Tax, $2\frac{1}{2}$ per-cent on premiums received	Prohibited	Recip. law; Fraudulent statements as in (5)
o. all Agts. reported)	Tax, 2 per-ct. on gross premiums; general Agent at \$50
o.	Sundry Fees; $1\frac{1}{2}$ per-cent gross premiums
o.	Tax, $2\frac{1}{2}$ on prems.; State life co.'s also pay $1\frac{1}{2}$ additional tax on surplus above $4\frac{1}{2}$ reserve	Prohibited (colour, age, benefits and rates)	...
o.	Tax, 1 per-cent on premiums; License \$200
l	Tax, 2 per-cent on premiums, less losses
ertificate	...	Prohibited	...
o.	Tax, 2 per-ct. on gross prems. reducible to 1 per-cent if net receipts are invested in State	...	Reciprocal Law
o.	Sundry Fees & License; License \$300	Prohibited as in 20	Reciprocal Law
o.	Tax, $2\frac{1}{2}$ per-cent gross premiums	Prohibited	Reciprocal Law

The late Professor Sylvester.

THE sympathetic allusion to the death of James Joseph Sylvester, made by the President at the monthly meeting in March, must have come, not as a reminder merely, but even as a surprise to many members of the Institute. Sylvester's name stands so high in the roll of distinguished mathematicians, his active connection with the science of life contingencies ceased so many years ago, that we of the present generation need make no apology if we have almost forgotten that connection. Yet for ten years Sylvester was actuary of a Life Office; for nearly forty years he was an Honorary Member of our Institute; and, therefore, though we are unable to attempt any critical account of his life's work and of its influence, yet it would scarcely be fitting if no reference to his long and brilliant career were made in the pages of this *Journal*.

Born in 1814, Sylvester read for the Mathematical Tripos at Cambridge, and was Second Wrangler in 1837. On account of his religious scruples—for he was always a staunch Jew—he was unable to take his degree or to compete for a fellowship. He accordingly left Cambridge at once, and accepted the Professorship of Natural Philosophy at University College, London. Almost immediately he commenced the long series of contributions to mathematical literature which have made his name so famous in the scientific world. At first, probably on account of the position he occupied, it seemed that his attention would be directed towards Physics; for among his earliest papers were "The Analytical Development of Fresnel's Optical Theory of Crystals", and "The Motion and Rest of Fluids" (*Philosophical Magazine*, 1837–8). But very soon the real bent of his genius, which was towards Pure Mathematics, asserted itself. In rapid succession, he produced a series of brilliant papers on Sturm's Functions, Invariants, Elimination, &c., which were published, for the most part, in the *Philosophical Magazine* and the *British Association Reports*, and led to the ready recognition of his talents by the Royal Society, of which he was elected a Fellow in 1839.

He left London in 1844 to become Professor of Mathematics at Virginia. But his absence from England was very brief. He returned in 1845, and for a time discontinued the work of teaching, in which he had hitherto been engaged. It was at this stage of his career that Sylvester's connection with our profession

commenced. He was the first actuary of the Equity and Law, and for some years he was also Consulting Actuary to the Law Reversionary Interest Society. Moreover, he found leisure to study law, and was called at the Inner Temple in 1850, though he never practised. In 1848 the Actuaryship of the Royal Exchange Assurance became vacant, and Sylvester desired the post; but when he found that the late J. A. Higham, then next in rank in the office though only 28 years of age, was a candidate, he withdrew on the ground that he would not compete with one who had been his own pupil.

Sylvester's solitary contribution to the *Journal* is an elegant little note "On Multiplication by aid of a Table of Single Entry", published in 1854 (*J.I.A.*, iv, 236). As evidence of his influence in actuarial matters, it is a fact of interest that the table of whole-life with-profit premiums which he calculated for the Equity and Law over fifty years ago is still in use in that office. Sylvester, however, was not happy in his occupation during this period. He had no great love for actuarial and legal work, and he gladly relinquished it in 1855, when he was appointed Professor of Mathematics at the Royal Military Academy, Woolwich. He threw himself into his new, and more congenial, duties with characteristic enthusiasm, and, by the experience he gained at Woolwich, he was soon enabled to render a distinct public service. A committee was appointed to enquire into the working of institutions like the Royal Military Academy, and Sylvester, who was invited to give evidence, fearlessly recommended several drastic reforms, which were eventually adopted. It is a curious irony that, by the adoption of one of his own suggestions, he was compelled to resign his position at Woolwich in 1870, after fifteen years' service, on a totally inadequate pension. Fortunately, his claims to more generous treatment were brought before the notice of the House of Commons, and were gracefully recognized by Mr. Gladstone on behalf of the Government.

Mathematicians will probably agree that his fifteen years at Woolwich constituted the period of Sylvester's greatest intellectual activity. It was in 1864 that he read before the Royal Society his famous "Algebraical Researches", which were an epoch-making discovery in the Theory of Equations. To this succeeded a host of masterly contributions, on almost every branch of mathematics, to almost every important scientific journal of Europe. The most interesting of these to actuaries will probably

be the article on the Theory of Probabilities (*Phil. Trans.* 1865). What is especially admirable in all Sylvester's writings at this period is his evident enthusiasm for his subject. Indeed, his love for mathematical science and its methods, his belief in the value of these methods as a mental training, were alike unbounded. It may, perhaps, be of interest in this connection to recall his famous passage of arms with Huxley. The latter, in a lecture "On the Educational Value of the Natural History Sciences", had said, "I do not question for a moment that, while the Mathematician is busy with deductions *from* general propositions, the Biologist is more especially occupied with observation, comparison, and those processes which lead to general propositions. . . . The Mathematician deals with two properties of objects only, number and extension, and all the inductions he wants have been formed and furnished ages ago. *He is occupied now with nothing but deduction and verification.*" The last sentence was not long left unchallenged. At the meeting of the British Association, at Exeter, in 1869, Sylvester, who was President of the Mathematical and Physical Section, devoted his opening address to "A Plea for the Mathematician", which was really a vigorous reply to Huxley. In a passage of great eloquence, he maintained that "mathematical analysis is constantly invoking the aid of new principles, new ideas, and new methods, not capable of being defined by any form of words, but springing direct from the inherent powers and activity of the human mind, and from continually renewed introspection of that inner world of thought of which the phenomena are as varied, and require as close attention to discern, as those of the outer physical world—to which the inner one in each individual man may, I think, be conceived to stand in somewhat the same general relation of correspondence as a shadow to the object from which it is projected, or as the hollow palm of one hand to the close fist which it grasps the other: that it is unceasingly calling forth the faculties of observation and comparison, that one of its principal weapons is induction, that it has frequent recourse to experimental trial and verification, and that it affords a boundless scope for the exercise of the highest efforts of imagination and invention." Huxley made no reply, other than a kindly reference to "the artillery of our eminent friend", in a letter addressed to Tyndall, in 1870,* and, indeed, no reply

* Now published as a preface to Huxley's *Lay Sermons, Essays, and Reviews*.

seems possible to one who has any conception of the work of a mathematical genius of the order to which Sylvester belonged.

During some six or seven years after his retirement from Woolwich, Sylvester was without a professorship. His leisure, however, was not unfruitful. He continued his mathematical researches with undiminished zeal, and took up some branches, *e.g.*, Spherical Harmonies, to which he had hitherto contributed but little. He also began to take a more active interest in public affairs, more especially in educational matters. In 1872 he contested the vacancy on the London School Board, created by the resignation of Huxley, by whom, it is gratifying to note, his candidature was cordially supported.

In 1877, Sylvester again left England, this time to become Professor of Mathematics at the new Johns Hopkins University, Baltimore. The importance of his work there can hardly be exaggerated. He practically originated the study of mathematics in the United States. He founded, and was the first editor of, the *American Journal of Mathematics*, and he succeeded very quickly in gathering round him a band of enthusiastic students, many of whom are now among the leaders of scientific thought in their country. While at Baltimore, Sylvester always strove to bring the men with whom he came in contact into closer sympathy with England. At the University celebrations in honour of Washington's birthday, he delivered an address on Education, in which he deplored the fact that so many American students went to the German Universities rather than to Oxford or Cambridge. He attributed this, rightly or wrongly, to the religious exclusiveness of the English Universities,* and made an eloquent plea for the removal of these disabilities, from which, forty years before, he had himself suffered at Cambridge.

Only a few more years passed before this plea was answered in a manner that must have given Sylvester the keenest satisfaction. In 1883, he was elected Savilian Professor of Geometry at Oxford, and had thus at last the gratification of filling, at that University, the position corresponding to that held at Cambridge by his distinguished friend Cayley.

Unfortunately, the mathematical world was not much longer destined to enjoy the full measure of his power. His general health became less robust, and, with advancing age, his eyesight

* Our readers will remember that this address was delivered nearly twenty years ago, when, *e.g.*, it was practically impossible for any person not a member of the Church of England to become a Fellow of his College.

partially failed. He was released from the active duties of his professorship in 1892, a deputy being appointed, and came to London, where he spent most of his time at the Athenæum Club. A few months before the end, he appeared to recover somewhat, and actually resumed his mathematical researches. But this rally was not of long duration. He was seized with paralysis towards the close of February, and passed away peacefully in London on March 15, 1897.

This is not the place to dwell upon Sylvester's character or disposition—on the gentleness of heart that lay beneath the rugged exterior of his manner. But, in closing this brief and necessarily imperfect sketch of his career, it would be ungenerous not again to refer to his absolutely unselfish devotion to his work. It is to men like Sylvester that the world owes most, men who, single-hearted, resolute, self-less, pursue their labours for the sake of Truth alone. It is by the hand of such as he that Science may hope to win from the unknown her brightest gems of knowledge.

A. L.

ACTUARIAL NOTES.

1.—On Central-Difference Interpolation Formulæ. By JOHN SPENCER, A.I.A.

I. THE formula commonly employed for interpolation by means of central differences is, adopting the usual notation,

$$u_x = u_0 + x\left(a_0 + \frac{x}{2}b_0\right) + \frac{x(x^2-1)}{3}\left(c_0 + \frac{x}{4}d_0\right) + \&c. \quad (1)$$

where x lies between 0 and 1. (See *Text-Book*, Part II, p. 449).

Examining this expression, it will be seen that five equidistant values of u are required in order that d_0 may be calculated, of which values three are situated on one side of u_x and the remainder on the other side. The value of any other difference is similarly obtained from an odd number of the given values of the function.

In these circumstances, it would seem to be probable that slightly better results would be obtained if, instead of the above, an interpolation formula involving differences derived from an equal number of values of u on either side of u_x were employed, the *interval* containing the desired value being in this case central. Such a formula has been given by various authors, but it is not, perhaps, so familiar to actuaries as that given above, and the

writer therefore ventures to direct attention, briefly, to the formula, and to suggest one or two cases in which it may advantageously be used in practice.

The following slightly altered scheme of notation is adopted:

$$\begin{array}{rcl}
 & u_{-2} & \\
 & a_{-2} & \\
 u_{-1} & \beta_{-2} & \\
 & a_{-1} & \gamma_{-1} \\
 u_0 & \beta_{-1} & \delta_{-1} \qquad \beta_0 = \frac{1}{2}(\beta_{-1} + \beta_{+1}) \\
 & a_0 & (\beta_0) \gamma_0 (\delta_0) \epsilon_0 \\
 u_{+1} & \beta_{+1} & \delta_{+1} \qquad \delta_0 = \frac{1}{2}(\delta_{-1} + \delta_{+1}) \text{ \&c.} \\
 & a_{+1} & \gamma_{+1} \\
 u_{+2} & \beta_{+2} & \\
 & a_{+2} & \\
 u_{+3} & &
 \end{array}$$

Assuming as before that x is >0 and <1 we have, by the fundamental formula of the calculus of finite differences,

$$\begin{aligned}
 u_x = & u_{-2} + (x+2)\Delta u_{-2} + \frac{(x+2)(x+1)}{2} \Delta^2 u_{-2} \\
 & + \frac{(x+2)(x+1)x}{3} \Delta^3 u_{-2} + \frac{(x+2)(x+1)x(x-1)}{4} \Delta^4 u_{-2} \\
 & + \frac{(x+2)(x+1)x(x-1)(x-2)}{5} \Delta^5 u_{-2} + \text{\&c.}
 \end{aligned}$$

Stopping at 5th differences and substituting the above notation, this expression becomes

$$\begin{aligned}
 u_x = & u_{-2} + (x+2)a_{-2} + \frac{(x+2)(x+1)}{2} \beta_{-2} + \frac{(x+2)(x+1)x}{3} \gamma_{-1} \\
 & + \frac{(x+2)(x+1)x(x-1)}{4} \delta_{-1} + \frac{(x+2)(x+1)x(x-1)(x-2)}{5} \epsilon_0.
 \end{aligned}$$

But

$$\delta_{-1} = \delta_0 - \frac{1}{2}\epsilon_0$$

$$\gamma_{-1} = \gamma_0 - \delta_0 + \frac{1}{2}\epsilon_0$$

$$\beta_{-2} = \beta_0 - \frac{3}{2}\gamma_0 + \delta_0 - \frac{1}{2}\epsilon_0$$

$$a_{-2} = a_0 - 2\beta_0 + 2\gamma_0 - \delta_0 + \frac{1}{2}\epsilon_0$$

and

$$u_{-2} = u_0 - 2a_0 + 3\beta_0 - \frac{5}{2}\gamma_0 + \delta_0 - \frac{1}{2}\epsilon_0$$

Hence, after multiplication and re-arrangement of the terms,

$$\begin{aligned}
 u_x = & u_0 + xa_0 + \frac{x(x-1)}{2} \beta_0 + \frac{x(x-1)(2x-1)}{2 \cdot 3} \gamma_0 \\
 & + \frac{(x+1)x(x-1)(x-2)}{4} \delta_0 + \frac{(x+1)x(x-1)(x-2)(2x-1)}{2 \cdot 5} \epsilon_0 \quad (2)
 \end{aligned}$$

This formula may also be obtained, of course, from (1), by writing in that equation

$$a_0 = \frac{1}{2}(a_{-1} + a_0) = a_0 - \frac{1}{2}\beta_0 + \frac{1}{4}\gamma_0$$

$$b_0 = \beta_0 - \frac{1}{2}\gamma_0$$

$$c_0 = \gamma_0 - \frac{1}{2}\delta_0 + \frac{1}{4}\epsilon_0, \text{ \&c.,}$$

the connection between the two formulæ being thus exhibited.

To take a numerical example in order to show the relative accuracy of formulæ (1) and (2), we may calculate the value of $\log 1325$ from the following data:

		Δ	Δ^2
$u_{-1} = \log 120$	$= .0791812$		
u_0	$130 \quad .1139434$	347622	-25776
u_1	$140 \quad .1461280$	321846	-22213
u_2	$150 \quad .1760913$	299633	

Here $a_0 = 334734$ and $b_0 = -25776$

while $a_0 = 321846$ and $\beta_0 = -23994\frac{1}{2}$

By formula (1)

$$\begin{aligned} \log 1325 &= .1139434 + \frac{1}{4}(.0334734) + \frac{1}{3 \cdot 2}(-.0025776) \\ &= .1222312. \end{aligned}$$

Using formula (2)

$$\begin{aligned} \log 1325 &= .1139434 + \frac{1}{4}(.0321846) - \frac{3}{2}(-.00239945) \\ &= .1222145. \end{aligned}$$

The correct value is .1222159.

The above values have been calculated to 2nd differences, as such an approximation is, perhaps, adopted more frequently than any other in practice. The superiority of formula (2) over (1) in such cases is clearly shown, and the reason for this is apparent when it is borne in mind that four values of the function are taken into account in calculating β_0 the second central difference.

Moreover, although second differences are employed in the example, yet by using formula (2) the result is, roughly speaking, as accurate as would have been the case had effect been given to third differences.

The greatest value of $\frac{x(x-1)(2x-1)}{12}$ is $\frac{1}{125}$ nearly, this result obtaining when x is approximately equal to either .79 or .21, so that the effect of discarding 3rd and higher orders of differences will be slight when the successive differences diminish

rapidly. On the other hand, the maximum value of $\frac{x(x^2-1)}{6}$

the coefficient of c_0 in (1) is exactly eight times as great as that of the coefficient of γ_0 , and to neglect third and subsequent differences in using formula (1) would therefore be more serious.

It need hardly be pointed out that identical results will be produced by using the two formulæ when the differences of any given order are constant, provided, of course, that the calculations are carried up to this point.

It will be noticed that when x is equal to $\frac{1}{2}$, that is to say, when the desired value lies midway between two of those given, the terms in formula (2) involving γ_0 , ϵ_0 , &c., become zero, and the equation assumes the simple form

$$u_{\frac{1}{2}} = u_0 + \frac{1}{2}\alpha_0 - \frac{1}{8}\beta_0 + \frac{3}{128}\delta_0 - \&c. \quad (3)$$

This formula, which is a very convenient one for the bisection of an interval, may also be obtained by the method of separation of symbols as follows:

$$u_{\frac{1}{2}} = (1 + \Delta)^{\frac{1}{2}}u_0, \text{ or } u_0 = (1 + \Delta)^{-\frac{1}{2}}u_{\frac{1}{2}},$$

$$\text{also } u_{\frac{3}{2}} = (1 + \Delta)^{-\frac{1}{2}}u_1, \text{ or } u_1 = (1 + \Delta)^{\frac{1}{2}}u_{\frac{3}{2}},$$

therefore

$$\begin{aligned} u_{\frac{1}{2}} &= \frac{u_0 + u_1}{(1 + \Delta)^{\frac{1}{2}} + (1 + \Delta)^{-\frac{1}{2}}} \\ &= \frac{(1 + \Delta)^{\frac{1}{2}}}{1 + \frac{\Delta}{2}} \cdot \frac{u_0 + u_1}{2} \\ &= \left(\frac{1 + \Delta}{1 + \Delta + \frac{\Delta^2}{4}} \right)^{\frac{1}{2}} \cdot \frac{u_0 + u_1}{2} \\ &= \left(1 + \frac{\frac{\Delta^2}{4}}{1 + \Delta} \right)^{-\frac{1}{2}} \cdot \frac{u_0 + u_1}{2} \\ &= \left(1 - \frac{1}{8} \cdot \frac{\Delta^2}{(1 + \Delta)} + \frac{3}{128} \cdot \frac{\Delta^4}{(1 + \Delta)^2} - \frac{5}{1024} \cdot \frac{\Delta^6}{(1 + \Delta)^3} + \dots \right) \cdot \frac{u_0 + u_1}{2} \\ &= \left(\frac{u_0 + u_1}{2} \right) - \frac{1}{8} \cdot \left(\frac{\Delta^2 u_{-1} + \Delta^2 u_0}{2} \right) + \frac{3}{128} \cdot \left(\frac{\Delta^4 u_{-2} + \Delta^4 u_{-1}}{2} \right) - \&c. \quad (4) \end{aligned}$$

It will be seen, on reference to the scheme of notation, that this expansion is identical with formula (3).

II. Formula (2) may be applied, as an illustration, to deduce a new series of approximate values for μ_x , the force of mortality at age x .

Writing in the formula l for u , we have

$$l_x = l_0 + xa_0 + \frac{x(x-1)}{2} \beta_0 + \frac{x(x-1)(2x-1)}{1 \cdot 2} \gamma_0 + \&c.,$$

whence

$$\begin{aligned} -\frac{l_0 - l_x}{x} &= a_0 + \frac{x-1}{2} \beta_0 + \frac{(x-1)(2x-1)}{1 \cdot 2} \gamma_0 \\ &\quad + \frac{(x+1)(x-1)(x-2)}{2 \cdot 4} \delta_0 + \&c. \end{aligned}$$

If now we diminish x without limit, the expression becomes

$$-l_0\mu = a_0 - \frac{1}{2}\beta_0 + \frac{1}{1 \cdot 2}\gamma_0 + \frac{1}{1 \cdot 2}\delta_0 - \&c. \quad (5)$$

But

$$a_0 = -d_0$$

$$\beta_0 = \frac{1}{2}(\beta_{-1} + \beta_{+1}) = \frac{1}{2}(d_{-1} + d_{+1})$$

$$\gamma_0 = -d_{-1} + 2d_0 - d_{+1}$$

&c.

Therefore, by substituting these values in formula (5) we obtain the following approximations, stopping at β_0 and γ_0 respectively,

$$\mu = \frac{d_0 + \frac{1}{4}(d_{-1} - d_{+1})}{l_0} = q_0 + \frac{1}{4} \left(\frac{d_{-1} - d_{+1}}{l_0} \right) \quad (6)$$

$$\text{and} \quad \mu = \frac{2d_{-1} + 5d_0 - d_{+1}}{6l_0} \quad (7)$$

Formula (6) gives a fairly good approximation, but is not quite so simple nor so accurate (except at high ages) as the well-known value $\mu = \frac{d_{-1} + d_0}{2l_0}$. From it, however, we see that the difference between the force and the rate of mortality at any age is, roughly speaking, equal to $\frac{1}{4} \left(\frac{d_{-1} - d_{+1}}{l_0} \right)$.

On the other hand, formula (7) may be relied upon to a

greater extent, and it will be found to compare not unfavourably, as regards accuracy, with the more cumbrous formula,

$$\mu = \frac{7(d_{-1} + d_0) - (d_{-2} + d_{+1})}{12l_0} \quad . \quad . \quad . \quad (8)$$

The values of μ for decennial ages as calculated by formulæ (7) and (8) from the mortality table published in Part II of the Text Book of the Institute of Actuaries, are given in the following table, the true values being placed in column (3):

Age	FORCE OF MORTALITY.		
	(1) By Formula (7)	(2) By Formula (8)	(3) True Value
20	·00551	·00550	·00550
30	·00767	·00768	·00768
40	·00989	·00990	·00990
50	·01541	·01541	·01542
60	·02920	·02921	·02920
70	·06354	·06353	·06353
80	·14906	·14907	·14909
90	·36240	·36240	·36230

In comparing these values, the relative simplicity of formula (7) should be borne in mind.

Theoretically, formula (2) is not so suitable as (1) for the calculation of approximate values of μ , as, for the purpose, we need a formula, the most favourable application of which occurs when x is diminished without limit. Formula (1) satisfies this condition, whereas formula (2) is used to the best advantage when $x = \frac{1}{2}$. Practically, however, there seems little objection to formula (7), as the above results show.

II.—On a method of approximately calculating Net Premiums for Endowment Assurances on Two Joint Lives. By GEORGE J. LIDSTONE, F.I.A.

AN Endowment Assurance is commonly regarded as consisting of two portions, namely: a Pure Endowment and a Term Assurance, both of which depend upon the Mortality Table.

It is sometimes useful to regard the benefit from another point of view, and so to subdivide it that one portion may be

independent of mortality. This may be done by considering the endowment assurance to be made up of the following benefits:

- (a) A sinking fund assurance providing for payment of the full sum assured at the expiration of the specified term irrespective of any life contingency.
- (b) A varying temporary assurance, under which the sum assured in any one year is equal to the difference between (1) the sum assured by the endowment assurance policy, and (2) the reserve in respect of the sinking fund policy.

If a second life be now introduced it will be necessary to add a third benefit (c) and to subtract a fourth benefit (d), namely:

- (c) A varying term assurance similar to (b), but on the second life instead of the first.
- (d) A varying term assurance on the *joint lives*, the sum payable thereunder to be the reserve on (c) if the first life predecease the second, or the reserve on (b) if the second life predecease the first. [This benefit may be positive or negative, but will usually be negative].

The annual premiums for the complete benefit will be the sum of the annual premiums for the partial benefits (a), (b), (c) and (d). Now it is evident that the sum assured under benefit (d) will be numerically small, and hence the annual premium for that benefit must be very small. We may therefore obtain an approximate result by neglecting (d), and we shall then have

$$P_{xy\bar{n}} = P_{\bar{n}} + (P_{x\bar{n}} - P_{\bar{n}}) + (P_{y\bar{n}} - P_{\bar{n}}) = P_{x\bar{n}} + P_{y\bar{n}} - P_{\bar{n}} *$$

The following table shows the degree of accuracy obtained by the use of the formula. In constructing the table it has been assumed, for convenience, that the lives are of equal ages. The results in other cases will, however, be found to be similar, provided that the age at maturity is not very advanced. The error in the approximate value of $P_{xy\bar{n}}$ will usually lie between the corresponding errors in $P_{x\bar{n}}$ and $P_{y\bar{n}}$.

* The symbol $P_{\bar{n}}$ is here used symmetrically to denote the sinking fund, payable in advance, which will amount to 1 in n years. A different meaning, inconsistent with the principles of the Institute Notation, has been assigned to the symbol in Part I of the *Text-Book*.

Table of 100 $P_{x:n}$ (H^M 4 per-cent).

x	n	Approximate Value	True Value	Error.	n	x
20	10	8.673	8.680	-.007	10	20
	15	5.552	5.561	-.009	15	
	20	4.063	4.074	-.011	20	
	25	3.229	3.241	-.012	25	
	30	2.722	2.735	-.013	30	
	35	2.408	2.420	-.012	35	
	40	2.218	2.223	-.005	40	
30	10	8.839	8.844	-.005	10	30
	15	5.758	5.770	-.012	15	
	20	4.309	4.324	-.015	20	
	25	3.523	3.541	-.018	25	
	30	3.078	3.092	-.014	30	
	35	2.838	2.841	-.003	35	
	40	2.734	2.711	+.023	40	
40	10	9.137	9.148	-.011	10	40
	15	6.146	6.166	-.020	15	
	20	4.801	4.826	-.025	20	
	25	4.143	4.162	-.019	25	
	30	3.846	3.838	+.008	30	
	35	3.762	3.700	+.062	35	
50	10	9.885	9.913	-.028	10	50
	15	7.118	7.163	-.045	15	
	20	6.041	6.078	-.037	20	
	25	5.669	5.653	+.016	25	

It will be seen that, for the ages and terms most commonly occurring in practice, the formula gives a good working approximation, and an improvement can be secured with very little trouble, by mentally estimating (by means of the above or a similar Table) an interpolated value of the error.

III.—On an Approximation to the Rate of Interest yielded by a Bond bought at a Premium. By RALPH TODHUNTER, F.I.A.

A BOND redeemable in a given number of years, and bought at a premium on its redemption-price, might be dealt with in practice by writing down its book-value out of each dividend by an equal proportionate part of the premium, the remainder of the dividend being treated as interest. In the case of a bond

redeemable in n years, bearing interest at the rate j per annum (payable annually) on its redemption-price, and bought at a premium of k per unit on its redemption-price, the account (per unit) would work out as follows:

Year	Book-Value of Bond at Beginning of Year	Interest
1	$1 + k$	$j - \frac{k}{n}$
2	$1 + \frac{n-1}{n} \cdot k$	$j - \frac{k}{n}$
3	$1 + \frac{n-2}{n} \cdot k$	$j - \frac{k}{n}$
\vdots	\vdots	\vdots
$n-1$	$1 + \frac{2}{n} \cdot k$	$j - \frac{k}{n}$
n	$1 + \frac{1}{n} \cdot k$	$j - \frac{k}{n}$

giving an average rate of interest of

$$\frac{j - \frac{k}{n}}{1 + \frac{1}{n} \left(1 + \frac{2}{n} + \dots + \frac{n}{n} \right) k}$$

or

$$1 + \frac{j - \frac{k}{n}}{n+1} \cdot k$$

This expression will be found to give a very fair approximation (subject to certain limitations) to the true rate of interest yielded by the investment. Algebraically it may be obtained as follows:

The premium per unit paid for the bond must obviously be equal to the present value (at the true rate of interest) of an annuity for the number of years which the bond has to run of the *difference* between the nominal and true rates of interest. In symbols, if i be the true rate of interest,

$$k = (j - i) a_n$$

$$\begin{aligned} \therefore j - i &= ki \left\{ 1 - (1+i)^{-n} \right\}^{-1} = \frac{k}{n} \left\{ 1 - \frac{n+1}{2} i + \frac{(n+1)(n+2)}{6} i^2 - \dots \right\}^{-1} \\ &= \frac{k}{n} \left\{ 1 + \frac{n+1}{2} i + \frac{n^2-1}{12} i^2 - \frac{n^2-1}{24} i^3 + \dots \right\} \end{aligned}$$

If terms involving powers of i above the first be neglected, this equation gives, as a first approximation,

$$i = \frac{j - \frac{k}{n}}{1 + \frac{n+1}{2n} \cdot k}.$$

On inspection of the equation from which this result is obtained, it appears that the effect of neglecting the terms involving powers of i above the first is to give to i too *large* a value if k is *positive* and too *small* a value if k is *negative*. That this would be so appears from consideration of the method by which the approximation was arrived at in the first instance, for, on examination of the account set out above, it will be seen that the lower rates of interest contributing to the average are obtained (and that the larger book-values are involved) in the *earlier* years of the duration of the investment if k is *positive* and in the *later* years if k is *negative*.

An inspection of the algebraical equation will also indicate limits within which the approximation may be expected to give a good result. If $\frac{n-1}{6} \cdot i$ is small compared with 1—that is, speaking generally, if the term of the bond is short and the rate of interest yielded by the bond is not high—the value of $\frac{n^2-1}{12} i^2$ will be small as compared with that of $\frac{n+1}{2} i$.

In using the expression, the values of j and k must, of course, be calculated on the amount at which the bond is to be redeemed. For example, in the case of a $4\frac{1}{2}$ per-cent bond redeemable in 15 years at $112\frac{1}{2}$ and bought at $125\cdot93$, j must be taken as $\frac{4\cdot5}{112\cdot5}$, or $\cdot04$, and k as $\frac{13\cdot43}{112\cdot5}$, or $\cdot11938$. The substitution of these values in the expression will give $i = \cdot03012$, the true value being $\cdot03$.

It should be noticed, also, that the formula is not applicable to the case of a bond bought at a *discount*, unless there is a definite contract to redeem at a fixed date, and satisfactory security for the due fulfilment of the contract. If the bond is merely redeemable *at the option of the company*, on or after a fixed date, the bonus on redemption cannot be taken into account in estimating the yield on the investment.

The following table will indicate the degree of accuracy resulting from the use of the formula:

Approximate Rate of Interest yielded by a J per-cent Bond for 100 redeemable in n years at par and bought at 100+K, as determined (without the aid of interest tables) by the formula

$$i = \frac{j - \frac{k}{n}}{1 + \frac{n+1}{2n}k}.$$

$J(=100j)$	n	$K(=100k)$	Approximate Rate	True Rate
4	5	2.26	3.501	3.5
"	10	4.16	3.504	"
"	15	5.76	3.508	"
"	20	7.11	3.513	"
"	30	9.20	3.526	"
5	10	8.11	4.010	4
"	20	13.59	4.033	"
"	30	17.29	4.061	"
6	10	16.22	4.019	"
"	20	27.18	4.062	"
"	30	34.58	4.113	"

CORRESPONDENCE.

RATES OF MORTALITY IN WEST AFRICA.

To the Editor of the Journal of the Institute of Actuaries.

SIR,—I have lately had occasion to investigate the experience of the Office with which I am connected, in regard to European lives residing in the Congo Free State, and have the pleasure to place the results at the disposal of the Institute of Actuaries as a supplement to the recent contribution of Mr. A. E. Sprague on this subject. The remark made by Mr. King, in discussing that paper, that considerable value would attach to statistics relating to assured lives, has induced me to submit the accompanying table.

The observations are made according to policy years, and extend over a period of six years, terminating with the anniversaries in 1896, but no life was under observation for the special risk longer than five years. The lives, 141 in number, were almost entirely Belgians, mostly employed by the Congo Free State Government or the Congo Railway Company, and the average duration of the exposure to the climate risk was about $1\frac{1}{2}$ years. It will be seen that the mortality in the first year of observation was appalling, particularly in the age groups 30-39 and 40-49, and if this is a fair sample of the effect of the Congo climate on Europeans, it would appear that there is no premium practically obtainable which will cover the risk of the first

year's residence. It seems worthy of note that this experience confirms the conclusions arrived at by Mr. Sprague, (1) that the mortality in the first year of exposure to the Congo climate is far greater than in subsequent years, and (2) that the effect of the climate is much worse on the older than on the younger lives. Probably, the Congo Railway employés were not a very eligible class of lives as regards social status, and would not exercise a very intelligent care of their health, and in the case of the Free State officials, some few took part in semi-military expeditions and were killed by natives. It will also be observed that those few lives which entered upon, also completed the fifth year of observation, there being no deaths or withdrawals, nor have any further deaths been announced since the close of the observations. This seems to indicate either a process of acclimatization, or the survival of those who were from the first best able to stand the climate.

By grouping the observations of the 3rd, 4th and 5th years, the following comparison of rates of mortality is obtained.

	1st Year.	2nd Year.	3rd-5th Years.
All Ages .	15·38 per-cent	9·90 per-cent	7·62 per-cent

It has not been possible to trace to any appreciable extent the effect of the Congo climate on the subsequent mortality of the lives exposed to its influence, the withdrawal from further observation being, in nearly every case, coincident with the termination of the special risk. This was probably owing to the premiums having been paid by the employers so long as the lives remained in their service, and to the lives themselves not being able to afford to take over the policies on their return. The large number of withdrawals might, however, be taken to indicate that those who returned to Europe were not, as a rule, in failing health, as it would be expected that the policies would be kept up in any such cases. There was only one case where death occurred during the existence of the policy, and after cessation of the special risk.

Possibly an experience relating to Belgian lives may be inapplicable to our own countrymen, and I believe it is generally supposed that Englishmen withstand climatic influences somewhat better than other Europeans. Englishmen, however, do not seem to favour the Congo State much, and I cannot recollect a single English case being submitted to my Office. There are a few English lives residing in other parts of West Africa (*e.g.*, Niger Protectorate, Slave Coast, Gold Coast, Sierra Leone, and on the river Gambia) on the books, but the particulars are not readily available, nor is the number sufficiently great to be of practical use. Still, I cannot call to mind a single death amongst such lives during the last nine or ten years.

Whilst I do not suggest that the figures submitted herewith are in themselves more than merely interesting, I hope that perhaps other members may find it possible to contribute the experience of their Offices, in which case the aggregate results might prove practically useful.

I am, Sir,

Your obedient Servant,

L. K. PAGDEN.

81 Cornhill,
25 March 1897.

TABLE SHOWING RATES OF MORTALITY IN THE CONGO FREE STATE,
Observed amongst Assured Lives, and arranged in Policy-Years of Observation—

1890-1896.

Ages	FIRST YEAR					SECOND YEAR					THIRD YEAR					FOURTH YEAR					FIFTH YEAR					Rate of Mortality	Ages				
	Entrants	Deaths	Withdrawals	Existing	Years of Life	Rate of Mortality	Entrants	Deaths	Withdrawals	Existing	Years of Life	Rate of Mortality	Entrants	Deaths	Withdrawals	Existing	Years of Life	Rate of Mortality	Entrants	Deaths	Withdrawals	Existing	Years of Life	Rate of Mortality							
20-29	57	12	23	0	45.5	0.110	86	12	11	0	22.5	0.089	13	0	4	0	11	0.000	9	3	0	2	9	0.333	1	0	0	1	1	0.000	20-29
30-39	70	10	32	0	51	0.187	129	12	12	12	23	0.087	14	0	4	0	12	0.000	10	0	5	1	7.5	0.000	7	0	0	7	7	0.000	30-39
40-49	12	5	5	0	9.5	0.523	5	1	2	0	4	0.250	3	1	1	0	2.5	0.000	40-49	
50 & upds.	2	0	1	0	1.5	0.000	1	0	0	0	1	0.000	1	0	1	0	5	0.000	1	0	0	0	1	0.000	1	0	0	1	1	0.000	50 & upds.
All Ages	141	17	61	0	110.5	0.1538	63	5	25	2	50.5	0.0990	31	1	10	0	26	0.385	20	3	5	3	17.5	0.1714	9	0	0	9	9	0.000	All Ages

NOTE.—The ages are taken at the nearest age at commencement of year of observation.

The "Existing" include those lives which were still on the books at the close of the observations; in a few cases, where the special risk terminated before that date, the life was treated as observed up to the end of the policy-year for which the last extra premium was paid.

ON RATES OF MORTALITY AND WITHDRAWAL.

To the Editor of the Journal of the Institute of Actuaries.

SIR,—The interesting letters from Mr. Todhunter and Mr. Burn, upon the above general subject, which appeared in your last issue (pp. 273–80), demand from me some lines by way of response and acknowledgment.

DISCONTINUANCES.

Mr. Todhunter's letter appears to me to be an extremely valuable and opportune contribution to the discussion of this subject. I am in entire agreement with his arguments and conclusions as to the characteristic distinction between the rate of withdrawal (or non-renewal) and that of mortality, the former being in the nature of a discontinuous or periodic force, the latter of a continuous or momentary force. I am also disposed to think that the calculation of rates of non-renewal (as proposed by Mr. Todhunter) will be likely more truly to represent the real force of discontinuance than the computation of rates of withdrawal as ordinarily understood, and as worked out in my recent paper. I may add, that consideration of Mr. Todhunter's arguments, and the opportunities I have recently had of practically investigating a large body of data, lead me to attach less importance to the strict tabulation of withdrawals in their true years of duration, although I still think this a desirable course.

As regards the applicability of the Nearest Duration Method to the calculation of rates of non-renewal, I am also in accord generally with Mr. Todhunter, but subject to the reservation that, especially in the case where the "days of grace" are included in the term of the exposure, the method seems hardly to give satisfactory results in the early years of assurance. I think it will be found that some simple modification of the method can be devised, which shall give better effect in such a case to the incidence of the withdrawals in the early years, and this would be, I think, preferable (in the case where "days of grace" are included) to an application of the ordinary method over quarterly periods. It seems, however, very desirable to tabulate the withdrawals for statistical purposes at quarterly epochs during the early years of assurance, in order to give all needed data for investigations relating to the effect of withdrawal and of selection.

MR. KING'S INTER-VALUATION FORMULA FOR EXPOSED TO RISK.

Mr. Burn appears to be under some misapprehension as to the intention and purport of some remarks contained in my recent paper bearing on this interesting method. I had no intention of offering any general criticism upon Mr. King's method, and, indeed, I expressly stated at the outset (*J.I.A.*, xxxiii, 70) that, while "doubtless admirably adapted for the purposes designed by its author", this method would not (for reasons which had nothing to do with its merits) lend itself to my particular purposes. But in a special section of my paper, dealing solely with the various methods proposed for determining the ages at entry, I discussed, amongst

others, Mr. King's particular assumptions in this respect, and it was with regard to this alone that I arrived at the conclusion that, while the method "would probably give on the whole good results", it was "clearly inferior" (solely as regards the determination of the age at entry) "to the method of nearest ages" (*J.I.A.*, xxxiii, 134).

With Mr. Burn's remarks as to the general merits of Mr. King's method I am in hearty and entire accord, but I fail to see the force of his arguments, and I certainly do not agree with his conclusions, as to the possible error in the age at entry as tabulated. I agree with Mr. Burn that the deviation, in an individual case, between the true age at entry and that at which the risk is, by Mr. King's method, assumed to commence, is made up of two component parts:— (1) The interval between date of entry and nearest 31 December, which cannot exceed six months; (2) The interval between the 31 December so selected and the then nearest birthday, which also cannot exceed six months; and it is admitted, and has indeed been demonstrated by Mr. Whittall, that the date of entry and the birthday can so fall as to produce, by the combination of these two deviations, an interval of a year between the true and assumed entry ages. But Mr. Burn seems to suggest that the first of these deviations should be ignored, and the second only taken into account, in determining the error in the age at entry; and this (apparently) upon the ground that the life is *considered as entering on the selected 31 December*.

I would suggest that, in ascertaining the true deviations in the age, we are bound to give full effect to the facts, and are not entitled to shift the date of entry from the real point to an assumed or imaginary one.

Mr. Burn further says that I "saddle the whole of the error on to the age, without making any mention of the duration." But Mr. King's method, as Mr. Whittall has pointed out (xxxi, 185), "rejects durations of all sorts"; the only data tabulated being the age at commencement and the age at termination of the risk, between which the duration (inferentially) lies; and I can see no legitimate way of arriving at the difference between the real and assumed age at entry, in an individual case, other than by comparing the true age at entry and that assumed for commencement of the risk.

I would further suggest that Mr. Burn's argument confuses average and individual deviations, and that it is not strictly correct to say that *in an individual case* the date of entry is considered to be the nearest 31 December. The assumed date of entry is surely that recorded and tabulated, namely, the birthday nearest to the selected 31 December. In a great number of cases, the birthdays so chosen will tend to group equally over the six months preceding and following the selected 31 December, and we are thus entitled to say that, *on the average*, the cases are considered as entering at 31 December, but this conclusion is surely not applicable to an individual case.

It seems to me that Mr. Burn's argument could as legitimately be applied to the Nearest Age Method, pure and simple, by suggesting that the cases can all be "considered" as entering at their nearest birthdays, and that as the cases are all tabulated at those birthdays, there is really no possible error in the age at entry. But this would

be a *reductio ad absurdum*, and I think, therefore, there must be some fallacy in Mr. Burn's line of argument.

The particular case cited by him is one of those giving a deviation of a year in the age at entry, and if the case also withdraw during the period of observation, there may be a deviation of a year in the age at exit, in the same, or in a contrary, direction. The following table gives eight typical cases of extreme deviation:

Inter-Valuation Period (say) 1 January 1890-31 December 1894.

Date of Birth	Date of Entry	Date of Exit	Age at Assumed		Assumed Duration
			Entry	Exit	
(1) 30.6.'50	1.7.'90	1.7.'93	41	44	3
(2) 30.6.'50	1.7.'90	30.6.'93	41	43	2
(3) { 30.6.'50	29.6.'90	1.7.'93 }	40	44	4
{ 29.6.'50	30.6.'90	1.7.'93 }			
(4) 1.7.'50	30.6.'90	30.6.'93	39	42	3
(5) 1.7.'50	30.6.'90	1.7.'93	39	43	4
(6) { 1.7.'50	2.7.'90	30.6.'93 }	40	42	2
{ 2.7.'50	1.7.'90	30.6.'93 }			

The true age at entry is, in each of these eight cases, 40, the true age at exit is 43, and the true duration 3 years. If these ages and durations represent the correct application of Mr. King's formula, it appears to me impossible to avoid the conclusion that the age at entry, the age at exit, and the duration, can each be distorted by a full year more or less, and this does not at all bear out Mr. Burn's conclusion, that the ages taken "coincide throughout with the nearest age of the life."

It will be observed that the deviations in cases (1), (2), (3) are complementary to those in cases (4), (5), (6) respectively, and that the average of the whole gives a true result throughout, assuming that the numbers entering of each class are respectively equal.

The whole of the above remarks apply solely to cases entering during the period of observation. The large body of cases which pass through the commencement of the period as "survivors", and also those which pass through the termination of the period as cases "existing", are strictly tabulated according to their nearest ages at entry or exit, with a maximum deviation of six months, and it is only the cases entering, or emerging as withdrawals, during the period, that give rise to greater deviations either in the age at entry, the age at exit, or the duration.

As regards entrants, also, the maximum deviation is, when they pass into a new period of observation as "survivors", ingeniously and automatically, as it were, reduced to a deviation not exceeding six months. The method thus, in many respects, compares favourably with the nearest age method; but with greater possible individual deviations in respect of the "movement" of the period. Each deviation, large or small, has a compensatory deviation, with contrary

sign, and, so far as the cases with such compensating errors may be considered equal in their number and incidence, the method will give, in the aggregate, closely accurate results.

The cases of *death* are rather puzzling, because it is somewhat difficult to determine upon the proper standard of comparison to be applied in testing the accuracy of the age at exit. This may be done by comparing the age at death as tabulated ("nearest age to 31 December preceding death") with either—

- (1) The age at policy-anniversary preceding death;
- (2) The age at 31 December preceding death;
- (3) The age at birthday preceding death.

In the case of "survivors" dying within the period, Mr. King's method of dealing with the age at death seems to be the most appropriate, and the standard of comparison appears in this case to be the true age at 31 December preceding death, which gives in this class a maximum deviation of six months, and a curtate duration represented in calendar years.

In respect of cases both entering and dying within the period, the policy-anniversary standard (1) seems to be the most appropriate, but in these cases the deviation in the age at exit may considerably exceed six months. It has also been pointed out that in some cases the assumed age at entry may exceed the assumed age at death. This appears to arise from the method followed in estimating the age at death, which altogether ignores the policy-anniversary, and a simple modification of the method would seem (if otherwise admissible) to avoid this inconsistency of result. If in this particular class of cases entering and dying within the period, we take the age at death as the *nearest age to 31 December nearest to the policy-anniversary preceding death*, the age at exit, as assumed, can never be less than the assumed age at entry, and the difference will always represent (as it seems to me it should do) the true curtate duration of the policy. Under this suggested modification, however, the deviation in the age at exit, although somewhat reduced, can still exceed six months, and this seems to be unavoidable, in this particular class, by any modification of Mr. King's methods.

An alternative method would be to treat as a distinct class the cases which both enter and emerge (whether by withdrawal or death) during the period of observation; and to tabulate, in this class, the entrants, at nearest ages at entry; the withdrawals, at nearest ages at exit; and the deaths, at nearest ages to the policy-anniversaries preceding death. The deviations in the respective ages could then never exceed six months; and the death cases would be tabulated, throughout, at their true curtate durations. It seems to be permissible to treat these cases of entry and exit as a separate class, and by a distinct method, as these particular cases have (as Mr. King has pointed out, xxvii, 219) to be separately abstracted and classified, and are not derived directly from the valuation class-books; and it is, moreover, in this particular class that the maximum deviations arise.

Upon the whole, I consider that the method is admirably devised for the special purpose intended by its author, namely, the investigation of the mortality of an inter-valuation period at ages passed

through, and I cordially endorse Mr. Burn's conclusion that it is "undoubtedly the best which has as yet been suggested for this particular purpose."

TABULATION OF ENDOWMENT ASSURANCES.

Mr. Meikle, in his very interesting remarks in the course of the discussion upon my recent paper, called attention (xxxiii, 201-2) to some difficulties arising in the tabulation of endowment assurances, as regards the ages at maturity, and suggested that policies maturing at, say, 50 years of age, might, by certain applications of the Nearest Duration Method, be tabulated as maturing either at 49, 50, or 51. This is no doubt the case, but it will, I think, be found to depend upon the method adopted in estimating the ages at entry, and if the method be that of ages next birthday, or ages last birthday, or mean ages, the discrepant results indicated by Mr. Meikle will follow. I think it will, however, be found that where the *nearest age at entry* (Dr. Sprague's "commencing age") is combined with the *nearest integral duration*, the age at maturity will in all cases be represented accurately or to the nearest integer. So far as I know, the office practice as regards endowment assurances is to fix the maturity either (*a*) on a definite birthday, when the duration of the policy is fractional; or (*b*) on the policy-anniversary nearest to, or else next preceding, the selected birthday, when the duration is an integral number of years. In the case (*b*) it is evident that the deviation of the age at maturity (the duration being integral) will be identical with that of the age at entry, and if the latter is the nearest age at entry, the age at maturity, as tabulated, will be the nearest age at maturity. This may be 49 or 51 in the case of a policy *nominally* maturing at 50, but only where the policy really matures at a point nearer to 49 or 51 than to 50.

In the case (*a*) of payment on attainment of a birthday, if we assume that the office age next birthday is x and the true age at entry is either $(x-\phi)$ or $(x-1+\phi)$, where ϕ is a fraction $< \frac{1}{2}$, we have the following results respectively:

Policy payable on $(x+t)$ th birthday.

(1) True age at entry	$(x-\phi)$ or $(x-1+\phi)$.
(2) Nearest age at entry	(x) ,, $(x-1)$.
(3) True duration	$(t+\phi)$,, $(t+1-\phi)$.
(4) Nearest duration	(t) ,, $(t+1)$.
(5) True age at maturity	$(x+t)$,, $(x+t)$.
(2)+(4)=(6) Tabulated age	,, $(x+t)$,, $(x+t)$.

The assumed and true ages at maturity will thus be identical. The only case of possible doubt would be where the true age at entry was exactly half-way between two birthdays, so that $\phi = \frac{1}{2}$; in that case, care would have to be taken that, if the nearest age at entry is written up, the nearest duration is written down, and *vice versa*.

I do not imagine that Mr. Meikle is referring to this exceptional case, but rather suppose that there may be other varieties of office

practice, as regards duration and epoch of maturity, which are unknown to me, but which his much larger experience has brought under his notice, and in which the discrepant results referred to may arise. It would, however, appear that where the assurance matures either on a birthday or on a policy-anniversary, the combination of the nearest age at entry and the nearest duration will always give the age at maturity correctly, or to the nearest age.

DUPLICATES.

Mr. Meikle also refers to the fact that, by the Nearest Duration Method, cases upon the same life may be recorded as both "existing" and terminating by death. This arises where the period of observation is closed on the *policy-anniversary* in a certain calendar year, so that, in the case of two or more assurances upon one life, the assured, if dying between the policy-anniversaries in the closing calendar year, would appear as "existing" under some of the assurances, and as "dead" under others.

This difficulty appears, however, to arise solely from the limitations fixed in ascertaining the data, and not from the particular method of tabulation adopted.

I am, Sir,

Yours obediently,

THOMAS G. ACKLAND.

Croydon,

1 June 1897.

ON THE CALCULATIONS OF GEORGE BARRETT IN THE EARLY PART OF THIS CENTURY.

To the Editor of the Journal of the Institute of Actuaries.

SIR,—Some calculations of George Barrett have recently passed through my hands, in my capacity as one of the Librarians of the Institute, containing some original details and notes which, I think, may be interesting to the readers of our *Journal*. The calculations have been placed at the disposal of the Institute by the London Life Association, through the kindness of Mr. C. D. Higham.

As is well known, Barrett introduced the use of Commutation Columns into England. Very little is known of him beyond the particulars given in a paper by De Morgan in vol. iv. of the *Journal*. In the rough calculations under notice, by far the greater part are folio sheets, covered both sides to the edges with figures, and as the final values are alternate with the subsidiary elements of the calculations, the arguments being very indistinct, no purpose would be served in having the sheets bound into volumes, as the Institute Library already possesses Barrett's Final Tables based on these calculations, hereafter referred to.

Barrett was assisted by his sister and a niece in the work, though everything was finally checked by himself and initialled. He does not appear to have been so happy in a male helper, as there is a remark at the foot of one page—"Note. The person whom I got

"to finish this paper made a blundering job of it; he added instead of subtracting."

Readers of the *Text-Book* will remember that Barrett's Commutation Columns, equivalent to our D and N columns, were marked A and B, where $B_x = \sum A_x$. Barrett was in the habit of referring to them as Betsey and Nancy. There is a frequent reference, in brief directions to his assistants, to "Betsey's column and Nancy's first difference", and in one place, "those numbered with Black are the sum of the last addition of Betsey's, and will therefore be a proof of her additions being right if they agree." It may have been that Barrett's sister and niece were respectively named Betsey and Nancy, he naming the columns after them.

Barrett's great labour was a table of Commutation Columns for every possible combination of three joint lives, according to the Swedish Table of Observations (lives in general) at 4 per-cent.

The rough calculations present an enormous mass of figures on rough paper, evidently ruled by Barrett himself. The logs of the values of the A column are formed by the method described in the appendix to the second volume of Baily's work on Annuities, p. 42. The logarithms were then transferred to a second set of sheets in the same order, and the antilogs entered under each value.

The second set occupies 410 folios foolscap.

There are some dates on this batch, the first being, "The first prime was finished by me 14th Oct. 1802, G. BARRETT." The first prime being all the combinations of ages having differences 0 to n , where n had every value from 0 to 95. On the last folio is written—"Finis, G.B. Witness, Mrs. Purser, July 29 1804." With the very human addition, "When clock struck 6 in the evening"; also, "Finished transcribing the papers, Nov. 5th 1804, G. BARRETT." "Finished re-examining the papers, June 11th 1806.

"ANN-NALL BARRETT,

"GEORGE BARRETT."

Referring in the list of tables to the table for three joint lives, Barrett says "although computed at a rate of interest 4 per-cent, it can easily be made applicable to one of 3 or 5 per-cent by a method of employing it, which is well known to those conversant with the subject." A reliable method of easily making this transition would be a boon to Actuaries, but it does not appear that Barrett had in view any very exact method, for one of the Offices (the Provident), in answering his appeal for subscriptions, remarks that the approximation from his 4 per-cent Joint-Life Table to Northampton 3 per-cent, would be no more accurate than the methods in general use. It was a great pity that the Swedish Observations were used for this Table. Had the Northampton Table been used, the support offered by the Life Offices would, undoubtedly, have been much greater.

In Baily's paper there is a description of Barrett's device for forming the A column by means of movable slips having the values of λb_x and $\lambda(1+i)^n$ written on them. Barrett calls them rods, and there are the two bundles which he brought up to London to show to Baily. It does not appear that he used this system very extensively, as there is a note on one folio that he gave up the idea of using the

rods (on that particular table) as the resulting values were not so easily checked as when the table was formed in the usual manner. There are no papers which show that he used them for the three life table for which they were described as being specially applicable.

Barrett was very anxious to have the table so laboriously constructed, printed. It was out of the question for him to publish them at his own risk, and his attempt to get them published by subscription failed. He was never able to obtain enough names to warrant the commencement of the work of printing, and he refused Baily's suggestion to print part only of the tables.

The original draft of his prospectus is here with a blank dedication, afterwards filled in with the name of Lord Egremont, in whose service Barrett was a sort of steward.

This draft contains a paragraph stating his intention that "the whole of them will, after the printing is completed, be deposited in some public *Institution* in the Metropolis, not only for the benefit of those who may be disposed to pursue the enquiry further, but likewise for the purpose of correcting any error that may inadvertently have occurred in the printing. A similar method has been adopted with respect to some tables of logarithms, and is particularly desirable in all large works of this kind."

Barrett, however, changed his mind as to the desirability of letting his manuscript go, and the paragraph did not appear in the prospectus as printed. The number of guineas the work was to cost is left blank in the draft.

Together with the draft of the prospectus are several sheets of solutions of various problems in life contingencies, intended to accompany the tables. There is an outline of a method for tabulating the data for forming a life table. The schedule shown is identical in form with one used by Mr. Woolhouse for the Seventeen Offices Experience. The tables, as transcribed ready for printing, were bound up into six thick folio volumes, "handsomely bound", says Barrett. These volumes were presented to the Institute some time ago, also by the London Life Association.

Mr. Higham informs me that the Tables were purchased by the London Life Association on 6 July 1827, from Mr. Charles Babbage, for £150, probably for reasons of public spirit. Mr. Babbage to be allowed copies of De Parcieux's Tables for scientific purposes, but in case Mr. Babbage should connect himself with any Life Assurance Society, and be desirous of applying such Tables to the use of that Office, Mr. Babbage to pay to the Society the sum of £50 for the privilege of doing so.

I am, Sir,

Your obedient Servant,

FREDERICK SCHOOLING.

Prudential Assurance Company,

21 January 1897.

THE INSTITUTE OF ACTUARIES.

The Diamond Jubilee.

ADDRESS TO HER MAJESTY QUEEN VICTORIA, ON THE
COMPLETION OF THE SIXTIETH YEAR OF HER REIGN.

THE Editor has been good enough to suggest that a few words of Preface might fitly introduce the appended Reprint of the Address presented by the Institute to the Queen.

A suggestion may obviously travel through the entire gamut of appeal according to the position of its author and the nature of its subject, and, upon both grounds, I feel that the gentle hint of the Editor amounts virtually to a command.

With respect to the Lady whom we addressed, the most deftly finished language would prove intrusive and ineffective compared with the intrinsic worth of the occasion; and, indeed, our feelings upon the subject find their most appropriate utterance in Thomson's simple, and therefore adequate, line—

“Come, then, expressive Silence,
Muse Her praise.”

It is of deep interest, it may be passingly observed, to compare the authentic portraiture of the Elizabethan Reign, which Bishop Creighton has luminously presented in a recent volume, with the finer features and larger accomplishments of the Victorian Era.

With regard to the Body from which the Address emanated, the Institute is obviously a product of that large-minded and intellectual development, in closest union with social progress, which has eminently characterized Her Majesty's Reign. And, not for the purpose of exalting our position, but essentially with the object of indicating, in a specific instance, one of the Scientific and Social exhibitions of the character of Her Administration, we may justly point to the Institute of Actuaries as amply expressing, in its sphere, the constraining purport of Her Rule in the serviceable application of Scientific Method to an important series of Social facts, and in the establishment, on this basis, of a national and enlightened System of Assurance, destined more and more minutely to realize Her Majesty's Ideal in the Social advancement of Her Kingdom.

T. E. YOUNG.

TO THE QUEEN'S MOST EXCELLENT MAJESTY.

MAY IT PLEASE YOUR MAJESTY,

WE, THE PRESIDENT, COUNCIL, AND MEMBERS OF THE INSTITUTE OF ACTUARIES, Incorporated by your Royal Charter, beg leave humbly to approach your Majesty with our most respectful and hearty congratulations on the auspicious occasion of THE COMPLETION OF THE SIXTIETH YEAR OF YOUR MAJESTY'S PROSPEROUS AND BENEFICENT REIGN.

In common with the rest of your Majesty's subjects, we recognize with profound satisfaction the growth, during the last sixty years, of National greatness and all that tends to the consolidation of Empire; and we may be permitted to express the confident belief that nothing in the history of your Majesty's reign will be more memorable, or be found to have done more to strengthen the State, than the progress which, under your Majesty's fostering care, has been made in the development of Institutions tending to create and encourage provident habits. Among these are included Savings Banks, Building Societies, and the various forms of co-operation by aid of which small means are accumulated or economized; Friendly Societies providing against sickness and the wants of old age, with their wide-reaching ramifications and subtle influences for good; and, lastly, those more prominent and, in respect of their magnitude, more important Institutions—the Life Assurance Societies of the Kingdom—the guidance of which is our special privilege and concern.

In the grant to the Institute of Actuaries of a Royal Charter in the year 1884, by the terms of which there is confided to us the duty of extending and improving the data and methods of actuarial science, we gratefully acknowledge the increased dignity which has been bestowed on our profession, and we rejoice in this further proof of the anxious solicitude which your Majesty has ever shown, not only for the extension and development, but also for the wise direction on sound principles, of Provident Institutions calculated to promote the welfare of your people.

And, finally, we humbly pray that your Majesty's reign may be continued, to the great advantage and happiness of your Majesty's loyal subjects.

Given under the Common Seal of the said Institute,

T. E. YOUNG, *President.*

FRANK B. WYATT, } *Honorary*
A. F. BURRIDGE, } *Secretaries.*

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JOURNAL

OF THE

INSTITUTE OF ACTUARIES.

On Lost Policies, Certified Copy Policies, Certificates of Title, Possession of a Policy, Notice, Bankruptcy, and some other practical considerations with regard to the titles to Policies.
By T. B. SPRAGUE, M.A., LL.D., *Manager of the Scottish Equitable Life Assurance Society.*

[Read before the Institute, 26 April 1897.]

Lost Policies.

THE subject of lost policies is one that is constantly coming before Life Insurance Companies in different ways, and it is one regarding which it seems very desirable that they should all act on the same principles. At present, however, I believe there is a good deal of diversity of practice; and I think it may therefore be useful if I submit to my professional brethren the conclusions to which I have been led by my business experience and careful consideration of the various points involved.

I will first consider what should be done in the case of a policy which has become a claim by the death of the assured; when his executors claim payment, but cannot produce the policy, or give any satisfactory explanation of its absence, and the Office has received no notice of any dealing with the policy. It may, I think, be inferred from the judicial decisions relating to policies and other documents, that possession of a policy is not essential

for the purpose of establishing a title to it; and if there is no reason for suspecting that the policy is held by some person who has not given notice to the Office, it is clear that the claim must be paid, under proper precautions. The Office should have a statement by the executors, that a thorough search has been made for the policy, and that they are not aware of any dealings with it by the assured; and the claim should not be paid until after the lapse of a reasonable time, so as to afford an opportunity to any unknown holder of the policy, to come forward and claim payment. In particular, if by the terms of the policy the claim was made payable at a certain date, for instance, three or six months after the death of the assured, or after evidence has been produced of the death; and, by a regulation of the Office, adopted since the issue of the policy, the claim is made payable at an earlier date; then it will be prudent for the Office to delay payment until the date mentioned in the policy. Lastly, the Office may fairly insist on having an indemnity embodied in the discharge of the policy, against all losses, expenses, claims, and demands, which it may incur or sustain, under or by virtue of the policy, or in consequence of its non-production; and this indemnity should bear a 6d. stamp. If it should afterwards turn out that the assured had assigned the policy to some person who had not given notice to the Office, such assignee would have no claim against the Office, but would be entitled to claim payment from the executors out of the estate of the assured. The Office might be put to some expense; but if they have a proper indemnity, they can claim repayment of such expense from the executors.

It more frequently happens that the loss (or destruction) of the policy is made known to the Office in the lifetime of the assured, either upon the occasion of his asking the Office to make a payment in respect of the policy, or in consequence of his asking for a duplicate policy. If, when he applies for a payment of any kind—for a loan on security of the policy, or for the cash value of a bonus addition, or for the surrender value—he cannot produce the policy, his title is defective; and the question arises how the defect is to be cured. I believe that, in the circumstances supposed, some Offices would absolutely refuse to grant the assured a loan; but this, I think, is carrying caution too far, and is not fair to a man who gives a satisfactory explanation of the absence of his policy. In such a case, the Office should, I think, not consider simply how it may avoid all risk, but should take pains

to ascertain exactly what is the risk it will run, and how this may be satisfactorily met. I am not able to say how the application for payment of a bonus in cash is generally dealt with; but when an assured wishes to receive the surrender value of a policy which is said to be lost or destroyed, I believe it is not at all uncommon to delay payment, until the policy has become forfeited by non-payment of the premium, and then to make a so-called *ex gratiâ* payment to the assured. I doubt whether this precaution was ever of much real use, except so far as it allows time for any unknown assignee of the policy to come forward and give notice of his claim, by paying the premium, or otherwise. However this may be, the Non-forfeiture Regulations which are now so common, must have greatly lessened the value of the precaution; and it is, of course, absolutely useless in the case of an Office which states in its prospectus, that the fact of the policy becoming void by non-payment of the premium, does not involve the forfeiture of the surrender value; because in this case, no *ex gratiâ* payment can be made, but the surrender value must be paid to the person legally entitled to it.

It seems to be unnecessary to make any distinction between the three cases above mentioned; and whether the assured applies for a loan, or a cash bonus, or a surrender value, he may, I think, be recognized as the owner of the policy, and the desired payment made to him, if the Office has received no notice of any dealing with the policy, and is satisfied with his statements. The Office is clearly entitled to have formal evidence, by Statutory Declaration or otherwise, of the circumstances under which the loss or destruction of the policy took place, and of the searches which have been made for it; also satisfactory evidence that the assured has not assigned the policy or dealt with it in any way. It is also entitled to an indemnity; and its solicitor's charges in connection with the preparation of the declaration and the indemnity, should be paid by the assured and not by the Office. The only risk which the Office will run, is that the policy may have been handed, either with or without an assignment, or memorandum of deposit, to a person who has not given notice to the Office. In such a case, the holder of the policy has been guilty of negligence in not giving notice; and if the Office has not been guilty of any negligence, but has taken every proper precaution that prudence suggests, then the title of the Office will be preferred to that of the holder of the policy; and the payment it has made will be held to be valid, as against him.

Whether the Office is asked to make a cash payment or not, any application for the issue of a duplicate policy should, in general, be absolutely refused. Whatever evidence of the loss, or destruction, of a policy is given, it is generally still possible that it may have been dealt with in some way, without notice having been given to the Office; and most awkward questions might arise, and the Office might be involved in serious responsibility, if there were two policies, the original and the duplicate, in the hands of different parties. I am therefore of opinion that it is only in very rare and exceptional cases that a duplicate policy should be issued; for instance, if the policy has been lost through no fault of the assured, or his agent. Two cases of this kind have occurred in the course of my business experience. On one occasion, the assured stated that he had never received his policy; and after waiting several months to see if it could be traced, we were finally satisfied that it had never reached him, but must have been lost through the negligence of some person in the employment of the Office. In the other case, we were satisfied that the policy had been lost while in the custody of the Office. There are a few other cases in which an Office might, with perfect safety, issue a duplicate policy; for instance, when a policy has been partially destroyed by fire or otherwise, and the fragments are produced and given up. But it is very seldom that conclusive proof of the destruction of the policy can be given; and even then, the preferable course appears to be to issue a certified copy, with a memorandum written on it, stating the facts relating to the destruction of the original. In such a case, the certified copy will answer all the purposes of the original policy. In the more common case, where a policy is said to be lost, it sometimes happens—perhaps more often than not—that the policy is subsequently found, on a more thorough search being made for it. This happened lately in my own Office, shortly after the assured, and the assignee of a policy, had made a statutory declaration that both policy and assignment had been inadvertently burnt with other papers. In another case, the assured made a declaration that his policy was lost, and that he could not find it, although he had made careful search; but on his death his widow found the policy, and wrote to enquire if it was still in force.

Certified Copy
Policies.

The same objections do not, in my opinion, apply to the issue of a certified copy of a policy under proper conditions; and I will now consider what those conditions should

be. I believe it is (or was) the custom of a good many Offices, to regard the issue of a certified copy as practically equivalent to the issue of a duplicate policy. They only issue such a copy after full enquiry, and after the assured has made a statutory declaration; setting forth to their satisfaction all the circumstances relating to the loss of the policy, and the searches that have been made for it; and containing a statement that the assured has not dealt with his policy in any way. When a certified copy has been issued in these circumstances, the assured will naturally assume that, having done all that he was called upon to do, he has fully satisfied the Office, and that he and his representatives will never be required to do anything more to account for the non-production of the policy. On the other hand, the Office would, I think, find it very difficult to re-open the question, and call for further explanation, or even for an indemnity, when asked at a future time to make a payment in respect of the policy.

If it is desired to prevent the assured from making such assumption, and to reserve the rights of the Office, a memorandum might be written on the copy policy, stating the circumstances under which it was issued; and a copy of the statutory declaration attached; so that any person into whose hands the policy may come, will know exactly what the Office has done to satisfy itself. I am not, however, satisfied that this would so reserve the rights of the Office as to enable it at a future time to call for further evidence, or for an indemnity; and I will now describe what I consider to be a preferable course of procedure. The mere issue of a certified copy of a policy does not, by itself, confer on the assured any right that he did not possess previously, or alter in any way the responsibility of the Office. Suppose, for instance, that the assured, not having his policy in his possession, wishes to have a certified copy of it, to place with other important documents, so as to show the exact terms and conditions on which his life is insured. The policy itself may, perhaps, be deposited with the Office, or assigned to a creditor, in security of a loan; or it may be held by the trustees of a marriage settlement; and in each of these cases a certified copy may be issued without any other formality than a letter from the assured, stating his reason for wishing to have a copy of his policy. In order, however, to prevent any misunderstanding, or any improper use being made of the copy, it will be prudent for the Office to place a memorandum on it to the following effect:—"The possession of
 "this copy policy confers no rights on the assured or any other

“holder of it.” The copy should state what stamp the original bears, and should itself bear a 1s. stamp ; as otherwise it will not be available for any purpose whatever ; see the Stamp Act, 1891, §14 (4). If the original policy is stated to be lost, but the assured has no present intention of dealing with it, and does not ask to be recognized as the owner of it, precisely the same course may be followed ; the only difference being that the assured should be asked to state fully in writing the particulars of the supposed loss, and of the searches that have been made for the policy ; and when this is done, a certified copy, bearing the above-mentioned memorandum, may be issued without more ado.

It seems proper to charge an Office fee, say 5s., for the issue of a copy policy ; and whenever the Office grants a loan on security of a lost policy, or pays a cash bonus, as above described, it will be proper to issue a copy policy, if this has not already been done ; such copy being held by the Office in the former case, and by the assured in the latter after the surrender of the bonus has been endorsed on it.

Such a copy will, of course, not take the place of the original ; and a difficulty will generally arise if the assured wishes to sell his policy, or to obtain a loan on security of it from his bankers or others. What, then, should be done when the assured, not being satisfied with a certified copy of his policy, and having been refused a duplicate policy, wishes the Office to give him a document which will satisfy third parties as to his title ? This raises the general question of certificates of title.

Certificates
of title.

In general, a Life Office will not investigate the title to a policy which it has issued, until it is called upon to make some payment in respect of it. When the holder of a policy which has been dealt with, applies to the Office for a loan on security of it, or for a cash bonus, it becomes necessary to investigate his title ; and if this is found satisfactory, the loan is granted, or the cash bonus paid to him ; the only difference in practice being, that in the latter case the Office pays the expense of investigating the title, but in the former the holder of the policy is required to pay such expense. In neither case is it usual to give the policyholder any certificate that his title is in order ; and if he wishes to assign his policy to a third party, a fresh investigation of title becomes necessary. In these circumstances, it seems to me very reasonable that the policyholder should wish the Office to give him a certificate, stating that they have investigated his title and found it satisfactory ; and I can

see no good reason why the Office should refuse to do so. I have, however, found lawyers to be generally very reluctant to adopt a new idea of this kind; they fear that the Office might run some risk by granting such a certificate, but I have never found them able to say exactly wherein the risk consists. I accordingly recommended my Directors to adopt the system of granting certificates of title; and after full consideration, they resolved to do so, and the system has now been in operation for several years, without our having seen any reason to regret its adoption. The title is investigated by our Solicitor, of course at the expense of the assured or other claimant; and if found satisfactory, a certificate of title is issued. The wording of the certificate varies according to the special circumstances of each case, but the following may be taken as a specimen:—

“ *Policy*....., *on the Life of A. B.*

“ Whereas C. D. has claimed to be absolutely entitled to the
 “ above mentioned policy, and has deposited with the Society the
 “ Documents relating to the title to such policy which are
 “ included in the schedule hereunder written; and whereas the
 “ Directors of the Society have not received notice of any dealing
 “ with the said policy, except by the documents aforesaid; now I
 “ hereby certify that the said Directors are satisfied that the said
 “ C. D. is at present the absolute owner of the said policy.”

If it is the practice of an Office to grant certificates of title in the circumstances above described, there seems to be no good reason why it should not also grant a certificate of title, when a policy has been lost or destroyed, and the Office has satisfied itself as to the title of the assured. In this case, the certificate may be in the following form:—

“ *Policy*, *on the Life of A. B.*”

“ Whereas A. B., who effected with the * * * * *
 “ Life Assurance Society a policy on his own life No.
 “ and dated for the sum of
 “ pounds, alleges that the said policy has been lost by him, and
 “ has furnished to the said Society a statutory declaration
 “ accounting for such loss to the satisfaction of the Society;
 “ And whereas the Society has received no notice of any dealing
 “ with the said policy; Now it is hereby certified that no
 “ objection to the title to the said policy by reason of its non-
 “ production, will at any time be made by the said Society;

“ provided always that this certificate is given without prejudice
 “ to the right of the Society to demand and obtain an indemnity
 “ in respect of such non-production, before paying any moneys
 “ on foot of the said policy.”

Even if the Office is not in the habit of granting certificates of title in other cases, it may still be willing to grant a certificate such as the above in the case of a lost policy.

After the issue of a certificate of this kind, the Office could not object to the title of the assured or a subsequent assignee, on the ground of the non-production of the policy; but it could still object on any other ground: and if it should be subsequently ascertained that the assured had made a false declaration, and that the policy, instead of being lost, was in the possession of an assignee or deposittee, the Office may object to the title of the assured or the subsequent assignee, on the ground that it had received notice of a dealing with the policy; and then proceed to investigate the competing claims in the usual way.

Possession of a Policy. The above remarks will, I think, sufficiently indicate the principles upon which we should act, when we are asked, either by the assured, or by an assignee, to make a payment in respect of a missing policy, or to grant a duplicate policy; but my subject naturally leads me to consider some other points. For instance, what rights does the possession of a policy confer on a person who has not a regular title to it? and what course should the Office take, when, the assured being dead, it has notice that the policy is held by a person who declines to part with it, but is not in a position to give a discharge for the policy money?

Case 1.—If a man on his death-bed, wishing to bequeath to a friend or relative, the money that will be payable under a policy, hands the policy to him; this will not be sufficient to give such person any interest in the policy, and the executors of the assured can compel him to deliver up the policy to them.

Case 2.—The policy may be a settlement policy, effected for the benefit of the assured's wife and children, so that he has no power to deal with it for his own purposes; but he may nevertheless have obtained a loan, on the supposed security of the policy, from some person who does not understand its nature, and who demands repayment of his loan before he parts with the policy.

In both these cases, it seems probable that the Office might safely pay the policy money to the executors, without delivery of the policy; of course taking an indemnity from them against all

other claims and expenses. But the Office may very properly decline to run any risk in the matter, and insist on the executors taking the necessary steps to obtain possession of the policy.

Case 3.—Another possible case is, that the policy may be in the possession of a solicitor, who claims a lien over it for his costs. According to the text-books, this case differs from the foregoing in this respect, that the solicitor cannot be compelled to deliver up the policy, except upon his costs being paid to him, or lodged in Court if his account is disputed. In these circumstances, if the executors and the solicitor cannot come to terms, the proper course will probably be to give notice to the solicitor, that the claim will be paid to the executors at the end of a certain time, unless he takes some steps to prevent the Office paying.

Case 4.—If the assured has fraudulently assigned his policy, first to A and afterwards to B, it seems to follow from the provisions of the Policies of Assurance Act, 1867, that the one who is the first to give notice to the Office, will have the better title, whether he has the policy itself or not.

The Act says that “the date on which such notice shall be received” (that is, a written notice of the date and purport of an assignment made after the passing of the Act, 20 August 1867), “shall regulate the priority of all claims under any assignment.” I am inclined to think that in this case, if the assignee who holds the policy, was not the first to give notice to the Office, he can not be compelled to give up the policy, and that the Office should take the course suggested above for *Case 3*.

Case 5.—In the case mentioned (p. 376) under the first heading of this paper, the assured had been supplied with a certified copy of his policy, and had handed this copy to an assignee, from whom he had obtained a loan on security of the policy. The widow, who held the original policy, claimed the policy money; but the assignee, on being informed by the Office of the widow’s claim, soon satisfied her that his claim was preferable, and got the original policy handed to him.

Case 6.—We have seen in the foregoing cases that the mere possession of a policy, may give the holder no rights over it; but in England cases may occur in which possession of the policy confers valuable rights on a person who has no documentary evidence of title. The following is such a case which lately came under my notice. The assured, A, applied to B, for an advance to pay a debt, and promised to execute an assignment of his policy in favour of B. The policy was handed to B, who thereupon

paid the debt on behalf of A ; but when an assignment of the policy was prepared and sent to A, he refused to execute it. Meantime B had given notice to the Office of his claim over the policy. Subsequently A became bankrupt and died, and the trustee in bankruptcy claimed payment of the policy money ; alleging that B had no valid title, although he held the policy. Ultimately, the trustee, having taken legal advice, was satisfied that B had a valid charge ; and the policy money was paid on the joint receipt of B and the trustee, a separate cheque being sent to B for the amount of his claim. In Scotland a deposit of title deeds gives a lender no charge over the property.

Notice. Another point on which I will make a few remarks is that of notice to the Office. We have seen above that in many cases, the presence or absence of such notice will make a very great difference, when the Office is asked to make a payment, and the policy is not forthcoming. The importance of giving notice does not depend on the provisions of the Policies of Assurance Act quoted above, but rests on general legal principles. If, before the Act was passed, the Office had notice from a person, A, that he claimed an interest in a policy it had issued, the Office could not safely have made a payment in respect of that policy to any other person, without informing A previously, and giving him the opportunity of coming forward and proving his claim. The above mentioned Act has, I imagine, made no change in this ; but it has this effect, that, of two competing assignments made after 20 August 1867, that one shall prevail of which the Office first received statutory notice. I do not think that this is a desirable change in the law, as it may now happen that the claim of the assignee, whose assignment is prior in date, and who holds the policy, will be postponed to that of another assignee, whose assignment is of later date, and who has not possession of the policy, if from any cause the Office received statutory notice of this latter assignment before the other.

It has been decided that a memorandum of deposit, or an agreement to assign, is not an assignment within the meaning of the Act. A policy having been deposited with A, and an agreement to assign having been subsequently given to B ; and B having given notice to the Office before A did so, it was held that A's title must be preferred to B's. The decision would apparently have been different if B had had a regular assignment, and had given due notice to the Office. This distinction, as regards the effect of notice, between an assignment and a memorandum of deposit, seems to me a very undesirable one.

The Act provides that the notice to the Office must contain the date and purport of the assignment: and it may hereafter become necessary to decide what is the meaning of giving notice of the purport of an assignment. If a policy is assigned to the trustees of a marriage settlement, is it necessary to give notice of the exact trusts on which the policy is held? If it has been assigned to secure a debt, is it necessary to state the amount of the debt in the notice? It seems that the notice should, at all events, state whether the assignment is an absolute one, or only, in Scotch phraseology, an assignment in security (that is, a mortgage); but I believe that very commonly even this information is not given in the notice. In Scotland notice sometimes takes the form of sending to the Office a copy of the assignment; and in this case, there can be no doubt that notice of the purport of the assignment has been given.

The same Act provides that an insurance company shall be bound to acknowledge receipt of notice on payment of a fee of 5s. This is a very proper provision; but it would have been better if the Act had required the notice to be given in duplicate, which is the usual custom, and is convenient both to the Office and to the assignee. It is very commonly supposed by an assignee that, if he has given notice, and the Office has acknowledged his notice, his title is thereby admitted; and it seems therefore desirable to add to the acknowledgement a note such as the following:—

“This acknowledgement is given in conformity with the Policies of Assurance Act, 1867; and the Society, in giving it, makes no admission, and expresses no opinion whatever, in regard to the validity or effect of the deed mentioned in the notice. That deed, along with all other deeds (whether cancelled or not), that relate to the policy, must be produced to the Society when it is asked to make a payment under the policy: and should therefore be carefully preserved. All deeds of assignment must be properly stamped before the sum in the policy can be paid. See Act 31 Vict. Cap. VIII, §§ 19 and 20.”

When an Office has received notice of an assignment of one of its policies, it seems desirable to send to the assignee, as well as to the assured, any non-forfeiture notices that it is the practice of the Office to issue. This is done in my own Office; and the practice seems to give great satisfaction to assignees.

When a person is about to purchase a policy, or grant a loan on security of it, he will, as a matter of prudence, enquire of the

Office whether notice has been received of any dealings with the policy. The Office is under no legal obligation to answer such enquiry, but it cannot reasonably decline to do so. It may, however, after giving the desired information, say, "this information is given as a matter of courtesy only, and the giving of it is not to be construed as an admission of liability if the information should be in any respect incorrect."

Registration of Assignments. In Scotland a solicitor will sometimes send the original assignment to the Office, with a request that it may be registered; and it used to be the practice of Offices to write on the assignment an acknowledgement to the following, or a similar, effect: "Intimated of this date to the Society." When, in one case, this practice came under the notice of the Inland Revenue Authorities, some years back, they pointed out to the Office concerned that, by registering a deed in this way, they became responsible for seeing that it was duly stamped. The enactment to which they referred is now § 17 of the Stamp Act, 1891, which is as follows: "If any person, whose office it is to enroll, register, or enter in or upon any rolls, books, or records, any instrument chargeable with duty, enrolls, registers, or enters any such instrument, not being duly stamped, he shall incur a fine of £10." The production of the original assignment must, I think, be considered as notice of its contents; and these should be carefully noted by the Office for its own protection, although it carefully abstains from doing anything that might be interpreted as registering the deed. I do not think that an Office is bound to acknowledge a notice which is served on it in this way. But, if it receives also a certified copy of the deed, there can be no objection to making a note to that effect on the original deed; and this, by virtue of the Transmission of Moveable Property (Scotland) Act, 1862, constitutes a good acknowledgement.

Bankruptcy. In connection with the subject of notice, the effect of bankruptcy naturally comes under consideration. It is now settled law that in England an assignee in bankruptcy is bound, like other assignees, to give notice to the Office; and if he does not do so, and the Office is not aware of the bankruptcy, it can safely deal with the assured, as if he were not bankrupt. In Ireland the case is very different, as notice of the bankruptcy in the *Gazette* is held to be notice to the Office. It is therefore necessary in that country, before making any payment to the assured or his representatives, to ascertain whether he has been

bankrupt; and if the Office neglects to do this, or is deceived by the assured, and pays him, let us say, the surrender value of a policy, it will be liable to pay the amount a second time to the assignee in bankruptcy; and this will be the ease, even though the payment to the assured was made so soon after the notice in the *Gazette*, that it was not possible for the Office to see that notice. In Scotland also, "the confirmation of the trustee in a sequestration, operates as an intimated assignation of all debts due to the bankrupt"; but by a special provision in the Bankruptcy Act of 1856, if a debtor (for instance, an Insurance Office,) makes a payment to the assured in ignorance of the sequestration, he cannot be required to pay over again to the trustee: (Harvey's Lectures on Insurance Law in the *Transactions of the Actuarial Society of Edinburgh*, vol. iii, p. 295). This provision seems to me to put the matter on its proper footing; and in England, if the Office is aware of the bankruptcy, but has had no formal notice from the assignee in bankruptcy of his claim, I should doubt whether it could safely make a payment to the assured. In connection with this point, it may be useful to mention that I lately saw the report of a decision of a Canadian Court (in the case of *Crawford v. Canada Life Assurance Company*) by which the Office was held liable to pay the surrender value of a policy to the trustee in bankruptcy, although it had previously paid the amount to the assured. In that case, the trustee had written to the Office, notifying them of the bankruptcy, and claiming a certain policy on the bankrupt's life, but not the one which was subsequently surrendered.

It is a very common notion that policies may be transferred from one holder to another like shares—that, as it is the duty of the Secretary of a Company to receive transfers of its shares, and, if everything is in order, to enter the name of the transferee in the list of shareholders, in the place of the transferor's; so it is the duty of the Secretary of a Life Office to receive any assignment of a policy that is sent to him, and to enter the name of the assignee in the Office books as the holder of the policy. In accordance with this idea, Life Offices are often asked in whose name a particular policy now stands. My experience has led me to form the opinion that in such cases the best plan is to state plainly that policies can not be transferred like shares; that the Office is not concerned with the title until it is asked to make a payment in respect of the policy; that in that event, the claimant's title will be submitted to the Office solicitor;

"Transfer" of
a Policy.

and that, in the meantime, the Office will not undertake to read, much less to express an opinion regarding, assignments that may be sent to it, but will acknowledge any notices that are sent to it in duplicate with the statutory fee. If the claimant is not satisfied with this, his title can be investigated by the Solicitor of the Office—of course at his expense—and a certificate of title granted, as explained above.

Submission of Title to Solicitor. I am aware that some Offices—how many, I cannot say—do not submit every claimant's title to the Solicitor, but only the titles that they consider doubtful or complicated; but I think that, for various reasons, it is better that every title should, as a matter of course, be submitted to the Solicitor. If this is not done, there is a risk, that the person in the Office who examines the titles, may not have followed with the necessary promptitude, the constant changes that are introduced into the law by legislation, and by new judicial decisions; or that he may overlook some material point, with which he is not familiar, and thus pass a title which is really imperfect. I believe, in fact, that this must occasionally occur, unless such examiner is able to devote the whole of his time to legal questions; and even when the business of the Office is on so large a scale that this can be arranged, I believe that it is better that the examiner of titles should be a person who has also experience of other kinds of legal questions.

Since Life Offices were prohibited, first, by the Customs and Inland Revenue Act, 1888, § 19, and afterwards by the Stamp Act, 1891, § 118, from paying any money to a person who claims under an unstamped or insufficiently stamped assignment, it has, in my opinion, become more desirable than ever, that all assignments sent in to an Office, should be submitted to the Solicitor of the Office; and from the experience I have had in two Offices, of widely different constitution, I am able to say that habitual reference of all titles to the Office Solicitor, is not open to objection, either on the ground of delay, or on that of expense. It has also the incidental advantage that, when a troublesome legal question is put to the Manager by an assured or an assignee, he is able to say that it does not fall within his province to deal with legal questions, and that he cannot take the responsibility of doing so.

When it is the practice in an Office to refer all titles to its Solicitor, the Manager should, of course, make himself thoroughly familiar with the principles on which the Solicitor

deals with the different questions that arise. He should, however, carefully avoid taking any responsibility with regard to legal questions; and, with this object, he should consistently decline to give any opinion, or advice, on even the simplest question of the kind. If he once expresses an opinion on a simple point, he will often find that one question leads to another, and that it is much more difficult to decline to answer questions put to him, if he has already encouraged the enquirer by answering his prior questions. For similar reasons, I think it is unsafe for an Office to have its own printed form of absolute assignment, and give it to any policyholder who wishes to "transfer" his policy. It will be found in most cases that what is wanted is a form for mortgaging the policy; and it is very likely that the form of absolute assignment will be improperly used for this purpose, and that much trouble will consequently be caused at a later stage, to all parties concerned.

Whenever a policyholder asks for advice or assistance in dealing with a legal question, he should be told that he must consult a solicitor; and it may be added that, if desired, the Office solicitors will be instructed to advise or assist him, provided he will pay their professional charges in the matter.

Conclusion. It only remains for me to say that the foregoing remarks are intended to be made from the point of view of a Life Office Manager, who takes his law at second hand from the Office Solicitor, but claims the right to ask the Solicitor for explanations as to the principles on which his advice is based; and who sometimes will be prepared to advise his Directors to act contrary to such advice, when it would injure the reputation of his Office to insist on a technical point, the waiving of which would be attended with extremely little risk, or none at all.

As a specimen of the questions regarding which the Manager may ask the Solicitor for explanation, I will take the following: The Office has received notice of an assignment in favour of A.B., and at a later date, it receives a letter from A.B., withdrawing the notice, and stating that he has no longer any interest in the policy; and the Solicitor advises that this is not sufficient, but that the assignment must be produced. The Solicitor, if applied to for an explanation, will probably say that the deed of which notice has been given, may possibly involve the interest of third parties, in such a way that A.B. has no right to withdraw the notice. This explanation is clearly satisfactory. If, however, the Solicitor, after seeing the deed and making further enquiry, finds

that the deed was a simple mortgage to A.B., and that his claim upon it has been satisfied, but that there has been no formal re-assignment; he will probably report that, in order to complete the title of the assured, there must be a regular re-assignment of the policy executed by A.B. The Manager may then fairly ask what risk his Office will run, if such a re-assignment is dispensed with; and, judging from my own experience, I think he is not likely to receive a satisfactory reply, and may therefore hold that, in such a case, a re-assignment should not be insisted upon.

DISCUSSION.

THE PRESIDENT (Mr. T. E. Young) said that Dr. Sprague commanded their admiration and respect, not more for the combination of theoretical and practical wisdom he displayed, than for the constant and unstinted pains he showed, in submitting to the members for their guidance in the perplexing problems which their work presented, the results to which the unrivalled fulness of his knowledge and thoughtful experience had directed him. In all the questions discussed in the paper, and occurring in their daily practice, they should be guided obviously by the principle that their object was not precise technical accuracy in every case: but the reasonable solution, from a practical and business point of view, of the difficulties which might confront them. As heads of offices they had to act as sensible business men, working upon commercial principles, where minute technical difficulties must often be discreetly swept aside, under the broader guidance of business sagacity and judgment derived from practical experience and directed to business ends. Minor risks must constantly be fearlessly and thoughtfully disregarded, where larger interests and considerations seemed supreme. With respect to notices of assignment, whether in the specific form of a notice or in the production of the deed, he thought with Dr. Sprague that they should intimate that their registration was simply that of a notice which did not express or imply any opinion upon legal ownership; and upon that point he might mention that for the guidance of assignees, and with a disclaimer of responsibility, he invariably specified any lacunæ in the chain of assignments and reconveyances which might be apparently disclosed by the title column. He also, like Dr. Sprague, referred the constant enquirers respecting their legal position to a solicitor, especially having regard to the present position of the stamp laws: and he had long had occasion to discontinue the use of printed forms of assignment, as a fertile source of much possible misconception. He presumed that most of the members adopted the practice he followed, immediately a policy had lapsed, of intimating the fact to the assignees, so that an opportunity might be afforded of maintaining their security. With regard to the decision upon titles in the office, he naturally acted in comparatively simple cases upon his own judgment and knowledge; but, in cases of a more complex character, he

always bore in mind the opinion of an eminent solicitor, that by a reference to legal advisers, the company retained the right of a claim for damages against the firm if an erroneous judgment was expressed, which was not generally feasible where the adviser was a legal official upon the staff. He had never seen any valid reason raised against the grant, under suitable conditions, of certificates of title; more especially bearing in mind that their object was to assist in every reasonable manner the constituents of the company. Upon the incidental subject of indemnities he was no doubt uttering what to most of them would be a truism, but he (the President) could not avoid expressing the surprise he had frequently felt at the very easy manner in which indemnities were often accepted without any enquiry. It should obviously be the case that, before an indemnity was received, adequate investigation, chiefly from independent sources, should be pursued with a view of deciding whether the covenants which the indemnifier undertook to incur were financially adequate. The question of lost policies was a fertile and perplexing one, and during his tenure of office he had not issued duplicates in half-a-dozen cases. He had rarely known an instance where a document stated to be lost had not ultimately been recovered—one of his most recent experiences being that, when a claim occurred, the original policy (stated on solemn statutory declaration to have been hopelessly lost many years ago) was presented by the claimant enfolded within the duplicate itself. Where so exceptional a concession as a duplicate had been granted, he had been legally advised that the duplicate should be in the precise form of the original, should bear the original date, and should be accompanied by an endorsement under seal declaratory of the nature and occasion of the grant itself. His own views were distinctly tending to the definite conclusion that duplicate policies should not be issued; and it had sometimes occurred to him whether it might not be feasible, among the multitudinous new schemes which were circulated by the companies, for an office under suitable conditions, both pecuniary and otherwise, to offer to assume the custody of the policy it granted, issuing at the outset a formal memorandum that the policy was deposited with the company in fiduciary charge.

MR. H. W. MANLY, after expressing his regret at Dr. Sprague's absence, and its cause, said that the author had not suggested any startling new departures, but his principal object was to endeavour to obtain greater uniformity in practice among the offices. He, for one, was very grateful to Dr. Sprague for having written that paper, and for putting down, not only his thoughts, but directions in many cases; because, when the cases had to be dealt with in practice, one did not always think at the exact moment what was the best thing to do. It was desirable to have a paper of that kind at hand, to refresh the memory as to what procedure to adopt. With regard to the grave question of issuing duplicate policies, he had always been strongly averse to doing so; in fact, it had never been done with his advice. One office with which he had been connected had had a clause in the deed of settlement, providing for the issue of duplicate policies under certain circumstances; but it was guarded by the condition that the board of directors should be satisfied with the evidence produced that

the policy was absolutely lost or destroyed. Two policies were issued under that clause while he had been connected with the office. In one case it was perfectly safe, because the ashes of the policy were produced, on which could be traced the printing of the policy. The other was the case of a client of a director, but no harm ever came of it. There could be no objection to issuing a copy of the policy, if that copy contained the endorsement which Dr. Sprague suggested, that the possession of the copy policy gave no right to the assured or any other holder of it. He did not see why Dr. Sprague should be so particular in suggesting that a shilling stamp should be put on it, because he did not think that a copy policy should be available for any purpose whatever. There was always a chance that a copy policy might be accepted by an assignee carelessly, and taken as if it were an original document. In the paper, the author had described an occasion when possibly it might have led to some trouble. He referred to case 5. Dr. Sprague said "In the case mentioned under "the first heading of this paper, the assured had been supplied with a "certified copy of his policy, and had handed this copy to an assignee, "from whom he had obtained a loan on security of the policy"; and when he died, the widow found the original policy. Probably she had taken very good care to lock it up from her husband, with the intention that he should not use it. Fortunately, the assignee satisfied the widow that his claim was preferable, and so obtained possession of the policy. He felt some hesitation with regard to the question of certificates of title. He could not see any objection to the second form proposed by Dr. Sprague; and the first form, where the certificate was to the effect "Now I hereby certify that the said "directors are satisfied that the said C.D. is at *present* the absolute "owner of the said policy", perhaps did not commit the office too much. But he was very much surprised in reading further on in the paper, with reference to the Policies of Assurance Act—"The above mentioned Act has, I imagine, made no change in this; but it has this "effect, that, of two competing assignments made after 20 August "1867, that one shall prevail of which the office first received statutory "notice. I do not think that this is a desirable change in the law, as "it may now happen, that the claim of the assignee, whose assignment "is prior in date, and who holds the policy, will be postponed to that "of another assignee, whose assignment is of later date, and who has "not possession of the policy, if from any cause the office received "statutory notice of this latter assignment before the other." It appeared to him that it was only in virtue of the Policies of Assurance Act that one might safely give a certificate of title of that character; without that Act, it would have been an extremely dangerous thing to do. Still, as they were now all endeavouring to simplify in every way the contract of life assurance, and to do as much as they could for their policyholders, he saw no objection to issuing a guarded title of the character suggested. With regard to the notices of assignment, where Dr. Sprague referred to that plan which was sometimes adopted of persons sending the deed itself to the office to be registered, he seemed to imply that that should be treated as a notice. His own practice had always been to decline to receive that as a notice, saying they required to have a written notice in the form of the Act. At

the same time he agreed that it was very desirable that, such a document having been presented to the office, a notice of that document should be inserted in the register. Similarly, with regard to the question of bankruptcy, although it was now settled that in England a trustee in bankruptcy must give a written notice to the office; still, on the other hand, he did not think that the Policies of Assurance Act did away with the common law that, if the office did receive notice in any form, it was bound to take note of it; and for that reason he had for many years stopped the practice of searching for encumbrances, searching for bankruptcies, or otherwise doing any act by which the office might be charged with notice; and, although a subscriber to a trade protection association, he had insisted that the *Gazette* should not be sent to the office, in order to prevent the possibility of a claim being set up hereafter, on the ground that the receipt of the *Gazette* was notice of its contents. No doubt, it would be very difficult to prove that he had received that special number, but he thought it better not to run even that risk.

MR. T. G. C. BROWNE referred to the author's infinite capacity for taking pains; and, as an example, mentioned that, in his acknowledgment of a notice of assignment, he had taken the precaution of stating not only that the deeds would be required, but that a deed, along with all other deeds, whether cancelled or not, should be produced; and he also took the opportunity of pointing out the necessity for having all deeds properly stamped. It was most desirable that those two points should be brought before the public. Dr. Sprague remarked, "I am not able to say how an application for "payment of bonus in cash is generally dealt with in the case of a "lost policy." In reply to the question whether there was any decided case which, in the absence of notice of any charge, would protect the office in the event of its paying the cash value of bonuses, without obtaining production of the policy, his (Mr. Browne's) own office was advised as follows:—"An insurance company is not bound by "an assignment of a policy of which it has not received notice, an "assignment not being perfected till notice has been given. If, "therefore, the assured's representatives claim under a policy, of the "loss or destruction of which they give evidence (and this evidence "must be carefully considered in each case), the company have no "alternative but to pay. There are some recorded cases, in which "the representatives of the assured have sued on lost policies; and in "all such cases the Courts have compelled payment. We do not "think that, having regard to the prospectuses which the company "issue"—that is, simply the undertakings which were given in their prospectuses and in their bonus notices to pay cash bonuses—"they "can make any distinction between payment on death and a cash "payment for bonus, although there is a distinction in law, a cash "payment of bonus being practically a purchase." It had been the practice in his own office to act upon that opinion in the case of lost policies in dealing with cash bonuses, of course with a proper indemnity. Dr. Sprague made some remarks about *ex gratiâ* payments. They did not apply to offices which had non-forfeitable conditions; but in those offices which were antiquated enough not to have adopted non-forfeitable conditions, the question of *ex gratiâ*

payments frequently cropped up. It was obvious that no *ex gratiâ* payment could be made, if any understanding had existed beforehand that it might be made in certain events, such as the lapsing of the policy. If any such understanding was come to with the office beforehand, the so-called *ex gratiâ* payment would cease to be one, and would inherit all the defects of title of the policy when it subsisted. With reference to *ex gratiâ* payments, he ventured to bring before the Institute a case which once came under his observation. It had very little to do indeed with lost policies, but it was a peculiar case. A mortgaged to an insurance company certain leasehold property and a policy on his life in the same office, and subsequently sold to B, the equity of the redemption of the leaseholds, but not the equity of the policy. A was to continue to pay the premiums, but B was to concur in any arrangements that A should make for the release of the policy from the mortgage, provided it did not result in the rate of interest being raised on the mortgage, or the mortgage being called in. A became bankrupt and was unable to pay the premiums, and the office gave notice to B that, unless the policy was kept up, the loan would have to be called in. B, therefore, paid certain premiums to keep the policy alive. Meanwhile it had been assigned by A's trustee in bankruptcy to C, and B brought an action against the latter, claiming a lien on the policy for the premiums paid by him. Mr. Justice Chitty held that, under the circumstances, B was only a volunteer; and gave judgment for the defendant, a decision which was confirmed on appeal. Pending the action the policy finally lapsed, and the office applied, *ex gratiâ*, the surrender-value in reduction of the loan for which B had made himself liable, the amount of such surrender-value being less than the premiums paid by B. C brought an action against B and his mortgagees, claiming payment of the surrender-value that was paid to B, *ex gratiâ*; but he entirely failed, it being held that the office was free to make a gift to B if it chose. The Judge was very strong in his remarks that it was entirely a free gift; and this, after the seemingly harsh decision on B, that in paying the premiums he was a volunteer and had no claim for a lien on the policy. They all had had experience of the very circumstantial statements which were made about lost policies, and he might mention that on one occasion they had a statutory declaration from a man that on a specific occasion, which he quite well remembered, he had burnt his policy along with some other papers, but a few months afterwards the policy was found. That showed that people, in perfect good faith, would make the most inaccurate statements in the most solemn form. He was glad they had the weight of Dr. Sprague's great authority for giving certificates of title. They knew how very shy lawyers were to allow it; but their objections were altogether inarticulate. If the system of conveyancing were so bad that it did not admit of such a reasonable procedure as giving a certificate of title, then the sooner it was mended the better. A case came before his notice a short time ago, where a question arose about the possession of a policy. A claimed under a mortgage of which due notice had been given, and there was a subsequent mortgage given to B; and his representatives,

who had the policy, claimed not only under the mortgage, but also as having a lien on the policy for certain premiums that B had paid to maintain it. B's representative declined to give up the policy. C claimed under a third mortgage. He mentioned the case as it was one in which his office had taken advantage of the Payment into Court Act. They had paid the money into Court and had left the parties to fight it out among themselves—they would look for the decision with some interest. He was also glad to see that Dr. Sprague approved of sending all cases—he assumed he meant assigned cases—to the company's solicitors for their investigation. There was no more troublesome person in the world than the amateur lawyer. However familiar he might be with some small department of the law, he had not that grasp of general principles, nor that familiarity with the constant movement that was going on in legal ideas, to be anything but very timid; and he was perfectly justified in being timid; but that did not do away with the fact that he was a most troublesome person to deal with. It facilitated business very much to send every assigned case to the company's solicitors. There was a saying that Scotchmen had a method of answering a question by asking another. It appeared to him that Dr. Sprague had learned something from his prolonged residence in Scotland, by the last paragraphs of his paper. He proved to the complete satisfaction of every member of this Institute, that the actuary was not created for the purpose of answering legal conundrums put to him by the office solicitor; but at the same time he took care not to bar his right to ask that gentleman very searching and shrewd questions indeed if he felt inclined to do so.

MR. A. H. BAILEY said that he had had more than half a century's experience of that kind of work, and thought the practice adopted by all well-conducted offices was very much in accordance with what Dr. Sprague said in his paper. He (Mr. Bailey) could remember discussions before the Act of 1867 was passed, and a suggestion was then made (which he very much regretted was not adopted, as it would save a great deal of trouble to all connected with life assurance societies) that policies of assurance should be assigned as shares were assigned, on the printed form provided by the office. They should be treated just as the Bank of England treated Consols.

MR. HART thought that, with regard to Mr. Manly's remark as to the copy policy, it might be required to be produced in Court, and then a shilling stamp might be necessary.

MR. R. TODHUNTER referred to Dr. Sprague's statement that "the foregoing remarks are intended to be made from the point of view of a Life Office Manager." He thought there were other points of view from which the subject of the examination of titles might be discussed—the point of view of the public, and the point of view of men like himself, upon whom practical work in connection with the question often fell. From the point of view of the public, he could hardly believe that there was no objection to the practice of submitting every title to the solicitors. Very often a certain amount of undeserved odium was thus brought on the office, and an impression was often produced that the office was attempting to interpose legal obstacles. Then there was also the question of expense. The result

of submitting a simple title to the solicitor often was to compel the claimant to employ a solicitor. He did not know whether Dr. Sprague intended to confine the cases to be sent to the solicitors to assigned cases. As a matter of fact, certain questions might arise on probates; and although, of course, even in a simple assignment some pitfall might lurk, yet as a matter of fact in their ordinary private affairs they would not hesitate to take the risks of acting upon such an assignment. But he would admit that, when one started examining deeds, it was difficult to know where to stop; and it would no doubt be considered desirable that some limitations should be imposed upon the examination of titles in the office. Then taking the point of view of the men who actually had to assume a certain amount of direct responsibility. The ultimate responsibility, of course, fell upon the manager; and, from his point of view, it was not unnatural that he should desire to be as safe as possible. It was conceivable that by the aid of telephones and typewriters a manager might carry on the whole business practically without incurring the necessity for any skilled labour. That was done to a large extent in business houses now, and also to some extent in Government offices—one skilled man conducting the business of a whole department, and a number of typewriters and clerks being employed at low salaries to carry out his instructions. But that system was not conducive to the production of a succession of capable actuaries. If that were done where were the future managers to come from? Men in his position were trained by the Institute to deal with those questions. They were required by the Institute to go with some thoroughness into those questions of law as they affected assignments of life policies, and they were prepared to take a certain amount of responsibility in those matters. They desired to find some sphere of usefulness, and here was one. Even from the point of view of the manager, he did not think Dr. Sprague's argument was conclusive. Even solicitors were not infallible; and, as a matter of fact, nearly all the badly drawn and improperly stamped deeds which came before them, had been erroneously drawn or stamped by solicitors. It was quite true that solicitors who represented life offices were as a rule specially trained; but he could hardly believe that, in many of those busy legal offices, a number of those things would not be left to subordinates; and, if that were so, he did not see why the office could not act as safely upon the opinion of a properly trained assistant actuary, as it could upon the men in the solicitors' offices.

MR. DAWES (a visitor) said his chief interest in the paper was the legal point of view. There were one or two statements with which he could not entirely agree. In the matter of the indemnity and the 6*d.* stamp, that was a trifling detail, but, of course, unless an indemnity was under seal it would be of very little use in many people's opinion; and if it was under seal, a 10*s.* stamp would be required. Another very interesting point was that of the transfer of policies. Dr. Sprague did not suggest it, but he says the common notion is that policies may be transferred like shares. A great many people had that idea. Policies of assurance issued in South Africa had, he believed, got printed endorsements on them which could be filled up; and

they did pass from hand to hand. Last week a case was brought to his knowledge, in which an English policy was endorsed with simply the signature of the assured. This was duly witnessed, and he (Mr. Dawes) was told to put a proper assignment on at the top of the signature, and give notice to the office. When he raised a question on the matter, he was told "Oh, it is the proper thing to do; it is quite the common thing out there." He asked what sort of document they wanted put on, because it seemed to him a curious thing to put on the deed after the signature was there. Office solicitors were bound to give their opinion according to the law; and that was not always exactly palatable to the managers of insurance companies, who would rather like the solicitor to say something else. What one wanted was one or two offices who would be good enough to turn themselves into leading cases in order to get some questionable point settled. The difficulty was to find volunteers. The last point raised by Dr. Sprague, was as to the production of the deed. Naturally one wanted to see the deed, to see if a third party was coming in. With regard to re-assignment, one wished, of course, to facilitate business as far as possible. He had always been of opinion that a letter of withdrawal was quite sufficient, without having a formal re-assignment. There was no legal estate in a policy, as there was, for instance, in a freehold. His difficulty, when deeds were not forthcoming, was the fear that they were insufficiently stamped, or not stamped at all; and there was just a possibility of their turning up afterwards and giving a lot of trouble. He did not know of any case where the Inland Revenue had succeeded in recovering the £50 penalty; but one was always followed by a sort of haunting dread that, when a deed was not produced, there might be that possibility attached to it. One was, therefore, bound to be on the safe side, and to ask for everything to be produced, even at the cost of giving trouble. If the manager of the office took upon himself to say "There is no particular risk about this, and, therefore, I will waive it", that was his business. He thought it was wrong to blame a solicitor for not giving the advice the manager wanted, if it were not in accordance with the law. That should not be expected, because he was afraid that managers were only human, and if anything turned up afterwards, they (or their successors) would be a little inclined to blame the solicitor.

MR. A. F. BURRIDGE said he agreed with the majority of the speakers, in thinking that the duplicate policy should not be encouraged; but there might arise a case in which a duplicate policy might almost of necessity be issued. If the original policy were lost or mislaid while in the custody of the office itself, or during transit from the office agent, he took it that the assured had a right to demand a duplicate policy. Again, on the question of paying cash bounties on a lost policy, he quite agreed with Mr. Browne: it was a custom in his office to do so under proper limitations and, of course, taking an indemnity. Carrying the point one step further, it might be interesting to quote a case which came under his notice some time ago, of a lost policy being the subsequent object of an absolute assignment, and being sold to a third party, who was perfectly satisfied with the defective title as it stood. In that case it was true a

certificate of title, or what was equivalent to a certificate of title, was given—not exactly in the form given by Dr. Sprague, but a letter was written, stating that the office would not raise any objection when the policy became a claim on account of the loss of the policy, provided no further notices of dealings with the policy reached the office. That he took to be a very limited certificate, not applying to the title, but merely applying to the fact of the loss of the policy. It was, therefore, interesting to note that, either by public auction or private contract, such a limited certificate was accepted and a purchaser found for the policy. In the case of bankruptcy, of course, the Official Receiver must give notice to the office. If he did not do so, he believed the law was that the office was free to pay to the representatives of the assured, or to the assignees of the policy as shown by the books. A case came under his notice some few months ago, in which, after an office had so paid a policy on the life of a bankrupt, the bankruptcy not having been notified, the Official Receiver stepped in; but he was too late, and he could not substantiate his claim. Mr. Todhunter's remarks on the treatment of legal questions, as to whether they should be dealt with inside the office or sent to the office solicitors, were very interesting. He thought Dr. Sprague's meaning was perfectly clear, that he would limit the reference to the solicitors to the cases where the policy was the object of assignment, and he would not think it necessary to do so if it were a mere question of probate. Dr. Sprague said in his paper, referring to the difficulties arising under the Stamp Act of 1891, "It has, in my opinion, become more desirable than ever, that all assignments sent into an office should be submitted to the solicitor of the office; and from the experience I have had in two offices, of widely different constitution, I am able to say that habitual reference of all titles to the office solicitor, is not open to objection, either on the ground of delay or on that of expense." He would suggest to Mr. Todhunter that the question of expense should not stand in their way. There was no better maxim than "cheap law is bad law"; and, therefore, they should not be deterred from seeking advice by considerations of cost. It was desirable in the discussion that the object of Dr. Sprague should not be misunderstood. Dr. Sprague did not wish to elicit an opinion from his legal adviser which was not in strict accordance with the law; on the contrary, he desired to obtain the strictly legal statement of the case, and then, with his well-known acumen, he probed his lawyer as to the reasons of his decision, and would himself distinctly assume responsibility, or obtain the sanction of his directors for assuming direct responsibility, for waiving any strictly legal point which he did not think it necessary to enforce. The object of the paper had their warm sympathy. It was to simplify their contract as much as possible, and endeavour to show the public that, in holding a policy of assurance, they were holding one of the simplest and most secure contracts they could obtain.

The PRESIDENT expressed the thanks of the meeting to Dr. Sprague, for his latest contribution.

Dr. SPRAGUE, in reply, writes:—

It was a great disappointment to me, that I was unable to go to London and read my paper myself, as I had intended; but the discussion seems to be so well and fully reported, that I believe I understand the views of the various speakers as well as if I had been present. I am particularly pleased that the reading of the paper has called forth from other actuaries of experience so much interesting information, as to the practice of their respective offices, and as to their own opinions; and I am glad to learn that there is not so much diversity of practice as I had supposed. There is such a general agreement among the speakers with the principles laid down in my paper, that there are comparatively few points on which I shall have any further remarks to make.

I see that the President does not share my opinion that all assignments sent to the office in proof of a claimant's title, should be submitted, as a matter of course, to the solicitor; but that, when a title is comparatively simple, he acts upon his own judgment. He has therefore in every case to consider and decide whether the title is sufficiently simple for him to deal with it himself, or whether it should be sent to the solicitor. It is, I think, a much more convenient course to pay the solicitor a salary for advising on titles and other legal matters, and then to send him every assignment, so as to obtain from him a written report on every title, whether simple or complicated. In this way, the actuary (or manager) is freed from all direct responsibility as to legal matters, and a sharp line is drawn between his duties and those of the solicitor. When an arrangement of this kind is made, it would be inconsistent for the actuary to volunteer information, as the President says he does, regarding "lacunæ in the chain of assignments and reconveyances." It is much better, I think, that the actuary should not do this; but that he should confine himself to recording and acknowledging the notices served upon him, and not even compare these notices, either with each other or with the deeds when they are sent in.

As regards indemnities, it is, I imagine, very rarely indeed that an office has to make a claim on a person who has granted it an indemnity; and, as it would often be considered vexatious and oppressive, if the company insisted on having proof that the person giving the indemnity is good for the amount, I think it is better to accept the indemnity without making any enquiry. I do not know why an indemnity not under seal, should be considered of very little use, as Mr. Dawes says; and I should be glad to have information on the point. Upon one occasion, a solicitor acting for us took the indemnity in the form of a regular penal bond, and thus, in my opinion, put the claimant to much unnecessary expense.

I cannot agree with Mr. Manly, that it is only by virtue of the Policies of Assurance Act that we can safely give a certificate of title in the form contained in my paper. If there are two competing absolute assignments, and the office has only received notice of the one which is later in date, it would, quite independently of the Act, have been safe in recognizing the assignee under the later assignment as the owner of the policy, provided he produced his title and it was found satisfactory. If it subsequently received notice of the

prior assignment, it might probably, by virtue of the Act, still recognize the assignee under the later assignment as the owner of the policy ; but this seems to me to be a case in which it would be better to decline to grant a certificate of title. Although Mr. Manly seems to take exception to what I say as to the production of a deed being notice of its contents, I think we are really of the same opinion. If a deed is produced, and left with the office for a time, this is certainly a notice of the existence of the deed, and should be registered as such ; but it is not such a notice as the office is bound to acknowledge under the Policies of Assurance Act. I entirely agree with Mr. Manly's remarks as to bankruptcy.

I think Mr. Browne's remarks form a valuable contribution to the discussion, and I have been greatly interested by his account of the case where there are three claimants to the policy money, the second of whom holds the policy. I trust that, when the case is decided, Mr. Browne will report the result to the Institute. I am not sure that Mr. Bailey is right in saying that it would save a great deal of trouble to all connected with life offices, if policies could be assigned like shares or consols ; for I am inclined to think that the trouble would not be got rid of, but only shifted from one shoulder to the other. Policies would still be mortgaged as at present : and different persons would, as now, be interested in the same policy in different ways ; and would give notices to the office, which it would be bound to register and keep in view. Even if the assignment of policies was assimilated to the transfer of consols, distringas would be served on the company, by persons having different interests in the same policy ; and it would, before paying the sum assured or the surrender value, have to investigate the title of the claimant. In one respect such a system would be greatly preferable to the present one : it would enable a man to give notice to the office, by means of a distringas, that he has an interest in a policy ; and when that interest ceases, to withdraw his distringas, without the necessity of preserving and producing to the office at some future time the cancelled deed which gave him the interest. But this advantage would be gained in a much simpler way by providing that a creditor, or other person, instead of giving notice to the office of a deed, might protect his rights by simply giving notice that he is interested in the policy. Apart from this, I differ with Mr. Bailey in thinking that we should not so much consider what will save us trouble, as what will conduce to the convenience of our constituents. It is quite a new idea to me that the public may object to their titles being submitted to the solicitor of the company. Mr. Todhunter does not say distinctly why this is ; but I infer from what he says, that he and persons in his position are more easily satisfied than the office solicitor would be ; in fact, that they are likely to pass, without question, titles to which the solicitor would raise objections. In the absence of further explanation, this seems to me to be a strong argument in favor of referring all titles to the solicitor. If a well-founded objection is taken to a title, it must be disposed of, whether it has been raised by the solicitor or an official in the office ; and I do not understand why it should be more necessary for the claimant to employ a solicitor in the one case than in the other. Perhaps when

Mr. Todhunter refers a title to the solicitor, he always instructs him to write to the claimant; but this is not necessary or desirable in the case of a simple title. In conclusion, I have to thank Mr. Burridge for correcting some misconceptions as to my meaning on the part of Mr. Todhunter and Mr. Dawes.

The Mathematical Law of Mortality. By DR. THEODOR WITTSTEIN of Hanover. Translated by D. A. BUMSTED, F.I.A.

[A translation of an essay by DR. WITTSTEIN on the Mathematical Law of Mortality appeared in this *Journal* (vol. xxiv, p. 153). DR. WITTSTEIN afterwards published a pamphlet in continuation of the subject, in which he used the method of Least Squares for the determination of two of the constants in his formula, and applied the formula to the graduation of the H^M Table of Mortality. The following is an abridged translation]:

A COROLLARY TO THE METHOD OF LEAST SQUARES.

§ 18. IN applying the method of Least Squares a peculiar difficulty is often experienced whenever the function, of which we have a series of observed values, is not linear with respect to the unknown constants to be determined and is therefore either of a higher algebraic order or is transcendental.

§ 19. We can scarcely conceive of this difficulty ever being entirely obviated, but we have succeeded in proving that it is possible in respect of a certain class of functions, namely when we can transform the given function, by ordinary operations, into another which is linear, as for example,

$$y = e^{a+bx} \text{ into } \log y = a \log e + bx \log e$$

$$y = \sin(a + bx) \text{ into } \arcsin y = a + bx$$

where the new functions have become linear in regard to the unknown constants a and b . In cases of this kind it is always possible to determine exact values of these constants from the new functions, provided we bear in mind the following proposition.

§ 20. Theorem. If a series of observed values A &c. of a function y is given, having the weights w &c. we may, instead of making the sum of the squares of the errors of y a minimum, with perfect accuracy determine the unknown constants contained in y by making the sum of the squares of the errors of an

arbitrary function of y , $f(y)$ a minimum, ascribing at the same time to each observation a weight w' , such that,

$$\frac{w}{w'} = K \left(\frac{df(A)}{dA} \right)^2 \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where K denotes an arbitrary positive constant which is the same for all the observations.

The proof is very simple. If the sum $\Sigma w (y-A)^2$ and the sum $\Sigma w' [f(y) - f(A)]^2$ (where Σ includes all the observations) are each to be a minimum for the same values of the constants contained in $f(y)$, this is only possible when each term of the one sum is either equal to the corresponding term of the other sum or differs from it by a positive factor which is the same for every term. Denoting this factor by K , we must therefore have

$$\frac{w(y-A)^2}{w'[f(y) - f(A)]^2} = K$$

or

$$\frac{w}{w'} = K \left(\frac{f(y) - f(A)}{y - A} \right)^2 \quad . \quad . \quad . \quad . \quad . \quad (2)$$

from which the above expression (1) follows at once, as we may correctly substitute for the ratio of the small differences $f(y) - f(A)$ and $(y - A)$ the ratio of their differentials. For it is a fundamental principle in applying the method of Least Squares that the errors of observation must be nearly infinitesimal in comparison with the observed quantities themselves.

§ 21. Since the weights have only a relative significance we may, for the purpose of simplifying the calculations, so assume the arbitrary constant K that if the expression

$$\left(\frac{df(A)}{dA} \right)^2$$

contains a constant factor which has the same value for all the observations, we have the product of K into this factor = 1. If there be no such factor we take $K = 1$.

Thus if, in the above example of § 19 the function

$$y = e^{a+bx} \quad \text{weight} = w$$

is transformed into

$$\log y = a \log e + bx \log e, \quad \text{weight} = w',$$

we get

$$w' = wA^2$$

with which weight the constants of the new function $a \log e$ and $b \log e$, must be calculated.

Similarly if in the second of the above examples the function

$$y = \sin(a + bx), \quad \text{weight} = w,$$

is transformed into

$$\text{arc sin } y = a + bx, \quad \text{weight} = w'$$

we get

$$w' = w(1 - A^2).$$

APPLICATION OF THE FORMULA TO A SECOND EXAMPLE.

§24. The example given in the first part of this paper (see *J.I.A.* Vol. xxiv) was incomplete inasmuch as the method of Least Squares was not applied to the solution, and that example cannot therefore be regarded as an entirely suitable test of our formula. We therefore give another example in which the method of Least Squares is fully applied.

Table D contains the figures from which the H^M Mortality Table was calculated and we have added Col. (5) which gives the weights of the probabilities of dying calculated according to the formula of §4 $w = \frac{l}{q(1-q)}$ the maximum weights being taken as 100 and only whole numbers being used.

Table E contains Woolhouse's graduation, which we use for comparison, and Table F shows the results of our calculation which we will now explain.

§25. In order to apply our formula (No. 17, §13)

$$q_x = a^{-M-x^n} + \frac{1}{m} a^{-mx^n} \quad . \quad . \quad . \quad . \quad (3)$$

to the observations in Table D it is necessary to determine 4 constants, of which we will first consider M and m .

With respect to M we can draw the same conclusion as in §11. Table D ends with two living at age 96, both of whom died in the following year. It is possible that with a greater number of observations there might be some survivors at age 97, and we will therefore take the limiting age at 98, or make $M=97$. It will be observed that this agrees with Woolhouse's Table E.

To determine the second constant m , we seek firstly the minimum probability of dying. This is found at age 16 in Table D whence it would follow according to §14 that $m=5$. But this deduction is very unsafe in view of the small numbers observed at this age; as appears also from the fact that $q_{16}=0$; which is a value of which no use whatever can be made. It is

further to be noticed that the observations under consideration deal with persons who were accepted as healthy and that this quality must be assumed in respect of the children also. We therefore think it would be safer to put $m=6$ by which the minimum is brought down to age 14. In this also we agree with Woolhouse's Table E.

Inserting these two constants in equation (3) it becomes

$$q_x = a^{-(97-x)^n} + \frac{1}{6} a^{-(6x)^n} \quad . \quad . \quad . \quad . \quad . \quad (4)$$

§ 26. The other constants a and n must be determined by the method of Least Squares, and we must first decide how and to what extent the observations in Table D can be used.

The calculation is based as before, not upon the observed deaths, but upon the observed probabilities of dying, so that we make direct use of our formula for q_x . We will therefore determine the two unknowns a and n so that they may reduce the sum of the squares of the errors of these probabilities, each multiplied by its weight, to a minimum. The weights can be calculated directly by the formula in § 4, without using the values in Col. 5 of Table D.

Now we know that in getting this formula it is assumed that the observed numbers of living and dying are large, and that if it be applied to numbers which are not large only approximate values of the weights are obtained. This is of itself no disadvantage for in this case, as in many others, the final results are scarcely affected when approximate values of the weights are used instead of the true weights. But when the deaths = 0, the formula is not applicable and we must limit our calculations to the observations for ages 17 to 93 inclusive. Our task therefore consists in determining two unknown constants from 77 observations and their weights.

§ 27. The following is the course usually prescribed for solving this problem, as the function q_x is transcendental in respect of a and n .

We must first, as in § 11, choose arbitrarily from the observations two ages, and insert in our function the corresponding pairs of values of x and q_x . Two equations are thus obtained from which we find a and n . In order that the solution may be possible we must be careful to choose the ages sufficiently high to secure that the second part of our function which pertains to infant mortality may vanish; then the calculation can be proceeded with exactly as in § 11.

§ 28. Since the second part of our formula, which relates to infant mortality, gives a value which is small in comparison with that given by the first part, we may in the meantime neglect that second part and deal only with the function $q = a^{-M-x^n}$ to which we attribute the weights given by the formula of § 4.

We have already in § 11 (14) deduced from the above equation the following:

$$\log \log \frac{1}{q} = n \log (M-x) + \log \log a$$

which is linear with respect to the constants n and $\log \log a$; the weights corresponding to it may be denoted by w' .

Denoting in future the observed values of the probability of dying by q , we get by means of the proposition in § 20

$$\frac{df(\Lambda)}{d\Lambda} = \frac{d \log \log \frac{1}{q}}{dq} = \frac{(\log e)^2}{q \log q} \quad \text{and} \quad \frac{w}{w'} = \frac{K(\log e)^4}{q^2 (\log q)^2}$$

and finally taking $K(\log e)^4 = 1$ (§ 21): and substituting for w its value, we have

$$w' = \frac{lq}{1-q} \left(\log \frac{1}{q} \right)^2$$

Since according to this new function the expression for the error of observation takes the form

$$n \log (M-x) + \log \log a - \log \log \frac{1}{q}$$

we must now make

$$\Sigma w' \left\{ n \log (M-x) + \log \log a - \log \log \frac{1}{q} \right\}^2$$

a minimum, where Σ extends to all the 77 observations: Then differentiation with respect to n and $\log \log a$ gives us the two equations

$$n \Sigma w' \log (M-x) + \log \log a \Sigma w' = \Sigma w' \log \log \frac{1}{q}$$

$$\begin{aligned} n \Sigma w' \left(\log (M-x) \right)^2 + \log \log a \Sigma w' \log (M-x) \\ = \Sigma w' \log \log \frac{1}{q} \log (M-x) \end{aligned}$$

and the solution of these gives the values of n and $\log \log a$. We thus get

$$a = 1.41863 \quad n = .63507$$

Substituting these in the formula for q (No. 4) the probabilities of dying are obtained. These however are not the final probabilities given in Table F, since agreeably with what was said at the beginning of this Section they require another correction with which we will now deal.

§ 29. The values of the constants a and n in the last § were calculated from an incomplete formula for q : the complete one being:

$$q = a^{-(M-x)^n} + c$$

$$\text{where } c = \frac{1}{m} a^{-mx^n}$$

This may be written

$$q - c = a^{-M-x^n}$$

where c indicates a given constant for each observation.

The weights w , which belong to this new function $q - c$, are identical with those of the function q in the last § as can easily be seen from the theorem in § 20.

The calculation on the basis of this new function is as follows:

§ 30. Proceeding as in § 28, we deduce from the above expression for $q - c$

$$\log \log \frac{1}{q-c} = n \log (M-x) + \log \log a,$$

which is linear in respect to n and $\log \log a$.

The weights w' corresponding to this new function are not strictly speaking identical with those of § 28. For applying § 20 it is seen that

$$\frac{dfA}{dA} = \frac{d \log \log \frac{1}{q-c}}{d(q-c)} = \frac{(\log e)^2}{(q-c) \log (q-c)}$$

$$\text{and } \frac{w}{w'} = \frac{K (\log e)^4}{(q-c)^2 [\log (q-c)]^2}.$$

Substituting for w its known value, and taking $K (\log e)^4 = 1$, we obtain for w' a different value from that in § 28. But the effect of the difference must be very insignificant on our values and moreover as it only concerns weights which themselves are more or less approximations, we may quite disregard it. We retain therefore the values of w' in § 28 unaltered.

The quantity to be reduced to a minimum will therefore be the following,

$$\Sigma w' \left(n \log (M-x) + \log \log a - \log \log \frac{1}{q-c} \right)^2.$$

Differentiating this with respect to n and $\log \log a$ we get the two equations,

$$n \Sigma w' \log (M-x) + \log \log a \Sigma w' = \Sigma w' \log \log \frac{1}{q-c}$$

$$n \Sigma w' [\log (M-x)]^2 + \log \log a \Sigma w' \log (M-x) = \Sigma w' \log \log \frac{1}{q-c} \times \log (M-x),$$

solution of which gives the final values of the two unknown quantities, n and $\log \log a$.

On comparing these two equations with those at the end of § 28, we notice that the terms on the left side are identical and therefore will not require recalculation. On the right side those terms only need be recalculated in which the quantity c has a significant value; as our calculations extend only to five decimal places, this was found to occur at the ages 17 to 34; the remaining terms can be taken unaltered from § 28. We see therefore that the correction of § 28 given by the calculations of this § does not greatly affect the result.

The solution of these equations gives

$$a = 1.41790 \quad n = .63549$$

and the probability of dying at any age x is,

$$q = 1.41790^{-.97-x^{.63549}} + \frac{1}{6} \times 1.41790^{-.6x^{.63549}}$$

By this formula we have calculated the numbers given in Columns 4, 5, 6 of Table F. We may observe that the numbers in Col. 5 calculated to 5 decimal places were identical with the values of the quantity c , except in a few isolated cases in which the difference was unity in the 5th decimal place. This proves that we were justified in adopting the value of c found in § 28.

Finally, from the probabilities of dying (Table F, Col. 6) we have calculated in the usual manner the numbers living (Col. 2) and the deaths (Col. 3); using as a radix the arbitrary number of 10,000 living at age 10, which is the basis of Tables D and E.

§ 31. It must strike anyone that the values of the constants a and n , which form the basis of Tables C and F, namely

$$a = 1.42423 \qquad n = .63033$$

$$\text{and} \qquad a = 1.41790 \qquad n = .63549$$

differ from each other by very small amounts and that their differences sometimes act in opposite directions and neutralize one another. Table C is founded on observations of Officials, who were not subject to medical examination at entry; while Table F is founded on the observation of men who were proved to be healthy at entry. Nevertheless the differences between the two values of a and of n do not amount to so much as 1 per-cent and the greatest part of the difference in the tables is in the value of the constant M , that is the limiting age.

We may hence conjecture that possibly the quantities a and n are constants in the absolute sense of the word; like, for example, the accelerating force of gravity—the mechanical equivalent of heat, &c., and that the above are only approximate values. The consequence of this would be that in all tables of mortality—the instance before us being that of males after the exclusion of the disturbing influence of infant mortality—the probabilities of dying would be absolutely identical and the tables would differ only in the limit of age; so that the probability of dying would always be the same when $\omega - x$ is the same. Or in other words, one of these tables of mortality would be immediately altered to another when we shift Col. 4 along Col. 1 in the proper direction for as many years as are equal to the difference between the limits of age in the tables. Columns 2 and 3 then follow in the usual way.

If this property, which is only approximately true in the cases we have considered, were found to be generally true, it is evident that the construction of mortality tables would be facilitated and simplified to an extraordinary degree. In this connection it may be noted that as soon as a and n are accurately determined, the value of the constant M and therefore the limiting age, can be found by the formula (obtained from (13) in § 11),

$$M = x + \left(\frac{\log \frac{1}{q}}{\log a} \right)^{\frac{1}{n}}$$

The following is an example of the use of this formula.

If we assume the accurate value of the constants a and n to be the arithmetical means of the above say,

$$a = 1.421 \qquad n = .633$$

we can then, without knowing M , calculate all the probabilities of dying for the successive values of $M-x$, and thereby form a normal table wanting only the ages. In order to apply this normal table to the observations in Table A we select from the latter the probability of dying which has the greatest weight, namely

$$x = 36 \qquad q = .009384;$$

and then inserting these numbers in the above formula we have $M = 95.5$, whereas we had fixed $M = 95$ as a basis for Table C.

Similarly if we select from Table D the probability of dying which has the greatest weight, namely

$$x = 35 \qquad q = .008236$$

and substitute these numbers in the same formula we have $M = 97.2$; while for Table F the value $M = 97$ had been assumed. The Mortality tables derived from constants thus determined will obviously differ very little from our Tables C and F, and will correspond almost as closely to the actual observations.

We will not pursue this further, as we are proceeding upon entirely hypothetical ground. More extensive observations are needed before we can decide whether, and how far, the above-mentioned conjecture holds good.

D. H^M Table of Mortality.

(1) Age	(2) Numbers exposed to Risk of Death	(3) Numbers Dying	(4) Probability of Dying in a Year = $(3) \div (2)$	(5) Relative Weights	(6) Unadjusted Numbers Living	(7) Numbers Dying
0	21	0	0.0000
1	47.5	0	0.0000
2	65	1	0.0154
3	79	0	0.0000
4	93.5	0	0.0000
5	119	1	0.0084
6	117.5	1	0.0068
7	186.5	0	0.0000
8	239.5	0	0.0000
9	309.5	1	0.0032
10	379	3	0.0079	...	10,000	79
11	434	0	0.0000	...	9,921	0
12	491.5	2	0.0041	...	9,921	40
13	578	2	0.0035	...	9,881	35
14	731	3	0.0041	...	9,846	40
15	908	2	0.0022	...	9,806	22
16	1,129	0	0.0000	...	9,784	0
17	1,421	6	0.0042	8	9,784	41
18	1,810.5	11	0.0061	7	9,743	59
19	2,414	17	0.0070	8	9,684	68
20	3,293.5	19	0.0058	13	9,616	56
21	4,578.5	32	0.0070	15	9,560	67
22	6,397	40	0.0063	23	9,493	59
23	8,534	66	0.0077	25	9,434	73
24	10,936	75	0.0069	37	9,361	64
25	13,622.5	70	0.0051	61	9,297	48
26	16,339	113	0.0069	54	9,249	64
27	19,170.5	124	0.0065	68	9,185	60
28	21,837	171	0.0078	64	9,125	71
29	24,588	181	0.0074	77	9,054	67
30	27,112.5	224	0.0083	75	8,987	74
31	29,213	215	0.0074	91	8,913	65
32	31,232	260	0.0083	86	8,848	74
33	32,969	274	0.0083	91	8,774	73
34	34,535.5	300	0.0087	91	8,701	76
35	35,818.5	295	0.0082	100	8,625	71
36	36,840.5	326	0.0089	96	8,554	75
37	37,360	357	0.0096	90	8,479	81
38	37,804.5	389	0.0103	85	8,398	87
39	38,112.5	405	0.0106	83	8,311	88
40	38,195	377	0.0099	89	8,223	81
41	37,838	396	0.0105	83	8,142	85
42	37,258.5	399	0.0107	80	8,057	87
43	36,534.5	387	0.0106	79	7,970	84
44	35,693	421	0.0118	90	7,886	93
45	34,735.5	429	0.0124	65	7,793	97
46	33,660.5	421	0.0125	62	7,696	96
47	32,502	460	0.0142	53	7,600	107
48	31,228	440	0.0141	51	7,493	106
49	30,055.5	459	0.0153	46	7,387	113

E. H ^M Table.				F. H ^M Table.					
WOOLHOUSE'S GRADUATION				NEW ADJUSTMENT					
(1) Age x	(2) l_x	(3) d_x	(4) q_x	(1) Age x	(2) l_x	(3) d_x	(4) q_x	(5)	(6)
0	0	14,137	2,380	0.0017 + 0.1667 = 0.1684		
1	1	11,757	679	0.0018 + 0.0560 = 0.0578		
2	2	11,078	359	0.0018 + 0.0306 = 0.0324		
3	3	10,719	220	0.0019 + 0.0186 = 0.0205		
4	4	10,499	147	0.0020 + 0.0120 = 0.0140		
5	5	10,352	105	0.0021 + 0.0080 = 0.0101		
6	6	10,247	79	0.0022 + 0.0055 = 0.0077		
7	7	10,168	71	0.0023 + 0.0048 = 0.0071		
8	8	10,097	52	0.0024 + 0.0028 = 0.0052		
9	9	10,045	45	0.0025 + 0.0020 = 0.0045		
10	10,000	49	0.0049	10	10,000	41	0.0026 + 0.0015 = 0.0041		
11	9,951	40	0.0040	11	9,959	37	0.0027 + 0.0011 = 0.0038		
12	9,911	33	0.0033	12	9,922	37	0.0028 + 0.0008 = 0.0036		
13	9,878	28	0.0029	13	9,885	35	0.0029 + 0.0007 = 0.0036		
14	9,850	28	0.0028	14	9,850	35	0.0030 + 0.0005 = 0.0035		
15	9,822	28	0.0029	15	9,815	35	0.0032 + 0.0004 = 0.0036		
16	9,794	32	0.0032	16	9,780	35	0.0034 + 0.0003 = 0.0037		
17	9,762	37	0.0039	17	9,745	37	0.0035 + 0.0002 = 0.0037		
18	9,725	47	0.0048	18	9,708	37	0.0037 + 0.0002 = 0.0039		
19	9,678	56	0.0057	19	9,671	38	0.0038 + 0.0001 = 0.0039		
20	9,622	61	0.0063	20	9,633	40	0.0040 + 0.0001 = 0.0041		
21	9,561	64	0.0067	21	9,593	41	0.0042 + 0.0001 = 0.0043		
22	9,497	65	0.0068	22	9,552	43	0.0044 + 0.0001 = 0.0045		
23	9,432	64	0.0068	23	9,509	44	0.0046 + 0.0001 = 0.0047		
24	9,368	62	0.0066	24	9,465	46	0.0048 + 0.0001 = 0.0049		
25	9,306	62	0.0066	25	9,419	48	0.0051		
26	9,244	61	0.0067	26	9,371	50	0.0053		
27	9,183	64	0.0069	27	9,321	52	0.0055		
28	9,119	65	0.0072	28	9,269	54	0.0058		
29	9,054	67	0.0074	29	9,215	56	0.0061		
30	8,987	70	0.0077	30	9,159	59	0.0064		
31	8,917	70	0.0079	31	9,100	61	0.0067		
32	8,847	72	0.0081	32	9,039	64	0.0070		
33	8,775	73	0.0083	33	8,975	66	0.0074		
34	8,702	74	0.0085	34	8,909	69	0.0078		
35	8,628	76	0.0088	35	8,840	72	0.0082		
36	8,552	77	0.0091	36	8,768	75	0.0086		
37	8,475	81	0.0095	37	8,693	79	0.0090		
38	8,394	82	0.0098	38	8,614	81	0.0095		
39	8,312	84	0.0101	39	8,533	85	0.0100		
40	8,228	84	0.0103	40	8,448	88	0.0105		
41	8,144	86	0.0105	41	8,360	93	0.0110		
42	8,058	86	0.0107	42	8,267	96	0.0116		
43	7,972	89	0.0111	43	8,171	99	0.0122		
44	7,883	91	0.0116	44	8,072	104	0.0129		
45	7,792	95	0.0122	45	7,968	108	0.0136		
46	7,697	100	0.0129	46	7,860	112	0.0143		
47	7,597	104	0.0137	47	7,748	117	0.0151		
48	7,493	108	0.0144	48	7,631	121	0.0159		
49	7,385	112	0.0152	49	7,510	126	0.0168		

D. H^M Table of Mortality.

(1)	(2)	(3)	(4)	(5)	(6)	(7)
Age	Numbers exposed to Risk of Death	Numbers Dying	Probability of Dying in a Year = (3) ÷ (2)	Relative Weights	Unadjusted Numbers Living	Numbers Dying
50	28,855·5	476	0·0165	41	7,274	120
51	27,510·5	479	0·0174	37	7,154	124
52	26,208·5	446	0·0170	36	7,030	120
53	24,785	426	0·0172	33	6,910	119
54	23,426	444	0·0190	29	6,791	129
55	22,170·5	509	0·0230	23	6,662	153
56	20,746	479	0·0231	21	6,509	150
57	19,377·5	463	0·0239	19	6,359	152
58	18,116·5	455	0·0251	17	6,207	156
59	16,890·5	428	0·0253	16	6,051	153
60	15,672·5	488	0·0311	12	5,898	184
61	14,392·5	468	0·0325	10	5,714	186
62	13,261	459	0·0346	9	5,528	191
63	12,147·5	454	0·0374	8	5,337	200
64	11,021·5	443	0·0402	7	5,137	206
65	9,984·5	435	0·0436	5	4,931	215
66	9,009·5	421	0·0467	5	4,716	220
67	8,081	396	0·0490	4	4,496	220
68	7,214	399	0·0553	3	4,276	237
69	6,375·5	389	0·0610	3	4,039	246
70	5,622	315	0·0560	2	3,793	213
71	4,953	308	0·0622	2	3,580	222
72	4,378	349	0·0797	1	3,358	268
73	3,771·5	297	0·0788	1	3,090	243
74	3,228	340	0·1053	1	2,847	300
75	2,693	254	0·0943	1	2,547	241
76	2,253	240	0·1065	1	2,306	245
77	1,848·5	201	0·1087	0	2,061	224
78	1,531	188	0·1228	0	1,837	226
79	1,257	171	0·1360	0	1,611	219
80	995	140	0·1407	0	1,392	196
81	782	125	0·1599	0	1,196	191
82	609·5	105	0·1723	0	1,005	173
83	464	96	0·2069	0	832	172
84	339	61	0·1799	0	660	119
85	254·5	55	0·2161	0	541	117
86	184	40	0·2174	0	424	92
87	128·5	28	0·2179	0	332	72
88	91·5	26	0·2842	0	260	74
89	57·5	11	0·1913	0	186	36
90	43·5	10	0·2299	0	150	34
91	32	10	0·3125	0	116	36
92	20	9	0·4500	0	80	36
93	10·5	7	0·6667	0	44	29
94	4	0	0·0000		15	0
95	3·5	1	0·2857		15	5
96	2	2	1·0000		10	10
97					0	
98						

E. H^M Table. WOOLHOUSE'S GRADUATION				F. H^M Table. NEW ADJUSTMENT					
(1) Age x	(2) l_x	(3) d_x	(4) q_x	(1) Age x	(2) l_x	(3) d_x	(4) q_x	(5)	(6)
50	7,273	116	0·0160	50	7,384	131	0·0177		
51	7,157	120	0·0167	51	7,253	136	0·0187		
52	7,037	123	0·0175	52	7,117	140	0·0198		
53	6,914	129	0·0186	53	6,977	146	0·0209		
54	6,785	134	0·0197	54	6,831	151	0·0221		
55	6,651	140	0·0210	55	6,680	156	0·0234		
56	6,511	146	0·0225	56	6,524	162	0·0248		
57	6,365	152	0·0240	57	6,362	167	0·0262		
58	6,213	160	0·0256	58	6,195	172	0·0278		
59	6,053	166	0·0275	59	6,023	178	0·0295		
60	5,887	175	0·0297	60	5,845	183	0·0313		
61	5,712	183	0·0320	61	5,662	188	0·0332		
62	5,529	192	0·0346	62	5,474	193	0·0353		
63	5,337	200	0·0375	63	5,281	198	0·0375		
64	5,137	207	0·0404	64	5,083	203	0·0399		
65	4,930	214	0·0434	65	4,880	207	0·0425		
66	4,716	220	0·0466	66	4,673	211	0·0452		
67	4,496	224	0·0499	67	4,462	215	0·0482		
68	4,272	228	0·0532	68	4,247	219	0·0514		
69	4,044	232	0·0573	69	4,028	221	0·0549		
70	3,812	237	0·0622	70	3,807	223	0·0587		
71	3,575	243	0·0681	71	3,584	225	0·0628		
72	3,332	250	0·0749	72	3,359	226	0·0672		
73	3,082	255	0·0829	73	3,133	225	0·0720		
74	2,827	258	0·0912	74	2,908	225	0·0772		
75	2,569	253	0·0984	75	2,683	222	0·0829		
76	2,316	246	0·1064	76	2,461	220	0·0892		
77	2,070	237	0·1147	77	2,241	215	0·0960		
78	1,833	226	0·1232	78	2,026	210	0·1035		
79	1,607	214	0·1331	79	1,816	203	0·1117		
80	1,393	201	0·1447	80	1,613	194	0·1208		
81	1,192	189	0·1580	81	1,419	186	0·1309		
82	1,003	172	0·1714	82	1,233	175	0·1420		
83	831	154	0·1859	83	1,058	164	0·1544		
84	677	135	0·1989	84	894	150	0·1683		
85	542	114	0·2099	85	744	137	0·1838		
86	428	94	0·2197	86	607	122	0·2014		
87	334	77	0·2312	87	485	107	0·2212		
88	257	61	0·2393	88	378	92	0·2440		
89	196	50	0·2532	89	286	78	0·2701		
90	146	41	0·2795	90	208	62	0·3004		
91	105	33	0·3127	91	146	49	0·3361		
92	72	25	0·3513	92	97	37	0·3787		
93	47	20	0·4158	93	60	26	0·4305		
94	27	13	0·5073	94	34	17	0·4956		
95	14	9	0·6370	95	17	10	0·5813		
96	5	4	0·8163	96	7	5	0·7053		
97	1	1	1·0000	97	2	2	1·0000		
98	0			98	0				

ACTUARIAL NOTE.

On the Calculation of the Present Value of a Series of Payments-certain when the Reproductive Rate of Interest differs from the Remunerative Rate. By GEORGE J. LIDSTONE, F.I.A.

IT not infrequently happens that it is required to calculate the present value of a series of payments on the basis that interest at one rate of interest (j) is to be realized by a purchaser or lender for a specified term on the whole of his capital invested, while instalments of capital are accumulated at rate i , being the rate at which it is assumed that reinvestments can be made.

Formulae have been given for special cases, but it is believed that no general expression, applicable to any series of payments, has hitherto been published. The writer recently obtained such a formula which it is thought may be of some general interest.

First let it be required to find what sum should be paid t years hence in consideration of a present payment of 1; it being stipulated that, at the expiration of n years from the date of the advance, the lender is to be in the same position as if he had been paid interest at the rate j on the whole of his capital for n years, although he could only make investments at rate i .

The amount of the original advance is 1

The amount of the annual interest j
accumulated for t years at rate i is $js_t^{(i)}$

The present value at rate i of the difference
between interest at rate j and rate i
for the remaining $n-t$ years is $(j-i)a_{n-t}^{(i)}$

Total amount to be paid to lender $1 + js_t + (j-i)a_{n-t}$
 $= 1 + j(s_t + a_{n-t}) - ia_{n-t}$
 $= 1 + j \cdot s_n \cdot v^{n-t} - (1 - v^{n-t}) = v^{n-t}(1 + js_n)$

The present value of 1, receivable at the expiration of t years, will therefore be the reciprocal of the last expression, i.e.,

$$\frac{(1+i)^{n-t}}{1+js_n} \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

$$= v^t \cdot \frac{(1+i)^n}{1+js_n} \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

$$= v^t \cdot \frac{1+is_n}{1+js_n} \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

The last form suggests that the formula might have been obtained from general considerations.

Considering now expression (1), it will be seen that the only term involving t is $(1+i)^{n-t}$ which is equal to the *accumulated amount* at the end of n years of a payment of 1 made t years hence.

Similarly in expression (2), the term involving t is v^t which is equal to the *present value* of 1 receivable t years hence.

It is therefore evident that the present value of any series of payments under the conditions named (n remaining constant) may be found in either of the following ways:

(a) By multiplying the *present value* of the series of payments (calculated at rate i) by $\frac{(1+i)^n}{1+j\bar{s}_n^-}$;

(b) By multiplying the *accumulated amount* of the series of payments (calculated at rate i) by $\frac{1}{1+j\bar{s}_n^-}$.

These results are perfectly general, and may be tested by applying them to special cases. For example, the result in the case of an annual payment of 1 for n years will be

$$\frac{\bar{s}_n^-}{1+j\bar{s}_n^-} = \frac{1}{\frac{1}{\bar{s}_n^-} + j}, \text{ the usual formula.}$$

As another example, take the case of a bond of 1 redeemable at par in n years and bearing interest meanwhile at the nominal rate x . The price to be given for the bond, under the special conditions mentioned above, will be found at once to be

$$\frac{1 + x\bar{s}_n^{(i)}}{1 + j\bar{s}_n^{(i)}}.$$

[Compare *Institute of Actuaries' Text-Book*, Part I, page 134.]

THE INSTITUTE OF ACTUARIES.

EXAMINATIONS OF THE INSTITUTE, APRIL 1897.

EXAMINATION FOR ADMISSION TO THE CLASS OF ASSOCIATE
(PART I).*Examiner*—PROF. S. L. LONEY, M.A.*Supervisors*—MESSRS. C. D. HIGHAM and GEOFFREY MARKS.*First Paper.*

1. Find, correct to a farthing, the bankers' discount on £345. 6s. 7d. due 57 days hence, at $\frac{1}{4}$ per-cent per annum.

2. Find the G.C.M. of

$$2x^3 - 7x^2 - 8x - 35 \quad \text{and} \quad 2x^3 + 9x^2 + 16x + 21.$$

Find also the square root of $62 - 14\sqrt{13}$.

3. Solve the equations ;

$$(1) \quad x - ay + a^2z = a^3,$$

$$x - by + b^2z = b^3,$$

and

$$x - cy + c^2z = c^3;$$

$$(2) \quad \sqrt{2x+1} + \sqrt{7x-27} = \sqrt{3x+4}.$$

4. What is meant by the expressions "A varies as B" and "A varies inversely as B."

The weekly expenses of a boarding-school are partly constant and partly vary as the number of scholars. If they are £45 when the number is 40, and £65 when the number is 80, find what they are when the number of scholars is 90.

5. A man holds certain amounts of a 5 per-cent stock and of a 6 per-cent stock, and he sells out from both when the former stands at 102 and the latter at $101\frac{1}{4}$, and invests in a $5\frac{1}{2}$ per-cent stock at 103. He then finds that his income is unaltered; compare the amounts of the stocks that he held, brokerage being reckoned in each case at $\frac{1}{8}$ th per-cent.

6. Find the number of combinations of n things, taken r at a time.

How many games of lawn tennis, in which each side consists of a lady and a gentleman, can be arranged at a party consisting of 7 ladies and 4 gentlemen?

7. One shilling, lent on condition that one penny per shilling be paid monthly as interest, accumulates at compound interest. Find the amount of the debt at the end of 10 years, given that $\log 6 = \cdot 7781513$, $\log 65 = 1\cdot 8129134$, and $\log 1\cdot 4841 = \cdot 171452$.

8. Prove that $a^x = 1 + x \log_e a + \frac{x^2}{2} (\log_e a)^2 + \dots$ to infinity.

Deduce the Logarithmic Series, and show how the values of logarithms to base 10 are calculated.

9. Find the present value of an annuity, for n years, of £P, payable annually, the first payment to be made in one year's time, interest being reckoned at $\frac{r}{100}$ per unit per annum.

Deduce the value of a perpetual annuity.

10. In a certain competition the chances are 7 to 3 against one competitor A, 11 to 5 against B, and 13 to 7 against C. What is the chance that one of these three wins?

11. Sum to n terms the series

$$(1) \quad 1 + 13 + 61 + 253 + \dots$$

and
$$(2) \quad \frac{1}{1.2.3} + \frac{1}{2.3.4} + \frac{1}{3.4.5} + \dots$$

12. If a straight line be divided into any two parts, prove that the square on the whole line is equal to the sum of the squares on the two parts, together with twice the rectangle contained by the two parts.

Second Paper.

1. An Insurance Company purchases, at a price of £1,020, a 5 per-cent Bond for £1,000, redeemable 2 years hence at par.

Trace the transaction through the books, and show the Ledger Account from the date of purchase until the repayment of the Bond, interest being payable half-yearly.

2. The value of a pound of gold is 30 times that of a pound of silver, and the weights of equal quantities of gold and silver are as 19 to 10; find the value of a bar of silver equal in bulk to a bar of gold worth £2,500.

3. What is the price of meat per lb. if, on a reduction of 25 per-cent in the price, 8 lbs. more than before are obtained for a sovereign?

4. Prove that there are two values of p for which the equation $ax^2 + 2hx + b + p(a'x^2 + 2h'x + b') = 0$ has equal roots.

5. Assuming the truth of the Binomial Theorem for a positive integer, prove its truth, with a certain condition, for negative and fractional indices. Find the sum of the series

$$1 + \frac{2}{12} + \frac{2.5}{12.18} + \frac{2.5.8}{12.18.24} + \dots \text{ to infinity.}$$

6. Two cyclists travel, one from A to B and the other from B to A, each at a uniform speed. They start together, and one reaches B in $2\frac{1}{2}$ hours and the other reaches A in 3 hours 36 minutes after they meet. How long was each on the journey?

7. Prove that the arithmetic mean of any number of positive quantities is greater than the geometric mean.

Find the greatest and least values of the expression

$$\frac{x^2 - 2x - 3}{2x^2 + 2x + 1}, \quad x \text{ being real.}$$

8. Find the present value of £P due n years hence, compound interest being reckoned at £ r per unit per annum.

At 4 per-cent compound interest find what sum should be paid down now to receive a freehold estate, worth £400 per annum, in 12 years' time, given $\log 2 = \cdot 30103$, $\log 13 = 1 \cdot 1139434$, and $\log 6246 = 3 \cdot 7956$.

9. A man with a capital of £10,000, on which he receives interest at 5 per-cent per annum, spends annually £900; prove that all his capital will be spent before the end of the 17th year, given $\log 2 = \cdot 30103$, $\log 3 = \cdot 4771213$, and $\log 7 = \cdot 8450980$.

10. The probability of the happening of an event in one trial being known, find the probability of its happening 1, 2, 3, times exactly in n trials.

Find the chance that in 7 throws with a pair of dice (each having six faces marked 1, 2, 3, 4, 5 and 6 respectively) the sum of the readings of the faces turned up will be 9 in exactly 3 of these trials.

11. From a heap of playing cards, which originally consisted of three of each suit, one card has been lost. From the remaining eleven cards five are drawn at random, and are found to be two hearts, two clubs, and a spade. If from the remaining six cards two be now drawn at random, prove that the chance of at least one of them being a diamond is $\frac{5}{7}$.

12. Draw a tangent to a given circle from an external point.

Show how to draw the common tangents to two given circles.

EXAMINATION FOR ADMISSION TO THE CLASS OF ASSOCIATE (PART II).

Examiners—MESSRS. F. T. MASON BYERS, G. J. LIDSTONE,
W. P. PHELPS, and E. R. STRAKER.

First Paper.

1. Obtain in symbols the annuity to repay a loan of X in n years, with interest at i per unit, the Sinking Fund for which is to accumulate at j per unit.

What are the constituent parts of each payment?

What is the Redemption payment at the end of t years?

2. A loan of £10,000, bearing interest at the rate of 4 per cent per annum, payable half-yearly, is to be repaid by 40 equal half-yearly payments, including interest and instalment of principal. Having given that $(1.02)^{-20} = .67297$, find:

- (a) The amount of the half-yearly payment.
- (b) The amount of principal included in the first and in the twenty-first half-yearly payment respectively.
- (c) The total amount of principal repaid, after payment of the twentieth half-yearly sum.

3. (a) Find the present value of an annuity-certain of 1 per annum for n years, payable by instalments k times a year, interest being convertible m times a year.

(b) A, B, and C, contribute equal sums towards the purchase of a perpetuity. Find the number of years certain that each person, or his representatives, should successively enjoy it, C having the absolute reversion, so that they may all benefit equally.

4. Explain exactly what is meant by

- (a) Exposed to risk.
- (b) Select Mortality Table.
- (c) $H^{M(5)}$ Table.
- (d) "Existing" in a Mortality Experience.

Discuss two of the principal methods that have been suggested for dealing with the withdrawals in a Mortality Experience.

5 Having given a complete table of net annual premiums for whole-life assurances, how would you construct therefrom a table of l_x ?

6. Define the following expressions:

- "Expectation of life."
- "Most probable after lifetime."
- "Vie probable."

Prove that the value of a life-annuity is less than the value of an annuity-certain for the term of the curtate expectation of life.

7. Demonstrate the following, and give a verbal explanation of each:—

$$(a) \quad P_z^{(m)} = P_x + \frac{m-1}{2m} P_z^{(m)} (P_x + d).$$

$$(b) \quad {}_nV_x^{(m)} = {}_nV_x \left[1 + \frac{m-1}{2m} P_z^{(m)} \right].$$

By means of your verbal interpretations, deduce the corresponding formulæ for Endowment Assurances maturing t years after entry.

8. (a) Show that the value of a life annuity payable m times a year, the first payment due after the interval $\frac{1}{2m}$, is approximately equal to $(a_x + \frac{1}{2})$ whatever value is assigned to m .

(b) Find a formula for the value of an assurance payable one calendar month after death.

9. Prove that, upon the hypothesis that $l_x = k s^x g^{c^x}$, the value of an annuity on three joint lives aged respectively x, y, z , calculated at the rate i , is equal to the value of an annuity on a single life aged w , at the rate i' , where $i' = \frac{1+i}{s^2} - 1$, and $w = \frac{\log[c^x + c^y + c^z]}{\log c}$.

10. Obtain, in terms of commutation symbols and the rate of interest, an expression for the annual premium for a deferred annuity to commence at age 65 on a life now aged 40, the premium to be returnable in case of death before 65, and the annuity to be payable half-yearly, and to be complete.

11. State the form of the table of logarithms known as the "Logarithms of Gauss", and demonstrate how, by means of such a table, the logarithms of the sum or difference of two numbers, the logarithms of which are known, may be ascertained.

How would you apply this property in constructing a table of curtate expectations?

12. What course or courses are open to a Life Assurance Company in the case of defective title or conflicting claims to moneys which have become payable under a Life Policy?

Second Paper.

13. A Bond of £100, bearing interest at the rate of 4 per-cent per annum, payable half-yearly, and redeemable at the expiration of 30 years, at a premium of 10 per-cent, is bought for £114. Find approximately the rate of interest realized by the investor, having given $a_{\overline{60}|}$ at $1\frac{3}{4}$ per-cent = 36.964.

14. Show generally that, in a case such as that referred to in question 13, the difference between

- (a) The half-yearly interest on the Bond, and
- (b) Interest on the purchase-price, for half a year at the true rate realized,

will be equal to the Sinking Fund which, accumulated at the true rate, will replace the loss of capital on the redemption of the Bond.

15. Assuming that you are about to construct a table of Mortality from the records of an Assurance Institution, state any method you would adopt, giving the formula for the "exposed to risk", and show the headings of a schedule to contain the necessary observations.

16. Find the following probabilities:

- (a) That (x) will die before (y) .
- (b) That (y) will be alive at the end of the t th year succeeding the death of (x) .

17. Discuss the main principles underlying the Institute notation, and explain the following symbols:

$$\begin{aligned} & a_{\overline{n}|i:x,y,\dots,m}^r \\ & a_{\overline{n}|i}^x \\ & A_{x,y,z}^{\overline{1}|i} \\ & A_{x,y(t)}^{\overline{2}|1} \\ & {}_{n+0}V_x \end{aligned}$$

18. You are given a table showing the values of

q_x	applicable to male lives resident in India,
q'_x	.. female
q''_x returned to Europe after residence in India.

and you are requested to find the annual premium, payable for 20 years only, for a reversionary annuity payable to a woman aged x after the death of her husband aged y . Assuming that during their joint lifetime the couple will reside in India, but that, on the death of the husband, the wife, if living, will return to Europe, describe the process you would employ.

19. State Simpson's rule for finding the value of an annuity on three joint lives by means of annuities on single lives and two joint lives. Apply the rule to obtain the value of $a_{x,y,z}^{\overline{1}|i}$, where $x < y < z$.

20. Demonstrate the formula for the value of a reversionary life interest to pay the purchaser interest at the rate i after it falls into possession, and in the meanwhile, interest at the rate j , where j is greater than i .

In this case, what would be the redemption-money at the end of the year in which the life tenant dies?

21. Express $A_{x,y,z}^{\overline{2}|1}$ in terms of assurances determining on the first death.

22. What is the essential difference between Life Assurance Contracts and Contracts of Fire and Marine Assurance?

Some policies are expressed to be "indisputable." Can such policies be disputed? If so, on what grounds?

23. Irrespective of any expressed conditions, how are policies legally affected by the suicide of the life assured—

- (1) While insane?
- (2) While sane?

24. Discuss any four classes of investments (excluding life interests and reversions), which you consider specially suitable for the purposes of an Insurance Office, stating what you consider to be the advantages and disadvantages of each class, and the proportions in which they may be held, having regard to the special circumstances of the Office.

EXAMINATION FOR ADMISSION TO THE CLASS OF FELLOW
(PART III, SECTION A).

Examiners—MESSRS. T. G. ACKLAND, J. CHISHOLM, F. E. COLENZO,
H. W. MANLY, F. SCHOOLING, and H. C. THISELTON.

First Paper.

1. What is the meaning of the following expressions :

- (a) Chose in action ;
- (b) Cestui que trust ;
- (c) Tacking of mortgages ;
- (d) Consolidation of securities ;
- (e) Bonâ fide purchaser for value without notice ;
- (f) Freehold ?

2. When, and upon what evidence of title, can a Life Assurance Company safely pay (1) the sum insured by, (2) the surrender-value of, and (3) a Cash Bonus on, a Policy

- (a) To a Trustee under a deed of arrangement with Creditors ?
- (b) To the Trustees of a Marriage Settlement ?

3. Draft a concise form of Mortgage of a Life Policy to secure the repayment of an advance of money with interest.

4. State generally the circumstances under which a Life Assurance Company would be justified in paying the Insurance Money into Court without any fear of having to pay the costs.

5. In what cases should a provision be inserted in a Life Policy to the effect that the Funds of the Company shall be alone answerable, and that no Member of the Company shall be liable in respect of any claim upon it ?

6. State the different ways in which a Registered Friendly Society may be determined or dissolved.

7. On March 31st, 1875, the National Debt was stated to amount, in round figures, to £769,000,000 ; on March 31st, 1895, it was stated to amount, in round figures, to £657,000,000. Describe the various means which have been in operation to bring about this reduction.

8. Explain the principles governing the issue of notes by the Bank of England.

Against what security are they issued ?

In what manner does the Bank make a profit by its note issue ?

9. Describe the usual features of a monetary crisis, and illustrate your remarks by reference to any remarkable crisis which has occurred during the last 50 years.

10. State shortly the advantages and disadvantages of a world-wide system of international money. What is "Gresham's law"?

11. Explain the difference between a cheque and a bill of exchange.

What is the effect of crossing a cheque "& Co.", and what the effect of adding the words "not negotiable"?

12. Under what conditions and from what authorities can the following bodies obtain power to contract loans—

County Councils,
Boards of Guardians,
Corporations,
District Councils,
School Boards?

Second Paper.

13. If $\Delta x = h$, and $\Delta u_x = u_{x+h} - u_x$, show how to develop u_{x+n} in a series consisting of u_x and its successive differences.

14. What is meant by "the differences of 0"? Prove that

$$[n = n^n - n(n-1)^n + \frac{n(n-1)}{1 \cdot 2} (n-2)^n + \&c.]$$

15. If u_x be a function of x of the form $u_x = b_1 x + b_2 x^2 + \dots$ ad inf., show that it can be expressed in the form

$$u_x = b_1 \frac{x}{1-x} + \Delta b_1 \frac{x^2}{(1-x)^2} + \Delta^2 b_1 \frac{x^3}{(1-x)^3} + \dots$$

where $\Delta b_1 = b_2 - b_1$, and hence find approximately the value of the series

$$\frac{1}{10} - \frac{1}{11} + \frac{1}{12} - \frac{1}{13} + \dots$$

16. Show how, from n consecutive equidistant values of a function u_x , to find its approximate general expression.

Find a rational and integral function of x which, when $x=1, 2, 3$, shall assume the respective values 4, 6, 10.

17. Deduce formulæ for the value of a function $u_{x,y+n}$, where n is less than 5, in terms of

$$(1) \quad u_{x,y}, \quad u_{x,y+5}, \quad \text{and} \quad u_{x,y+10};$$

$$(2) \quad u_{x,y-5}, \quad u_{x,y}, \quad \text{and} \quad u_{x,y+5}.$$

and apply the formulæ so deduced to the computation of the value of $a_{40.53}$, given

$$a_{40.45} = 12.995$$

$$a_{40.50} = 12.205$$

$$a_{40.55} = 11.233$$

$$a_{40.60} = 10.038$$

18. Explain fully and clearly how the operation of Finite Integration is applicable to the finite summation of series, and state what you know of the finite integration of rational functions of x .

Prove by integration that the sum of the series $x^a + x^{a+1} + x^{a+2} + \dots + x^n$ is equal to $\frac{x^{n+1} - x^a}{x - 1}$.

19. Adjust the following values of q_x by means of the graphic method of graduation.

[The Candidate will be supplied with millimetre paper.]

Age	Exposed to Risk	Deaths	q_x
75	1133	92	·0812
76	1052	108	·1027
77	944	87	·0922
78	873	93	·1065
79	796	113	·1420
80	692	90	·1301
81	623	83	·1332
82	564	92	·1631
83	498	79	·1586
84	425	70	·1647

20. What assumptions as to a law of mortality are made severally by Gompertz and Makeham in their formulæ for the numbers living at a given age?

State how far these formulæ have been found to give results substantially agreeing with actual observation, and what advantages follow their adoption in practice.

21. State concisely the methods adopted in deducing the Institute of Actuaries H^M Life Tables, as regards (a) the recording of the data; (b) the computation of the numbers exposed to risk; (c) the graduation of the results. Do you consider that the methods severally adopted were capable of improvement, and, if so, in what respects?

22. Given values of $(m\mu)_{x+\frac{1}{2}}$, the central marriage rate, and of $(d\mu)_{x+\frac{1}{2}}$, the central death rate, at all ages, how would you proceed to deduce a Combined Marriage and Mortality Table showing the numbers living, dying, and marrying at all ages?

23. The benefits granted by a Pension Fund include (1) pensions at a given age; (2) fixed sums payable at death before attainment of pension age. How would you construct, from the experience of such a Fund, the necessary Tables for computing the values of the several benefits and contributions, the element of secession being taken into account?

24. Discuss concisely the advantages or disadvantages of the several methods of deducing the numbers exposed to risk, and the rates of mortality and of withdrawal, amongst assured lives, known as the Exact Duration Method, the Mean Duration Method, and the Nearest Duration Method.

EXAMINATION FOR ADMISSION TO THE CLASS OF FELLOW
(PART III, SECTION B).

Examiners—MESSRS. T. G. ACKLAND, J. CHISHOLM, F. E. COLENSO,
H. W. MANLY, F. SCHOOLING, and H. C. THISELTON.

First Paper.

1. Describe fully the plan proposed by Woolhouse for making a rapid approximate valuation.

2. State clearly how you would provide at a valuation for the immediate payment of claims on proof of death and title, and for the varying incidence of the premium income throughout the year. Is such provision in all cases necessary?

3. Explain the methods that have been suggested for classifying a large number of Joint Life Policies for valuation purposes. What are the relative advantages and disadvantages of each?

4. What are the arguments for and against the inclusion of guaranteed surrender-values and guaranteed paid-up policies in the insurance contract? Under what circumstances, and to what extent, is it necessary to take account of such guarantees in the valuation of a life office?

5. What steps would you take to test the probability of any proposed rate of bonus being maintained at future distributions of profit?

6. Discuss the advantages and disadvantages to the participating policyholders of the acquisition of new business. What limitations, if any, as to the amount of new business and the expense of obtaining it, should, in your opinion, be fixed?

7. Give your views on the merits or demerits of any four of the following investments for the funds of a Life Office:—

Railway Ordinary Stock.
Railway Contingent Preference Stock.
Loans to Poor Law Guardians.
Loans on Personal Security.
Life Interests.
Colonial Municipal Securities.
Freehold Ground Rents.
Leasehold Ground Rents.

8. What are the various methods adopted for the periodical valuation of (a) Stock Exchange Securities; (b) Reversions? State their advantages and disadvantages.

9. To what causes do you attribute the recent decline in the rate of interest realized by Life Offices? What steps would you recommend should be taken by a Life Office with a view to mitigating the effects of such decline?

10. Explain clearly why, in purchasing separately the reversion to a fund and the life interest in it, the sum of the present market values generally differs from the realizable value of the fund. Are there any circumstances under which the full realizable value of the fund might be offered?

11. You have been consulted by a firm employing some thousands of men with reference to a superannuation scheme for their workmen. The employers are desirous of adopting a plan under which the workmen and employers contribute equally to the fund. Draft a short report on the subject, and indicate what regulations you recommend with regard to workmen leaving the service of the firm from any cause.

12. Indicate the various points to which you would direct attention, if asked to advise, in the interests of all concerned, upon the terms of a proposed acquisition by a Proprietary Life Office of the business of a Mutual Life Office.

Second Paper.

13. How would you proceed to calculate the death strain in a Life Office? What practical inference would you draw from a statement that the actual deaths in a Life Office had for a number of years been much less than the "expected" according to the mortality table used in the valuations?

14. How would you compare the financial positions and bonus prospects of two Offices, if the only available data were the Board of Trade Returns?

15. How would you proceed to compare the relative advantages to a policyholder of two Offices, one giving a compound and the other a simple Reversionary Bonus, the Offices not charging the same premiums? "

16. State generally upon what principles the surrender-values of an Insurance Office should, in your opinion, be computed, having regard, among other things, to

- (a) the pressure of the Office expenditure upon new and renewal premiums respectively;
- (b) the medical selection of the lives assured;
- (c) the voluntary withdrawal of the lives surrendering;
- (d) the class of assurance.

17. How would you calculate the single premium for a survivorship assurance on (x) against (y), coupled with a deferred assurance on (x) after the death of (y) of the same amount, such deferred assurance being subject to the normal annual premium for the age of (x) at the death of (y) with a proviso that it is never to exceed the premium for age $x+t$?

Draft the premium clause for such a policy.

✓ 18. Deduce, by first principles, the differential coefficients, with respect to x , of:—

$$(1) \log_n x; \quad \text{and} \quad (2) \bar{u}_x.$$

19. If $u = \phi(x, y)$, where x and y are connected by the equation $f(x, y) = 0$, and y is regarded as a function of x , find the differential coefficient of u with respect to x . ✓

Show how to determine the maxima or minima values of u .

20. Integration may be regarded either as the inverse of differentiation, or as the process of finding the limit of the sum of a series of values of a differential, $f(x)dx$, when x varies from one assigned value to another. ✓

Explain the connexion between these two aspects of the process.

Integrate the following functions:—

$$\sqrt{a^2 + x^2}. \quad \log_e (x + \sqrt{x^2 \pm a^2}).$$

21. If u denote the definite integral ✓

$$u = \int_a^b \phi(x, c) dx$$

state the conditions under which

$$\frac{du}{dc} = \int_a^b \frac{d\phi(x, c)}{dc} dx,$$

and prove this equation.

Establish the identity of

$$\int_0^\infty v^t \cdot {}_t p_{xyz} \cdot \mu_{z+t} \cdot \bar{a}_{x+t} dt \quad \text{with} \quad \int_0^\infty v^t \cdot {}_t p_{x\bar{A}_{x+t, y, z}} dt.$$

✓ 22. Write down, in the form of an integral, an expression for the value of \bar{A}_{wxyz}^4 , and show how you would apply a formula of approximate summation in deducing the value of the benefit, according to a given table of mortality, and at a specified rate of interest.

The Candidate, having finally handed in his answers to the foregoing questions, will be supplied with a copy of "Jones on Annuities", Vol. I, and the Institute Logarithm Card, but he should state, in his answers to Questions 23 and 24, what mortality tables and rates of interest he would consider most suitable for adoption in practice.

23. A sum of £12,500 was invested in 1885 in India $\frac{1}{2}$ per-cents to provide annuities of £250 each to two ladies, born in 1830 and 1835. In October 1888, the $\frac{1}{2}$ per-cents were converted into 3 per-cents, the annuities being thereafter made up to £250 per annum each, by sales of stock as found necessary, the stock being now reduced in consequence to £12,000. Calculate the value of the reversion to the stock.

24. A person aged 30, whose life is insurable at the rate of £2 per-cent per annum, desires to obtain as large an advance as possible

from a Life Office, in consideration of the grant by him of an annuity forming a first charge upon a well-secured life interest of £800 per annum, to which he is entitled expectant upon his surviving an aunt aged 60. He wishes to retain for the term of five years the right of redeeming the annuity by payment to the Office of all sums disbursed by it, accumulated at compound interest.

Calculate the maximum amount which, in your opinion, may be safely advanced, and draft a letter conveying the terms to which you would advise the Directors to agree.

PROCEEDINGS OF THE INSTITUTE.—SESSION 1896-97.

First Ordinary Meeting, 30 November 1896.

The first ordinary meeting of the session 1896-97 was held at the Hall of the Institute, on the 30th day of November 1896.

The President (Mr. T. E. YOUNG) in the Chair.

The President delivered an inaugural address.

Second Ordinary Meeting, 21 December 1896.

The President (Mr. T. E. YOUNG) in the Chair.

A paper "On some Legal Points arising in Life Assurance Practice", was read by the author, Mr. A. R. Barrand.

The following gentlemen took part in the discussion:—Messrs. W. Hughes, L. M. Simon, H. R. Harding, J. E. Faulks, G. King, G. Crisford, and the President.

Third Ordinary Meeting, 25 January 1897.

The President (Mr. T. E. YOUNG) in the Chair.

Mons. O. Lepreux, Directeur-Général de la Caisse Générale d'Epargne et de Retraite, and President of the Permanent Committee of Actuarial Congresses, was unanimously elected a Corresponding Member.

A paper "On Rates of Mortality in certain parts of Africa", was read by the author, Mr. A. E. Sprague, M.A.

The following gentlemen took part in the discussion:—Messrs. C. D. Higham, G. King, T. G. Lyon, M.D., A. Hewat, J. R. Hart, J. Chisholm, and the President.

Fourth Ordinary Meeting, 22 February 1897.

The President (Mr. T. E. YOUNG) in the Chair.

A paper "On Governmental Regulation of Life Insurance in the United States of America", by Mr. Sheppard Homans, was read by Mr. Wyatt, one of the Honorary Secretaries.

The following gentlemen took part in the discussion:—Messrs. J. Chisholm, F. E. Colenso, H. R. Harding, M. N. Adler, and the President.

Fifth Ordinary Meeting, 29 March 1897.

The President (Mr. T. E. YOUNG) in the Chair.

A paper "On Mortality Experience of Assured Lives and Annuitants in France", was read by the author, Mr. G. F. Hardy.

The following gentlemen took part in the discussion:—Messrs. A. B. Adlard, Geoffrey Marks, T. G. Ackland, A. H. Bailey, H. W. Manly, F. T. Mason-Byers, and the President.

The President referred to the death of Professor Sylvester, one of the Honorary Members of the Institute.

Sixth Ordinary Meeting, 26 April 1897.

The President (Mr. T. E. YOUNG) in the Chair.

A paper "On Lost Policies, Certified Copy Policies, Certificates of Title, Possession of a Policy, Notice, Bankruptcy, and some other Practical Considerations with regard to the Titles to Policies", by Dr. Sprague, was read by Mr. A. F. BurrIDGE, one of the Honorary Secretaries, in the unavoidable absence of the author.

The following gentlemen took part in the discussion:—Messrs. T. G. C. Browne, H. W. Manly, A. H. Bailey, J. R. Hart, R. Todhunter, A. F. BurrIDGE, the President, and Mr. Dawes, a visitor.

The Fiftieth Annual General Meeting, 1 June 1897.

The President (Mr. T. E. YOUNG) in the Chair.

The proceedings at the Annual General Meeting will be found on page 434.

REPORT, 1896-97.

The Council have pleasure to report to the members upon the progress of the Institute during the session of 1896-97, the forty-ninth year that it has been in existence.

The increase in the number of members in the year has been 38, as compared with 13 in that which preceded it. At the end of the year in which the Institute was incorporated by the Royal Charter the number of members was 434, while five years later, at 31 March 1890, it was 601. Since that time it has grown as follows:

At 31 March	1891	to 620,
"	1892	" 645,
"	1893	" 674,
"	1894	" 734,
"	1895	" 775,
"	1896	" 788,
"	1897	" 826.

The following schedule shows the additions, changes, and losses in the membership, which have occurred during the year ending 31 March last.

Schedule of Membership, 31 March 1897.

	Honorary Members	Fellows	Associates	Students	Corres- ponding Members	Total
i. Number of Members in each class on 31 March 1896 .	2	177	223	374	12	788
ii. Withdrawals by						
(1) Death . . .	1	7	3	3	...	43
(2) Resignation	1	1	12	...	
(3) Default in pay- ment of Sub- scriptions	3	12	...	
iii. Additions to Membership	1	169	216	347	12	745
(1) By Election	1	...	1	81
(2) By Order of Council	77	...	
(3) By Re-instatement	...	1	...	1	...	
iv. Transfers	1	170	217	425	13	826
(1) By Examination: from Associates to Fellows	5
	...	5
(2) By Examination: from Students to Fellows .	1	175	212	425	13	826
	3
	...	3
(3) By Examination: from Students to Associates .	1	178	212	422	13	826
	16
	16
v. Number of Members in each class on 31 March 1897 .	1	178	228	406	13	826

The Council have, with great regret, to report the loss by death during the year of one Honorary Member, Professor Sylvester; seven Fellows, Messrs. H. Engelbach, J. A. Higham, W. King, E. Reboul, B. Newbatt, W. Smith, and W. Wallis; three Associates, Messrs. T. J. Martin, D. F. A. Legg, and R. Poston; and three Students, Messrs. Milton Haight, S. C. T. Ramsay, and E. A. Taylor.

Professor Sylvester, whose great mathematical attainments were so well known and appreciated, was elected an Honorary Member in 1860, and was a contributor to the *Journal*.

Mr. John Adams Higham became a Fellow of the Institute under the Charter in 1884. He was a frequent contributor to the *Journal*, the first

volume containing his article "On the Value of Selection as exercised by the Policyholder against the Company", and recent volumes a series of articles and correspondence on the subject of graduation.

Mr. Benjamin Newbatt joined the Institute in 1859, and evinced a warm and energetic interest in its proceedings. In 1890-1 and 1891-2 he ably filled the position of President.

The Accounts for the year show that the total funds on 31 March last amounted to £5,284. 4s. 0d., being a decrease during the year of £589. 18s. 2d.

This apparent decrease is more than accounted for by the amount expended on the Mortality Experience Investigation, namely: £877. 8s. 11d.

The Annual Subscriptions, together with admission and other fees, amounted to £1,564. 10s. 0d., being about equal to those of the previous year.

The total Income for the year was £2,080. 8s. 0d., and the total Expenditure £2,670. 6s. 2d. Of this latter sum it will be seen that sums amounting to £1,159. 11s. 2d. have been expended on the following objects, namely: The Mortality Experience Investigation £877. 8s. 11d., and the completion of Dr. Sprague's Select Life Tables £282. 2s. 3d.

The Revenue Account and Balance Sheet are given herewith (p. 433).

The stock in hand of the Institute publications on 31 March was as follows:

No. of Copies	Description of Work
220	<i>Text-Book</i> , Part II.
40	Mortality Experience Tables.
818	Government Joint-Life Annuity Tables.
904	Select Life Tables.
457	Logarithm Cards.
380	Messenger Prize Essay (Friendly Societies).
517	Index to 10 Vols.
59	" to 20 "
1,043	" to Vols. 21 to 30.
1,036	Parts of <i>Journal</i> .

The following papers were submitted at the sessional meetings of the Institute, namely:

- 30 *November* 1896.—An inaugural address by the President, Mr. T. E. Young.
- 21 *December* 1896.—"On some Legal Points arising in Life Assurance Practice"—Mr. A. R. Barrand.
- 25 *January* 1897.—"Rates of Mortality in certain parts of Africa"—Mr. A. E. Sprague.
- 22 *February* 1897.—"Governmental Regulation of Life Insurance in the United States of America"—Mr. Sheppard Homans.
- 20 *March* 1897.—"Mortality Experience of Assured Lives and Annuitants in France"—Mr. G. F. Hardy.
- 26 *April* 1897.—"On Lost Policies, Certified Copy Policies, Certificates of Title, Possession of a Policy, Notice, Bankruptcy, and some other practical considerations with regard to the titles to Policies"—Dr. Sprague.

For the Examinations held in the United Kingdom on 23, 24, 26, and 27 April last, 170 candidates presented themselves, namely:

67	for Part	I.
55	" "	II.
23	" "	III, Section A.
25	" "	III, " B.

Of these the following numbers were successful:

36	in Part	I.
26	" "	II.
12	" "	III, Section A.
11	" "	III, " B.

The following are the successful candidates, the names in each class being arranged alphabetically.

PART I.

Examiner—PROF. S. L. LONEY.

Supervisors—MESSRS. C. D. HIGHAM and G. MARKS.

Class I:

C. Hogg.		R. J. Kimber.
E. V. Townshend.		

Class II:

J. A. Cherry.		J. Patrick.
W. A. Curtis.		W. Penman.
H. B. Harding.		S. H. Pipe.
N. F. B. Osborn.		F. H. Sherriff.
J. R. Smeaton.		

Class III:

T. F. Anderson.		L. L. Littell.
R. F. M. Bigby.		G. H. Maunder.
P. C. Crump.		M. B. Neale.
J. Dalton.		J. M. Ramwell.
C. Davey.		G. W. Richmond.
G. W. V. Fisk.		F. C. Robinson.
S. Harper.		F. Sanderson.
W. R. Hines.		E. C. E. Sharpe.
B. Humphrey.		O. B. Shute.
J. B. Jepps.		F. P. Symmons.
H. L. Jupp.		D. J. Wood.
A. M. Knight.		R. H. Younger.

PART II.

Examiners—MESSRS. G. J. LIDSTONE, W. P. PHELPS,
E. R. STRAKER, and F. T. MASON BYERS.

Class I:

S. Macnaghten.

Class II:

A. V. Lane.		W. C. Sharman.
S. W. Newling.		W. A. Workman.
A. W. Woolfe.		

Class III:

H. L. Adam.	J. E. S. Kemp.
C. R. Coop.	F. W. Le Maitre.
A. W. Findlay.	A. J. Mascal.
T. Gossett.	A. McDougald.
G. Gillies.	W. E. Norton.
J. Goodwyn.	C. A. Penny.
F. S. Goggs.	H. J. Rietschel.
F. B. Galer.	E. Ryley.
Wm. Hayercraft.	H. Slade.
S. Hazel.	J. E. Shimmell.

PART III.

Examiners—MESSRS. H. W. MANLY, T. G. ACKLAND, J. CHISHOLM,
F. E. COLENSO, F. SCHOOLING, and H. C. THISELTON.

SECTION A.

Class I:

None.

Class II:

†K. W. Elder.	J. Spencer.
†O. Kentish.	*C. T. Weeden.

Class III:

S. J. H. W. Allin.	*D. C. Fraser.
H. H. Austin.	A. Lowndes.
E. J. Bull.	*M. Rees.
†R. H. Fellows.	C. E. Reeve.

SECTION B.

Class I:

None.

Class II:

†N. Miller.	*C. T. Weeden.
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Class III:

†J. M. Allen.	F. Marchbank.
A. C. R. Cockman.	B. C. Morgan.
*D. C. Fraser.	*M. Rees.
†G. E. May.	H. M. Trouncer.

†L. A. Wintle.

Those marked (*) passed in both sections, and with those marked (+), have now completed the examination for the Class of Fellow.

In the Colonies the Examination entries numbered 51, as under:

For Part	I, 24.
„	II, 18.
„	III, Section A, 3.
„	III. „ B, 6.

The results of the Colonial Examinations will be duly announced.*

* These results are given on page 440.

The Council had the pleasure of announcing in November last that Mr. James Chisholm had kindly offered three prizes of the value of Thirty Pounds, Fifteen Pounds, and Ten Pounds respectively, for the three best essays on "The Relation of the Actuarial Profession to the State." The competition is open to all Members of the Institute, except Members of the Council and those who have gained prizes for essays offered by or through the Institute. Essays are to be sent to the Honorary Secretaries not later than the 31st December 1897.

Dr. Sprague's Select Life Tables deduced from the Institute of Actuaries' Experience (Healthy Males), the additional calculations for which were made under the superintendence of Mr. H. C. Thiselton, were published in 1896.

The Council are also pleased to report that in October 1896, Mr. George King was appointed Honorary Editor of the *Journal*, in succession to Mr. Ryan, whose resignation of the post was announced in last year's report. Pending the appointment of his successor, Mr. Ryan had kindly undertaken to continue the duties of Editor.

It was thought desirable that there should be two Honorary Sub-Editors, and Messrs. A. Levine and H. A. Thomson were accordingly appointed to these posts.

During the past session a series of six lectures on Legal Subjects have been delivered by Mr. G. Wood Hill, Barrister-at-Law, before the Members of the Institute in Staple Inn Hall. The Council observe with gratification that these lectures have been well attended, and that they have doubtless been of material assistance to those about to enter for the final examinations.

Encouraged by the success which has attended these lectures, the Council are endeavouring to make arrangements for lectures on other subjects, of close interest to actuarial students, to be delivered during the next session.

Considerable progress has been made with the new Mortality Experience Investigation, which is being carried out jointly by the Institute of Actuaries and the Faculty of Actuaries. The total number of contributing Offices is 66. 60 Offices contribute their experience in respect of Assured Lives, and 37 of these contribute also their experience in respect of Annuity, while 6 Offices contribute their experience of Annuity only.

The investigation is being actively carried on under the superintendence of Mr. T. G. Ackland, who in July 1896 undertook the duties of Official Supervisor, with the assistance of a competent staff of clerks.

Latterly the work has been concentrated on the Annuity Experience, and the Council have every hope that very shortly the results of this experience will be ready for publication.

It has been arranged that the second International Congress of Actuaries shall be held in London, under the auspices of the Institute, on the 16th May 1898 and the four following days. Full particulars as to qualification and other information will be found in the number of the *Journal* for April 1897.

The Council are again much indebted to the Honorary Examiners, who have bestowed great care and labour on the important work entrusted to them.

1896-97.	£	s.	d.	£	s.	d.
Amount of Funds, 31 March 1896—						
Messenger Legacy Fund				288	2	1
Brown Prize Fund				235	1	5
Library Fund				298	5	0
General Fund				5,052	13	8
				5,874	2	2
Annual Subscriptions—						
Town Fellows	£166	19	0			
Country "	63	0	0			
Fellows admitted since 1884:						
(1) By Examination	211	1	0			
(2) By Ballot	31	10	0	472	10	0
Town Associates						
Country "	161	14	0			
Associates admitted since 1884:						
(1) By Examination	138	12	0			
(2) By Ballot	42	0	0	408	9	0
Students				426	6	0
				1,307	5	0
Two Annual Subscriptions compounded for				21	0	0
Entrance Fees—						
Associates				4	4	0
Students				85	1	0
				89	5	0
Examination Fees				147	0	0
Sales of Publications—						
Journal				131	2	6
Text-Book, Part. I.				28	8	6
" " II.				101	15	9
Logarithm Cards				0	18	2
Institute Life Tables				13	1	0
Government Annuity Tables				11	4	0
Select Life Tables				57	6	1
Hardy's Friendly Societies				0	8	0
				344	4	0
Less Cost of Binding				10	5	0
				333	19	0
Received for Use of Hall				21	0	0
Dividends and Interest				160	19	0
				£7,954	10	2

Balance Sheet, 31 March 1897.

LIABILITIES.	£	s.	d.	£	s.	d.
Messenger Legacy Fund (£211. 1s. 10d. Consols), cost	203	17	8			
Accumulated Dividends	89	17	5	293	15	1
Brown Prize Fund (£200 Metropolitan Railway 4 per-cent Debenture Stock), cost	200	0	0			
Accumulated Dividends	42	16	1	242	16	1
Library Fund				274	4	8
General Fund				55	0	0
Chisholm Prize Fund				4,473	8	2
London and Westminster Bank—Advance				150	0	0
				£7,954	10	2
ASSETS.	£	s.	d.	£	s.	d.
Consols (£2,490. 14s. 9d.), cost				2,386	10	0
Metropolitan Railway 4 per-cent Debenture Stock (£1,050), cost				1,185	11	3
Great Eastern Railway 4 per-cent Debenture Stock (£800), cost				1,031	10	3
Great Northern Railway Preferred Ordinary Stock (£600), cost				703	11	6
On Current Account at London and Westminster Bank				56	1	6
On Special Account (Ackland & Others)				62	19	6
Arrears of Subscriptions				63	0	0
				£7,954	10	2
Examined and found correct, 30 April 1897.						
E. H. HOLTY,						
GEO. R. JELLINEK,						
H. R. HARRISON,						
Auditors.						

PROCEEDINGS AT THE ANNUAL GENERAL MEETING.

The Annual General Meeting of the members was held at Staple Inn Hall, on Tuesday, 1st June, the President, Mr. T. E. Young, B.A., in the chair.

The Report of the Council (given on p. 427) having been read,

The PRESIDENT said:—In now formally moving the adoption and circulation of the report and accounts, I take the opportunity of first referring to the large expenditure which has characterized the session, and to the gradual reduction in the admission of students during recent years. Both features, I venture to think, constitute subjects of congratulation, though from different points of view. The greater portion of the enhanced outlay appertains to the cost of the Mortality Investigation and possesses merely a temporary character, since it will be refunded by the contributions of companies and the sales of the volumes to be issued. But another portion is happily of a more permanent nature, and embodies the unwearied efforts of the Institute to enlarge its scope of service and utility by furnishing appropriate educational advantages to its rising members, and thus conducing, in the admirable phrase of our original constitution, “to the elevation of the status of the profession.” This concentrated educational movement, in fulfilment of obligations implicitly accepted by the Institute, will command the sympathy and encouragement of all its friends. A scientific society, I conceive, is violating the essential conditions of its existence when it endeavours largely to accumulate funds. It only realizes its paramount duty when that accumulation embraces the specific purpose of conferring upon its younger members an ampler and higher professional education than their seniors were competent to command. The gradual, though recently slow, decline in the number of admissions to our ranks of students is not to my mind a feature of regret, so long as a reasonable supply is maintained, and so long especially as that supply largely consists of young men who have really discovered their true vocation in our craft, and does not comprise simply a number of youths unhappily attempting one sphere of work after another, under the deplorable stress of competition. It has always been to me a subject of pathetic concern to observe the large number of young men flocking to our hall, where the ambitions—even the most moderate ambitions—of so few can be realized. Our scope is limited, and is unfortunately becoming more and more contracted through the changing conditions of assurance administration; and touched, therefore, with commiserative feeling for these youthful seekers, I do not myself deplore the fact that the straight gate to actuarial life is less thickly thronged. During the course of the session three eminent members of the Institute have passed away. In Mr. John Adams Higham we possessed an actuary of stimulating freshness and originality of mind, and it has always appeared to me that his papers on Selection, in 1850 and in 1851, when the subject was comparatively unexplored, will always rank pre-eminently amongst the most sagacious and stable contributions to the practical literature of our work—“the literature of power”, according to the famous diction of Wordsworth. In Mr. Newbatt we beheld a happy example, in these days of nebulous thinking and indecisive speech, of a man who always gained a clear vision of definite views, and was then equally competent to embody those views in forms of lucid and incisive expression; while in Professor Sylvester we have lost the honour of living connection with an imperishable renown. The void which they have thus left in our community is not all a void, for the record of their earnest labours constitutes a power of encouragement and faith to their successors, and a scientific body is enriched, not only in proportion to the hopeful future of its present members, but no less in proportion to

its impressive accumulation of memories of strenuous examples in the past. In more definite and systematic pursuance of the educational effort which has inspired the Institute from its origin, it is of happy augury to notice that the recent lectures upon the "Law of Real Property" have attracted large attendances, and have been received, as I have been informed from numerous quarters, with genuine satisfaction and profit. The Council propose to continue that scheme and to extend it in the direction of lectures upon finance and upon other subjects directly germane to our work. We thus hope to aid our students in the most effective manner for successfully meeting the special demands upon their thought and capacity, which the administration of assurance now imperatively requires, by drawing them away from the comparatively sterile region of mere book knowledge, and confronting them with personal teaching by lecturers, practically conversant with their subjects, and placing them, as far as lies within our scope, in direct contact with actual commercial and financial problems. In intimate connection with this subject, I gladly refer to the character and range of the papers which have been discussed during the session, and the direct help which they are competent to afford to students in relation to this practical aspect of our professional labours. Another very noticeable and encouraging feature is observable in the fact that two members have for the first time contributed papers of value and service, whose example, I trust, will stimulate many others to reveal the latent ability which I am sure is possessed in abundance by our younger men, and thus both extend the boundaries of our knowledge and gain for themselves our cordial recognition of their personal capacity, and of their loyal devotion to the interests of their *Alma Mater*. A very admirable opportunity is presented in this direction through the medium of the prizes which Mr. Chisholm has kindly offered. The subject of the competition is no doubt somewhat difficult, but let me remind Associates and Students that even failure to obtain a prize will be amply compensated by the stimulus to thought and research which the enquiry will entail—results of enduring value, which will leave a permanent impress upon character and capacity, quite apart from the subsidiary and temporary question of success. I rejoice also that our range of honorary distinctions has been enlarged by the appointment of two Sub-Editors of the *Journal*. It has always appeared to me desirable, especially in a small scientific community, to possess as wide a hierarchy as possible of official positions, so that opportunities may exist for fulfilling the just ambitions of men of promise and achievement by assigning to them specific posts of usefulness in the service of our professional community. I would refer for a moment to the vast and honourable enterprise in which the Institute of Actuaries and the Faculty of Actuaries are jointly engaged, and you will be interested in learning a few details with regard to the Mortality Investigation with which, through the kindness of Mr. Ackland, I am able to furnish you. The annuity experience is recorded upon upwards of 32,000 cards, of which 9,000 relate to males and 23,000 to females, so that the female experience is about 72 per-cent of the total facts. All predictions of completion in so complex an investigation must necessarily be interpreted with very indulgent latitude, but it is expected that the number exposed to risk in connection with the formation of the Select Annuity Tables will be scheduled and completed within three or four months' time, when the l_x and d_x columns can be prepared in readiness for graduation. The aggregate annuity table is expected to be similarly completed during the course of our ensuing autumn session. The experience of assured lives is contained upon upwards of 1,100,000 cards, and it has been approximately estimated that the period of risk will comprise upwards of ten million years. The males number upwards of 1,000,000, and the females upwards of 80,000, so that the female experience is about $7\frac{1}{2}$ per-cent of the

combined experience, as compared with 72 per-cent of the annuity experience. No exaggeration of language, it is obvious, is involved in the statement that this enterprise will constitute a monumental addition to the public services, and the statistical usefulness of the Institute, and to me it is equally a pleasure, as it is a duty, to express my admiration of the vigorous and workmanlike manner in which Mr. Ackland and his assistants are prosecuting the work. Before closing, I indulge in the ambitious hope that the meeting of the International Congress to be held during the coming session should prove a signal success by reason of the earnest and vigorous co-operation of the members, and the presentations of papers worthy of a unique occasion. Assembling, then, for the first time in the historic home of actuarial science, no effort should obviously be lost that the gathering, both in its character and consequences, shall be in harmonious correspondence with the high traditions of the Institute, which will inspire its labours. In conclusion, I should be doing injustice to my feelings if I did not express a word of hearty congratulation to those gentlemen who have passed their examinations, and an equally hearty word of hope and encouragement to those who on this occasion have failed. The record now presented furnishes conclusive evidence, I think, that the Institute is worthily embodying in larger and wider, but consistent forms, the principles so wisely advocated by its founders, and like all organizations which are destined to be permanent, because they fulfil a definite office in the social system, the Institute is constantly and thoughtfully utilizing its experience for the purpose of maintaining and directing its ceaseless advance—a type at once of an effective machinery and of a source of power. I have now much pleasure in moving that the report and accounts be adopted and circulated.

Mr. GEO. KING, in seconding the adoption of the report, said that the chairman had so eloquently and completely touched upon all the principal points that he only need say a very few words. The report before them was, he thought, a very remarkable one. Every paragraph spoke of activity, and it was evident that the Institute was not losing anything of its vitality: in fact, it was becoming more vigorous and active as it grew older. At the present time "Jubilee" was in the air, and it was worth remembering that next year the Institute would reach its own jubilee. He thought it was a happy coincidence that that event should occur with the meeting of the Congress, and it ought to stimulate them to use all the more effort to make the Congress a success. The Council, he knew, would do all it could in that direction, but success would depend upon all the Fellows and Associates taking an interest in the Congress by becoming members and by contributing papers. The President had referred to the satisfaction occasioned by the fact that two of the papers read at the sessional meetings were by new contributors. With regard to that, he might add that there was another opening for new contributors, who, perhaps, had not a subject of sufficient magnitude to bring before a sessional meeting, but yet who had something worthy to communicate in the *Journal*. In the forthcoming number there would be several actuarial notes of very great interest, and he would invite the members to contribute any note of subjects that they might meet with in their studies or in their professional practice, even should it not be of sufficient importance to elaborate into a long paper. The question in which he had taken the greatest interest for many years was that of education, and he was glad to say that the Institute was now carrying on that work more vigorously than ever, the result being that there was a large development in student membership, and that the students passed on to the higher grades and became Fellows very rapidly. The Institute was doing all it could to assist in the movement of educating the younger members. The President had eloquently referred to the lectures which had

been given during the past session, and which would be given in succeeding sessions. The provincial students had sometimes complained that they did not get the full benefit of those lectures, as they were unable to attend, but he did not think he was committing any breach of confidence when he said that the Council was doing all it could to assist such students by publishing in a separate volume the lectures delivered during the past session by Mr. Wood Hill. In that way the lectures would be put in a permanent form, and the provincial men would get the full benefit of them. With regard to the examinations, he might say that the Examiners, as usual, had had a great deal of work to do, which he hoped would be acknowledged by a suitable vote. He might, perhaps, be allowed to refer to a matter in one of the reports of the Examiners, where it was stated "There is strong evidence that the candidates rely too much upon the information conveyed by their tutors, and do not read and study sufficiently in their preparation. We consider that it should be clearly understood that it is necessary that the student should have an intelligent knowledge of first principles in each of the subjects, and be able to grasp the full significance of the questions; he should not, in fact, depend so largely upon committing to memory a number of stock answers to general questions." He read that extract because it so entirely embodied his own views, and was so entirely in accordance with the somewhat lengthy experience he had had of the examinations. It must be remembered that it was not what we learn that makes our intellect; it is what we think. The learning was merely the raw material brought to the mill, and it must be turned into the manufactured article by pondering and thinking it over, and working it up in their own minds. An actuary in that way would not only increase his own knowledge, but he would increase his practical power of applying that knowledge; and he would strongly urge upon all, whether they be students or not, the necessity of thinking more, even if it should result in reading less. A little knowledge thought well over was more useful than a great deal of knowledge simply gathered together and put away without being made use of. He had much pleasure in seconding the motion for the adoption of the report.

The motion was then put from the chair and unanimously agreed to.

The ballot for the election of Officers and Council for the ensuing year then took place, Mr. Ackland and Mr. Byers acting as scrutineers, with the result that the following were unanimously elected:—

President.

THOMAS EMLEY YOUNG, B.A.

Vice-Presidents.

HENRY WILLIAM MANLY.
HENRY COCKBURN.

| WILLIAM HUGHES.
| GERALD HEMMINGTON RYAN.

Council.

ALFRED BARTON ADLARD.	GEORGE KING.
MARCUS NATHAN ADLER, M.A.	FRANCIS LAING.
HENRY WALSINGHAM ANDRAS.	HENRY WILLIAM MANLY.
THOMAS G. C. BROWNE.	GEOFFREY MARKS.
ARTHUR FRANCIS BURRIDGE.	*WILLIE OSCAR NASH.
JAMES CHISHOLM.	*PHILIP LEWIN NEWMAN.
HENRY COCKBURN.	*GERALD HEMMINGTON RYAN.
GEORGE STEPHEN CRISFORD.	*JAMES SORLEY.
ROBERT CROSS.	THOMAS BOND SPRAGUE, M.A.
JOHN JAS. WALKER DEUCHAR.	WILLIAM SUTTON, M.A.
ALEX. JOHN FINLAISON, C.B.	HERBERT CECIL THISELTON.
NIEL BALLINGAL GUNN.	SAMUEL GEORGE WARNER.
*RALPH PRICE HARDY.	WM. JOS. HUTCHINGS WHITTALL.
AUGUSTUS HENDRIKS.	FRANK BERTRAND WYATT.
WILLIAM HUGHES.	THOMAS EMLEY YOUNG.

* Not Members of the Council for 1896-7.

Treasurer.

JAMES CHISHOLM.

Honorary Secretaries.

FRANK BERTRAND WYATT. | ARTHUR FRANCIS BURRIDGE.

Mr. H. W. MANLY, in responding on behalf of the Officers just elected, assured the members that he considered it a great privilege to be associated with the Institute in any possible way. The work was a growing one, and the labour placed upon the Hon. Secretaries at the present time was exceedingly great. It was a delight to serve the Institute in attending its committees and furthering its interests in all directions; and it was an honour and distinction held out to all who showed themselves worthy as members of the Institute, that they might at some time become a portion of its governing body.

Mr. SEABLE then moved "That, as auditors for the ensuing year, Messrs. Jellicoe and Harding be re-elected, and that Mr. C. H. E. Reed be elected in the place of Mr. Holt, who retires in accordance with the by-laws."

Mr. SHEPPARD seconded the motion, which was agreed to.

Mr. G. H. RYAN, in proposing a vote of thanks to the Officers of the Institute, said that the members had listened to a very eloquent account of the good fortune which the Institute had experienced during the past year, save only in respect of the sad loss they had sustained by the death of old and valued friends. He did not propose to travel over the ground again, or to offer any pale reflection of the eloquent words which had fallen from the President. Indeed, it seemed to him that a fairer estimate of the extent of their indebtedness to the Council and Officers might be formed if they took a wider retrospect, and considered what the Institute had done, not only during the past year, but during the fifty years of its existence. To his mind the greatest achievement of the Institute lay in the fact, not that it had been able to supply the demand, but rather that it had created the demand, and enforced the necessity for such trained skill and services as its members were able to afford. So much so that they could now look round and claim that there was scarcely an insurance institute of any prominence which did not in one capacity or another avail itself of their services. That, he thought, was an achievement of which they might all feel proud, and which accounted in a very great measure for the feeling of affection which they all had for the Institute, and the gratitude which they bore to their

leaders, past and present, and to the far-seeing men who had laid the foundations—unambitiously at the time—of their Institute. Again, it was worth while to point out the extraordinary influence which the Institute had been able to exert, and still wielded, through the individual efforts of its members, upon the financial well-being and the sound development of the business of life assurance. He could not imagine what the condition of life assurance business would have been at the present time had the Institute of Actuaries not been founded fifty years ago. The work of the Institute had been performed unostentatiously and unobtrusively. They had not been overburdened by State help, nor had they basked in the sunshine of popular favour; nevertheless, the work which they had done was of the utmost value, and he had little hesitation in saying that if the achievements of the Institute were compared with those of other kindred societies, it would be found to occupy a most creditable if not a unique position among them all. For that result they were deeply indebted to the past and present Councils. He did not propose to deal specifically with the individual services of their distinguished President, Mr. Young, which they so thoroughly appreciated, or the support which the Council had rendered him, or upon the services of their hard-working and affable Hon. Secretaries; but he should call special attention to the great services performed by the editor of the *Journal* and his assistants, and the Hon. Examiners. The work of the officials was most arduous, and was of the utmost value to the Institute; and as “thanks” was the only currency with which they could repay them, he thought the members should, on an occasion like the present, not stint them in the thanks which they rendered them for the gratuitous services and the hard work performed in the Institute’s behalf. He had great pleasure in proposing “That a cordial vote of thanks be presented to the President, the Council, and the Officers of the Institute for their services during the past year.”

Mr. SORLEY, in seconding the motion, said the Institute was to be congratulated in having a President like Mr. Young. The monumental address he had delivered at the commencement of the session, and the eloquent and business-like manner in which he had summed up the work of the session that afternoon, would convince anyone that they had the right man in the right place at the head of the Institute. His great industry, his research, and his knowledge of so many branches of learning, which to them were but a name, would stimulate their exertions not only in actuarial fields, but in other fields as well. The services rendered by the other officers of the Institute had been alluded to by Mr. Ryan, and he endorsed all that that gentleman had said, particularly with regard to the very great labour which the examiners had taken on behalf of the Institute, and the great obligation under which it rested with regard to their services.

The motion was carried unanimously.

The PRESIDENT, in acknowledging the vote, said—I am sure I am acting as a faithful medium of the feelings of the Council, and of my colleagues, the other officials, in stating that in our affection for the Institute it is a delight to live laborious days in its service, but it is obvious that a kindly expression of a feeling, such as that which has been proposed and accepted, adds a very attractive distinction to the reward which we very heartily appreciate. I thank you for your goodness.

Mr. JOHN HOLLIDAY, M.A., moved a hearty vote of thanks to Mr. E. H. Holt, Mr. Geo. R. Jellicoe and Mr. H. R. Harding for their services as auditors during the past year, and Mr. Harcastle seconded the motion, which was carried unanimously.

Mr. G. R. JELlicoe having replied,

The PRESIDENT, in adjourning the meeting until November 29, called attention to the illuminated congratulatory address which it was proposed to present to Her Majesty on the completion of the sixtieth year of her reign.

COLONIAL EXAMINATIONS.

Examinations were held on 23, 24, 26, and 27 April, at Sydney, Melbourne, Adelaide, Montreal, and Toronto, with the following results:

PART I.

Twenty-four Candidates sent in their names, of whom eighteen presented themselves, and twelve passed as follows:

Class I:

Catchlove, C. H. L. (Adelaide).
 Dick, W. T. (Sydney).
 Donaldson, J. S. (Sydney).
 Kelly, J. J. (Sydney).
 Moore, J. P. (Sydney).

Class II:

Barry, D. (Melbourne).
 Touzel, P. D. (Melbourne).

Class III:

Dunn, W. J. (Sydney).
 Kaufman, H. N. (Montreal).
 Melville, C. E. (Melbourne).
 Smibert, C. R. (Melbourne).
 Williams, H. S. W. (Melbourne).

PART II.

Eighteen Candidates sent in their names, of whom twelve presented themselves, and four passed as follows:

Class I:

Little, J. F. (Sydney).

Class II:

Macauley, T. B. (Montreal).

Class III:

Diamond, G. F. (Sydney).
 Norris, C. A. (Melbourne).

PART III (SECTION A).

Three Candidates sent in their names, of whom two presented themselves, and one passed as under:

Class III:—

* Moors, E. M. (Sydney).

PART III (SECTION B).

Six Candidates sent in their names, of whom four presented themselves, and passed as follows:

Class II:

* Moors, E. M. (Sydney).

Class III:

† Bradshaw, T. (Toronto).
 † Day, W. R. (Sydney).
 Elliott, C. A. (Sydney).

Those marked (*) passed in both sections, and with those marked (†), have now completed the examination for the Class of Fellow.

Additions to the Library.

THE following works have been added to the Library since the publication of the *Journal* for October 1896 :

- By whom presented
(when not purchased).*
- Accountants and Auditors, Society of.**
List of Members, &c. 1897.
- Actuaries, Institute of.**
Constitution and Laws. 1869, 1873. *H. W. Manly.*
- Airy (G. B.).**
The Theory of Errors. *Purchased.*
- Allen (Joseph).**
There is Dust in John's Eyes ; or, American and British Life Insurance Offices contrasted. 1882. *L. M. Simon.*
- Associated Scottish Life Offices.**
First and Second Report of the Committee on the Mortality among Persons engaged in the Sale of Intoxicating Liquor. *Purchased.*
- Australian Mutual Provident Society.**
Forty-eighth Annual Report, 1897. *The Society.*
- Babbage (Charles).**
Specimen of Logarithmic Tables printed with different coloured inks on variously coloured papers. 1831. *Ralph P. Hardy.*
- Barclay (Robert).**
The Silver Question and Gold Question. *Purchased.*
- Barlow (Thos.).**
Memoir of the late Griffith Davies. 1895. *L. M. Simon.*
- Bateman (Thomas), M.D., F.L.S.**
Reports on the Diseases of London, 1804-1816. } *The London Life Association.*
- Becher (John Thomas), M.A.**
Friendly Societies' Tables. 1825.
- Beck (Richd. L.).**
Practical Hints on Life Assurance. 1854. *L. M. Simon.*
- Begbie (Dr. Jas.).**
Observations on the Mortality of the Scottish Widows' Fund and Life Assurance Society. 1847. *L. M. Simon.*
On the causes of death in the Scottish Widows' Fund and Life Assurance Society. 1860. *L. M. Simon.*
- Belgium.**
Bulletin de l'Association des Actuaire Belges. *L'Association.*
Bulletin du Comité Permanent des Congrès internationaux d'Actuaire. 6 copies. *The Committee.*
Compte Rendu des Operations et de la Situation de la Caisse Générale d'Epargne et de la Retraite. 1895, 1896. } *Belgian Government.*

*By whom presented
(when not purchased).*

- Bevan (B.).
A Practical Treatise on the Sliding Rule. 1822. } *The London Life Association.*
- Blane (Sir Gilbert), Bart.
Select Dissertations on Several Subjects of Medical Science. 1822. }
- Boole (George), F.R.S.
A Treatise on Differential Equations. Third edition. 1872. *W. Stott.*
- Bourne (J. P.).
Contingent Reversions. *Purchased.*
- Bourne's Handy Insurance Directory for 1895, 1896, and 1897. *Wm. Schooling.*
- Bourne's Handy Assurance Manual for 1894, 1895, and 1896. *Wm. Schooling.*
- Bowser (Wilfrid A.).
Friendly Societies' Valuation and other Tables. 1896. *The Compiler. Purchased.*
- Breuer (A.).
La Réserve Mutuelle ou l'Assurance-Vie de Pagotille. *The Author.*
- Brooke (Henry James), F.R.S.
Observations on a Pamphlet entitled "A View of the Rise and Progress of the Equitable Society." 1828. } *The London Life Association.*
- Brown (Saml.).
The Mutual Life Assurance Society, Report of the Actuary on the Rates of Ordinary and Extra Premiums, on the Profit and Non-Profit Scale. 1854. *L. M. Simon.*
- Buckley (H. Burton), M.A.
The Law and Practice under the Companies Acts 1862 to 1893, and the Life Assurance Companies Acts 1870 to 1872. Seventh Edition by the Author, and A. C. Clauson, M.A. *Purchased.*
- Burchell (Jas.) and Burchell (Wm.).
A Short Address to the Members of the Mutual Life Assurance Society. 1882. *L. M. Simon.*
- Burdett (Henry C.).
A Practical Scheme for Old-Age Pensions. (Reprinted from the *Times*). 1896. *Purchased.*
- Canada, Dominion of.
Report of the Superintendent of Insurance for the Year ending 31 December 1895. *T. B. Macaulay.*
- Christison (Dr. Robt.).
An Investigation of the Deaths in the Standard Life Assurance Company. 1853. *L. M. Simon.*
Another Copy. } *The London Life Association.*
An Investigation of the Deaths in the Standard Life Assurance Company. 1858. *L. M. Simon.*

- By whom presented
(when not purchased).*
- Coglan (T. A.).**
The Seven Colonies of Australasia, 1895-6.
Wealth and Progress of New South Wales. Vol. I. } *The Author.*
1895-6.
- Cotsworth (M. B.).**
The Direct Calculator. *Purchased.*
- Croudace (John).**
Interest Tables. 1822. { *The London Life Association.*
- Dunbar (Charles F.).**
Theory and History of Banking. *Purchased.*
- Equitable Life Assurance Society.**
Memorandum and Articles of Association, to be
submitted to an Extraordinary General Meeting
of the Society. 1892. *L. M. Simon.*
- Farley (Richard).**
Tables of Six-Figure Logarithms. 1859. { *The London Life Association.*
- Fidelity Mutual Life Association.**
Dedication of Building. Convention of Agents. 1896. *L. G. Fouse.*
- Fox (Joseph John).**
On the Vital Statistics of the Society of Friends. 1859. *L. M. Simon.*
- Germany.**
Repertorischer Assecuranz Almanach. 1897. *The Editor.*
Zeitschrift des Königlich Preussischen Statistisehen { *German Government.*
Bureaus. 1896.
- Hand in Hand Insurance Company.**
Bi-Centenary Notice, 12 November 1896. *The Secretary.*
- Hardy (Peter).**
Life Assurance, its Schemes, its Difficulties, and its
Abuses. 1852. *L. M. Simon.*
- Henry (Jardine).**
Letter to A. G. Finlaison on Report and Observations
on the Mortality of the Government Life An-
nuityants, 1860. 1861. *L. M. Simon.*
- Holland.**
Bouwstoffen voor de Geschiedenis van de Levensverze- { *The Algemeene
Maatschappij van
Levensverzekering
en Lijfrente.*
keringen en Lijfrenten in Nederland. { *Compagnie
"Utrecht."*
Catalogue de la Bibliothèque concernant les Assurances {
sur la Vie et les Sciences accessoires. {
Jaarboekje van de Vereeniging voor Lebensverzekering.
1897. *E. W. Scott.*

		<i>By whom presented (when not purchased).</i>
Hudson (G. E. Mogridge).		
Heredity in Life Assurance.		<i>Henry Cockburn.</i>
Life Assurance Companies' Returns for 1896.	{	<i>The Board of Trade.</i>
Life Tables.		
Manuscript.	{	<i>The London Life Association.</i>
Liverpool and London and Globe Insurance Company— Past, Present, and Future. 1874.		<i>L. M. Simon.</i>
London Life Association, Prospectus of the. 1866.		<i>L. M. Simon.</i>
Maingie (Louis).		
L'Assurance contre L'Invalidité et la Vieillesse en Allemagne. Brussels, 1897.	}	<i>The Author.</i>
Répartition et Capitalisation.		
Merriman (Mansfield).		
A Text Book on the Method of Least Squares. 1885.		<i>Purchased.</i>
Morgan (Arthur).		
Supplement to "the Short Account" of the Equitable Life Assurance Society. 1857.		<i>L. M. Simon.</i>
Address to the General Court of the Equitable Life Assurance Society. 1859.		<i>L. M. Simon.</i>
A Reply to the Statements circulated respecting the Equitable. 1865.		<i>L. M. Simon.</i>
Moxon (T. B.).		
English Practical Banking.		<i>Purchased.</i>
Neison (Francis G. P.).		
Mortality of Master Mariners. (<i>Quarterly Journal of Statistical Society of London</i>). 1850.		<i>L. M. Simon.</i>
Reports on the Madras Medical Fund. 1856.	}	<i>G. H. Ryan.</i>
Report on the Bengal Military Fund. 1854.		
Report on the Bengal Military Fund, with Tables. 1855.		
New South Wales.		
Insurance Institute of. Sessional Proceedings.		<i>The Institute.</i>
Official Year Book of the Scientific and Learned Societies of Great Britain and Ireland. 1897.		<i>Purchased.</i>
Park (James Allan).		
A System of the Law of Marine Insurances. 2 Vols. 1817.	{	<i>The London Life Association.</i>

*By whom presented
(when not purchased).*

Parliamentary Papers.

Acts.

Building Societies. 1894.
 Customs and Inland Revenue. 1893.
 Fatal Accidents Enquiry (Scotland). 1895.
 Finance. 1894-5.
 Friendly Societies. 1893-5.
 Industrial and Provident Societies. 1893-5.
 Local Government. 1894.
 Mortgagees Legal Costs. 1895.
 National Debt Redemption. 1893.
 Registration Acceleration. 1894.
 Trade Union (Provident Funds). 1893.
 Trustee. 1893-4.
 Voluntary Conveyances. 1893.

Purchased.

Bills.

Friendly Societies Act Amendment (2).
 Funeral Expenses of Children Insurance.
 Infant Life Protection.
 Insurance of Children Registration (Scotland).
 Land Transfer (3).
 Old Age Pensions (4).
 Trade Union Provident Funds.

Colonies.

New South Wales.

Annual Report on Vital Statistics for 1895 and previous years. *The Government Statistician.*
 Statistical Register of the Colony of, for 1895 and previous years. *The Government Statistician.*

New Zealand.

Registrar-General's Report on the Census of, for 1896. *N.Z. Government.*
 Friendly Societies. Nineteenth Annual Report of the Registrar. 1896. *The Registrar.*
 Official Year Book for 1896. *N. Z. Government.*
 Statistics of the Colony of, for the year 1895. *N. Z. Government.*

Victoria.

Friendly Societies. Eighteenth Annual Report of the Registrar. 1896. *The Registrar.*
 Statistical Register of the Colony for 1894. *The Government Statist.*

Western Australia.

Conversion and Consolidation of the Public Debts of the several Australasian Colonies. Report on proposed scheme of The Hon. Sir Philip Fysh, K.C.M.G., by the Government Actuary, Edgar T. Owen. *The Government Actuary.*

Friendly Societies.

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(*Messrs. C. & E.*

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Goschen, George (J.), M.P. The Theory of the Foreign Exchanges.	<i>Anonymous.</i>
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THE LIFE ASSURANCE COMPANIES OF THE UNITED KINGDOM.

Summary of the Life Assurance and Annuity Revenue Accounts.

[Extracted from the Parliamentary Return for 1896, published in 1897.]

I N C O M E	Ordinary Companies	Industrial Companies	TOTAL
	£	£	£
Balance at the beginning of the Year	196,010,383	13,290,052	209,300,435
Adjustment: For Balance transferred	+ 113,712	- 113,712	...
	196,124,095	13,176,340	209,300,435
Premiums	18,657,348	6,616,613	25,273,961
Consideration for Annuities	2,365,466	6,114	2,371,580
Interest and Dividends (less Tax)	7,576,282	436,303	8,012,585
Increase in value of Investments	401,206	53	401,259
Fines, Fees, &c.	11,350	1,115	12,465
Capital Paid-up	8,038	111,064	119,102
Customs Timber Measuring, &c.	2,843	...	2,843
Donations (Itinerant Methodists)	1,127	...	1,127
Transfers from other Accounts	43,421	37,952	81,373
Miscellaneous	20,772	...	20,772
	225,211,948	20,385,554	245,597,502

O U T G O	Ordinary Companies	Industrial Companies	TOTAL
	£	£	£
Claims	13,614,138	2,774,101	16,388,239
Cash Bonuses and Reduction of Premiums	1,008,220	3	1,008,223
Surrenders	973,737	24,353	998,090
Annuities	1,230,700	4,151	1,234,851
Commission	1,061,914	1,826,471	2,888,385
Expenses of Management	1,653,061	1,093,058	2,746,119
Bad Debts	4,697	92	4,789
Decrease in value of Investments	240,639	...	240,639
Interest on Capital and Dividends and Bonuses to Shareholders	573,562	333,644	907,206
Transfers to other Accounts	268,601	46	268,647
Capital withdrawn from Life Account (Imperial Life)	194,538	...	194,538
Miscellaneous	8,316	...	8,316
Balance* at the end of the Year	204,379,825	14,329,635	218,709,460
	225,211,948	20,385,554	245,597,502

* This Balance includes the whole of the Life and Annuity Funds (£212,888,819), and, in addition, the Capital of Companies whose business is limited to Life Assurance only.

Summary of the Balance Sheets (1896).

LIABILITIES	Ordinary Companies	Industrial Companies	TOTAL
	£	£	£
Paid-up Capital (including sundry Shareholders' Balances) . . .	11,094,403	1,227,159	12,321,562
Life and Annuity Funds . . .	199,331,074	13,554,745	212,888,819
Fire Funds of Companies trans-acting Life Business . . .	10,099,014	...	10,099,014
Marine Funds of Companies trans-acting Life Business . . .	632,400	...	632,400
Reserve Funds	4,083,548	...	4,083,548
Other Funds	812,685	189,883	1,032,568
Profit and Loss Balances . . .	3,953,383	...	3,953,383
Depreciation and Investment Balances	1,134,000	877	1,134,877
Globe Annuity (Liverpool and London)	1,102,800	...	1,102,800
Outstanding Claims	3,108,339	19,129	3,427,468
Outstanding Accounts	523,948	11,996	535,944
Temporary Loans	101,278	...	101,278
	236,309,872	15,003,789	251,313,661

ASSETS	Ordinary Companies	Industrial Companies	TOTAL
	£	£	£
Mortgages	82,761,335	669,995	83,431,330
Loans on Policies	10,741,082	38,457	10,779,539
„ Rates	22,307,015	5,992,768	28,299,783
British Government Securities . .	4,775,528	987,225	5,762,753
Indian and Colonial Government Securities	16,750,249	274,868	17,025,117
Foreign Government Securities . .	5,523,496	5,106	5,528,602
Debentures	33,494,926	1,767,037	35,261,963
Shares and Stocks	20,452,198	28,620	20,480,818
Companies' own Shares	622,882	...	622,882
Land and House Property and Ground Rents	16,528,743	3,968,239	20,496,982
Life Interests and Reversions . .	5,287,975	354	5,288,329
Loans on Personal Security . . .	1,483,801	7,304	1,491,105
Agents' Balances and Outstanding Premiums	4,976,735	521,206	5,497,941
Outstanding Interest	2,226,622	147,607	2,374,229
Cash, Deposits, Stamps, &c. . .	8,201,369	196,029	8,397,398
Customs Timber Measuring Balances, &c.	5,295	...	5,295
Book-Room Grant (Itinerant Methodists)	10,000	...	10,000
Deficiencies, Preliminary Expenses, &c.	160,621	398,974	559,595
	236,309,872	15,003,789	251,313,661

INCREASE (+) or DECREASE (—) in the Chief Items of this Year's SUMMARY as compared with the corresponding Items for the previous Year.

	Ordinary Companies	Industrial Companies
INCOME.		
	£	£
Premiums	+ 1,019,665	+ 233,686
Consideration for Annuities	+ 623,979	— 4,436
Interest and Dividends (less Tax)	— 182,543	+ 39,453
Net Result of Realization and Re-valuation of Investments	+ 108,630	— 960
OUTGO.		
Claims	— 821,886	+ 355,347
Annuities	+ 95,418	— 345
Surrenders	— 7,728	— 867
Commission	— 71,263	+ 117,581
Expenses of Management	+ 59,077	— 13,030
LIABILITIES.		
Paid-up Capital (including sundry Share- holders' Balances)	— 26,517	+ 121,074
Life and Annuity Funds	+ 8,415,837	+ 1,081,372
ASSETS.		
Mortgages (including Loans on Rates)	— 1,829,816	+ 778,016
Life Interests and Reversions	+ 482,288	— 79
Loans on Policies	+ 125,695	+ 4,609
British Government Securities	— 147,448	+ 193,275
Indian and Colonial Government Securities	— 136,359	+ 48,942
Foreign Government Securities	+ 920,707	+ 5,166
Debentures	+ 4,484,384	+ 169
Shares and Stocks	+ 4,225,828	— 567
Companies' own Shares	+ 2,173	...
Land and House Property and Ground Rents	+ 1,038,635	— 174,978
Loans on Personal Security	— 729	+ 21

NUMBER OF COMPANIES.

The total number of Companies appearing in the above Summary is 93, of which 82 have been classed as Ordinary, 6 as Industrial, and 5 appear in both Classes, the Returns of these Companies showing the Ordinary and Industrial business separately. The Returns of the "Absolute" are included for the first time.

During the year three names have been added to the official List of Companies, namely, the Scottish Accident Insurance Company, Limited; British Life Office, Limited; Vulcan Boiler and General Insurance Company, Limited; in which cases the Board of Trade have issued their warrant under the provisions of Section 1 of the Life Assurance Companies Act, 1872.

SUMMARY OF THE ASSURANCES IN FORCE, *as shown by the last Returns of the Companies*
ORDINARY BUSINESS.

	WITH PROFITS		WITHOUT PROFITS		TOTAL		Re-assurances Amount	Net Amount
	No.	Amount	No.	Amount	No.	Amount		
ASSURANCES.		£		£		£	£	£
Whole Term of Life	746,081	363,160,041	126,799	65,119,573	872,880	428,279,617	22,643,674	405,635,94
Limited number of Premiums . . .	37,916	22,777,220	6,720	3,141,308	44,636	25,918,528	1,002,930	24,915,59
Endowments . . .	783,997	385,937,264	133,519	68,260,881	917,516	454,198,145	23,646,604	430,551,54
Endowment Assur- ances . . .	1,753	392,851	14,663	3,075,330	16,416	3,468,181	3,000	3,465,18
Joint Lives . . .	190,171	87,091,671	40,543	13,266,783	530,714	100,358,454	1,558,470	98,799,98
Last Survivor . . .	13,933	2,943,208	2,308	1,130,718	16,241	4,073,926	397,896	3,676,03
Contingent . . .	1,042	834,663	1,127	1,391,994	2,169	2,226,657	368,486	1,858,17
Issue . . .	22	27,866	3,487	5,313,124	3,509	5,340,990	1,330,847	4,010,14
Miscellaneous . . .	5	14,878	972	3,903,432	977	3,920,310	1,115,160	2,805,15
	954	909,570	5,504	7,056,304	6,458	7,965,874	1,486,662	6,479,21
	1,291,877	478,151,971	202,123	103,400,566	1,494,000	581,552,537	29,907,125	551,645,41
ANNUITIES.								
Immediate	25,431	1,267,422	22,889	1,244,533
Deferred	7,212	227,254	12,105	215,149
	32,643	1,494,676	34,994	1,459,682

INDUSTRIAL BUSINESS—(Sickness and Friendly Society Contracts not included).

	WITH PROFITS		WITHOUT PROFITS		TOTAL		Re-assurances Amount	Net Amount
	No.	Amount	No.	Amount	No.	Amount		
ASSURANCES.						£	£	£
Whole Term of Life	14,777,605	139,908,321	1,000	139,907,321
Limited number of Premiums	460	8,528	500	8,028
Endowments	14,778,065	139,916,849	1,500	139,915,349
Endowment Assur- ances	96,486	1,443,882	...	1,443,882
Joint Lives	171,181	1,785,112	88	1,785,024
Contingent	255,884	4,041,872	...	4,041,872
Miscellaneous	4	1,910	400	1,510
	1	300	...	300
	15,301,621	147,189,925	1,988	147,187,937
ANNUITIES.								
Immediate	63	3,185	...	3,185
Deferred	2	123	...	123
	65	3,308	...	3,308

The above figures are based on Returns deposited, for the most part, during the past five years, and are, therefore, merely an approximation to the amount of contracts in force at the present time. In the case of one Company, namely, the Customs Fund, the amount of business at a more recent date has been included. The figures of the Colonial and Foreign Companies have been excluded, as their Returns do not separately show the extent of business in the United Kingdom.

JOURNAL

OF THE

INSTITUTE OF ACTUARIES.

The Origin and Development of Scientific and Professional Societies, with their bearing upon the Institute of Actuaries and its associated Profession. A Presidential Address delivered before the Institute of Actuaries on the 29th of November 1897, by T. E. YOUNG, B.A., PRESIDENT.

I.—INTRODUCTORY.

IN adventuring upon a Second Presidential Address, the energy of hope and freshness which stimulated me in the former effort have vanished into trepidation of feeling. For I am unhappily conscious that I then exhausted any meagre resources I possessed of novel presentation of thought and research, and was reduced to virtual bankruptcy of serviceable suggestiveness which the interval of time has failed to annul.

I fear, moreover, to fancy that, violating the judicious attempt of recent years towards rendering this Address purely inaugural and introductory to Office, a righteous Nemesis may pursue me in the memories of future occupants of the Chair with the brand of retrogression and the imposition of needless burdens upon a post sufficiently endowed with honourable toil.

Notwithstanding these deterrent influences, I am emboldened to adopt my present design by an ardent desire to express, in a Valedictory discourse, some reflections upon the direction of the future development of the Institute, and particularly to unfold certain observations, which have long weighed upon my mind, affecting the appropriate education of our Students. A final motive, I confess, was supplied by the wish,—if I may daringly introduce the classical allusion,—

to complete a Trilogy of Addresses upon a subject so rich in scientific analogies and philosophic relations as our Professional work, when conceived under the universal generalisations of the Doctrine of Evolution. In an earlier Paper, I endeavoured to present the varied aspects of the commercial Business of Assurance in the revealing light of this teaching, and to exhibit its systematised congruence with the principles of that Scheme; in the Address last year, I attempted a parallel course relative to the Scientific practices and methods on which this Business is based; and, in my present essay, I purpose pursuing a similar design in briefly expounding the Origin and Development of Scientific and Professional Societies, with their bearing upon the Institute of Actuaries and its associated Profession.

For these manifold reasons,—individually, perhaps, inadequate, but, in totality, impressive to my mind,—I am confident you will indulgently forgive another trespass upon your time and thought.

II.—SCIENCE AND THE ARTS; AND THEIR ORDER OF DEVELOPMENT.

In discussing this subject, the signal distinction between a Science and a Profession, with the correlative demarcation existing between a Scientific Society and a Professional Body, demand preliminary consideration with a view to clearness and coherence of apprehension of their relevant structure and history.

I need hardly pause to mention that the primitive practical Arts of life constitute the rudimentary elements from which the Professions are ultimately organised under social and scientific agencies; and hence, in tracing the development of the Scientific and Practical forms of observation and application, we notice that the latter mode originates in unconnected and rudely experimental efforts which finally emerge in the more or less finished shape of the several segregated Professions,—either loosely allied with Scientific conceptions and methods or, in the higher Professions, distinctly animated and directed under their control.

It might, *a priori*, be conjectured, both from psychological and social considerations, that the Profession (or, to speak more precisely at this point, the Arts or Elementary rudiments and stages of its completed form) would chronologically precede the Science,—the practical and plastic Art embodying at first crude and simple scientific notions, vaguely and imperfectly grasped, so to speak, by connate common sense, without the remotest attempt

at precision of knowledge or even conception ; acquiring gradually a nicer and more adjusted character with the general advance of Science and its diffused knowledge ; until, ultimately, in the Professions (consolidated from the isolated Arts) we perceive, in greater or less degree, the organised result where Practice is associated with Science in the most intimate sympathy and union.

And this chronological priority of the Arts is the teaching of universal History.

It used frequently to be assumed* that the primitive knowledge of Mechanics, for example, incarnated in the massive architectural monuments of Antiquity, implied a vast scheme of Science which must have existed in those periods to guide the worker's brain and hand, but which had now become irrevocably lost. Deeper investigation, however, has disproved the plausibility of this suggestion ; and even at the present day many Arts are flourishing without the accompaniment of the Scientific principles on which theoretically they depend. The aphorism remains valid that the Principles which Art involves, Science subsequently evolves,*—the elementary principles of the latter being held in solution in the Arts without distinct separation ; while, in the Sciences themselves, these principles are explicitly presented as a co-ordinated system of mathematical and physical propositions. As a specific illustration in Science, I may point to the genesis of Geometry from rude practical measurements of the Earth where the words, $\gamma\eta$ and $\mu\acute{\epsilon}\tau\rho\omicron\nu$, form a compressed and vivid picture of this historical sequence.

Besides being chronologically anterior to Science, the Arts (or primitive contents of the Professions) are further differentiated from the Sciences by the circumstance that, while the former contain as a nucleus the simplest scientific truths in an incoherent and undeveloped state,—latent, it is true, in the execution of work but void of any enunciated form,—the Sciences consist essentially of a systematic series of generalised propositions, enlarging with the specialisation of intellect and the growth of preciser acquaintance with Nature.

And in subsequent stages, as I have indicated, the Arts acquire an exacter character with each conscious absorption of scientific principles into the machinery of their service as Professions.

Moreover, the object of the two provinces of Knowledge,—investigative and executive,—is essentially distinct. The essence of the Arts, and of the Professions in which they culminate, consists

* Whewell: *Novum Organon Renovatum*: cap. viii.

in direct applications to the affairs of life, while Science is a pure correlated scheme of general truths which possesses no necessary connexion, so far as the nature of the Science is concerned, with the requirements of technical work. A Science is self-contained and rests satisfied with its discoveries and their subsumption under Laws. Astronomy determines the uniformities of relation between Celestial Masses and the oscillations of the Tides, and, with enunciation of this dependency and its order in universal Time and Space (under prevailing conditions), the boundary of her domain is touched: it is the Art of Navigation (existing prior in crudely empirical form) that receives the intellectual bequest and transmutes the mental wealth into material results. So that the relation between a Science and the Arts (or, rather, of the Professions as their developed expression) may be generally defined as the accurate ascertainment and exact measurement, on the one hand, of the connexions between phenomena, concurrent and sequential, and the approximate application, on the other hand, to social and physical phenomena of the scientific truths already registered. The one is the Power; the other, the Instrument or train of mechanism which is adapted to production of practical effects.

Moreover, as a final note of distinction, a Profession, as an organised system of Arts, is distinguished from a Science by the vital accompaniment of a Teaching function. "To know" is the characteristic motto of the one; "To do" is the legend emblazoned upon the heraldry of the other: "knowledge",—of advancing precision and simplicity of form,—is the quest of Science; "works" expound the sphere and aim of Professional life.

III.—THE GENESIS OF A SCIENTIFIC AND PROFESSIONAL CLASS.

The origin of the Scientific and Professional Character, or, to speak more in harmony with our Subject, the genesis of the Segregation of a *Class* or *Classes* as the possessors and promoters of Scientific and Professional knowledge demands a passing notice.

Mr. Herbert Spencer* has adduced a multiplicity of examples in evidence of the contention that, during uncivilised eras, the medicine-man, and, in the primitively civilised periods, the priest, naturally became the Depository of Scientific truth and

* Principles of Sociology: Vol. III: cap. vi.

Professional application. Omitting the metaphysical and theological implications, the Ecclesiastic, besides alone possessing, in those uncultivated and troubled ages, the leisure competent for investigation and thought, was impelled to study physical phenomena in order to confirm his imputed dominion over Nature, as an objective support to his specialised functions. He secured the position of the supreme Man of Science; and, of his Professional superiority also, an illustration is furnished by the fact that the construction and custody of Bridges in Greece and Rome were exclusively associated with the Ecclesiastical Body. Mommsen has shown that the building of bridges, which were closely connected with the Temples, was relegated to the Sacred Colleges of Augurs and Pontifices; and that, in consequence of this assigned work or Profession, the Priests were termed "Pontifices", or "Bridge-builders", as the etymology of the word expresses. Milton, you will remember, with his refined and scholarly instinct in the employment of terms, uses the word in its original sense in the 10th Book of "Paradise Lost."*

From this combination of Scientific and Professional knowledge and activities in a single Class, a natural heterogeneity succeeded under the stress of wider and minuter learning, beyond the compass of individual mastery; the more detailed and exacting requirements of evolving Civilisation and the consequent complexities of Social relations; so that, in obedience to the enforced division of function and labour, the Man of Science and the Professional Man constituted, in time, two separate classes; while, at later stages, to which I now proceed, these Departments again assumed the more segregated forms of men devoted to individual Sciences and men attending to specific Professional pursuits.

IV.—THE GENESIS, AND CONDITIONS OF FORMATION, OF SCIENTIFIC SOCIETIES AND PROFESSIONAL BODIES: INTEGRATION.

We thus naturally and necessarily arrive from the Individual to the Collective stage, where the isolated units tend to combine and cohere into definite Associations of similar structure and function. Throughout the Organic and Inorganic Series of Nature, this Aggregation, or, as Mr. Spencer terms it, this process of Integration, is universally manifest; and as similarity,—if not, indeed, identity,—of Law also pervades and unifies human relationships, we find that individuals, attracted by

* Line 313.

corresponding character and impulses, conjoin into sympathetic associations under the genial compulsion of various influences, which I shall briefly describe, for the special cultivation of diverse domains of knowledge and the execution of distinct duties which the development of Corporate life compels.

Noting, therefore, as we proceed, the striking and interesting analogy of movement and order between the modes of combination in the material universe and the social sphere, we observe that the prime factor in the coalescence of individuals into Scientific and Professional Institutions consists in accordance of intellectual structure, and of specialised pursuit. But, besides this initial force and the mutual sympathy which it implies, confirming the tendency to union, various facts and conditions contribute a necessary impetus.

The individual element, however, again appears; for, as a matter of history, the formation of Scientific Societies has been simply the expansion of informal and private meetings of a few gifted men drawn together by the charm of a common purpose; recognising the need, with a view to effective energy, of united action and mutual stimulus; and animated with the larger aims, possible of realisation only by associated labour, which the widening sphere of knowledge and the intenser glow of intellectual light had awakened into hope. Intimately connected with these impulses, and indeed a primary condition of their practical exercise, has been the advent, after troubled eras, of National and Social freedom and peace which alone sustain the prospect of uninterrupted search and provide the serene atmosphere and continuous leisure on which Scientific prosperity depends.

Thus the earliest Society for the cultivation of Physical Science—the *Academia Secretorum Naturæ*,—was founded in Naples, in 1560, by Baptista Porta, as the consolidation of private gatherings of Scientific friends. Our Royal Society developed,—not as an instantaneous creation, but—as the social and genial enlargement of domestic conferences of cultivated men held prior to 1660,—comprising Dr. John Wilkins, Sir Wm. Petty, Dr. Willis, the Hon. Robert Boyle, and Mr. Christopher Wren,—who, attracted towards a fixed centre, sought to merge their isolated efforts in the promotion of method and research; labourers in different plots of the Intellectual Kingdom uniting their scattered holdings to form a more spacious and connected domain.

The origin of the Royal Academy of Sciences in France is ultimately traceable to informal assemblies in 1629, of scientific

colleagues for corporate concentration of energy, and among these early names we find a galaxy of wonder in themselves,—Descartes, Gassendi, Pascal, and Roberval. And although the Imperial Academy of St. Petersburg was originated under the autoeracy of Peter the First, and established by the fiat of Catherine the First in 1725, while the Royal Academy of Science and Belles Lettres of Berlin was created in 1700, by Frederick the First, still the former was instituted on the advice of individuals,—Christian Wolff and Leibniz,—and the latter was really guided in its origin by the counsel of Leibniz. These two Societies, therefore, and their Continental successors, may thus be fairly asserted to exemplify the same primitive method of origin: the spontaneous aggregation of individual workers constituting the nucleus of the associated splendour. In similar fashion, our own Institute commenced in the private meetings of professional friends who, abandoning, under the spell of a common pursuit, all independent action and unconnected research, gathered themselves into the unity of a Scientific Commonwealth whose generous expansion involves the history and tradition which inspire the hopeful vigour of our Corporate life to-day.

It will be found also to be the testimony of History that those Institutions, Scientific and Professional, have presented the most valid title to permanent solidity and renown which, once formed, have not instantly developed in complete equipment, but have proved the secure result of tentative and graduated effort,—secure by reason of apparent insecurity. The organism in Nature gains its exquisite adjustment with external conditions by successive adaptations attended with varying sufficiency and frequent check. The *Accademia del Cimento** (Academy of Experiment), established at Florence in 1657, with its noble watchword of investigation unfettered by preconception or dogmatic authority, was preceded, as an exemplar, by the *Accademia Secretorum Naturæ* whose brief period of service was extinguished by Ecclesiastical tyranny; and, in precedence and prediction of the stability of our Royal Society,—presenting a tentative type of conception to the mind and thus presuggesting possible difficulties and their solution,—we find the scheme of the Invisible College delineated by Boyle in 1646; the ideal Philosophic-Mathematic College, described by Evelyn in 1659; and the fabled New Atlantis of Lord Bacon washed and circled by the Southern Seas. The early and, indeed, the long-continued troubled fortunes of the Royal Society display an impressive and

* White: *History of the Warfare of Science with Theology*: Vol. I: cap. xii.

often pathetic lesson of the conquered hindrances that finally built up an unassailable Source of Power.* So chronologically was the Institute of Actuaries preceded by informal meetings of Actuaries which commenced prior to 1836 and consolidated into the Actuaries' Club in 1848. Experimental searches after an ultimate ideal involve the moral and mental discipline, born of failure and temporarily shattered hopes, and produce definiteness of structure and coherence of organisation; swiftness of resource; practised skill in devising appropriate mechanism; confidence derived from conquered defeats; confirmed hopefulness in final issues; and the gift of adaptation to changing circumstances,—qualities and powers which constitute the basis of triumphant and permanent Force.

I have referred, as an essential condition precedent to the foundation of Scientific Societies, to the secure establishment of political and civil order. A few illustrations will suffice. The decay of Athens as a political power ensued as a result of the Peloponnesian War, but the political descent was synchronous with the ascending glory of her Philosophical, Scientific, and Literary life, and the founding of Plato's Academy in B.C. 389 marked the advent of a period propitious, through civil peace, to intellectual advance. The calm succeeding the Civil Wars which ended in the restoration of the Stuarts formed the tranquil season which nurtured our Royal Society in its vigorous growth; and the famous Revival of Learning itself crowned the era immediately sequent to the final fall of the Byzantine Empire and the termination of the Middle Ages.

Other conditions on which the fortunes and progress of Societies are dependent may be roughly described as General and Special. The predominant intellectual and æsthetic character of the Greek, favoured by his National isolation after the Peloponnesian War, then found an unimpeded expression, as I have stated, in the creation of Philosophical Bodies; but the specific character of the Roman mind, expressing itself in external in place of internal activity, proved inimical to movement in a similar direction. No Academy can be traced in the Roman Empire; not merely in consequence of the National aptitude with its tendency to material aggrandisement and devotion to martial ascendancy, alien from that introspection and culture which are native to intellectual progress, but also by reason of the distracting turmoil of war and social complications forming effective barriers to combined mental adventure. In addition to

* Weld: *History of the Royal Society*: Vol. I.

the hindrances thus interposed by national characteristics and their material embodiment, the successive forms of Roman government,—and this political element possesses significant influence throughout the entire history of Scientific Societies,—furnished varying opposition to a quest which largely rests its sure foundation in consolidated political order. For while the Roman Republic despised the extension of learning as inconsonant with its own grosser structure,—the ornamental as compared with the useful,—the Empire, with equal though dissimilar animus, dreaded the enterprise of Arts and Letters as a possible precursor of political revolution, since freedom created by dedication to learning and Science extends with resistless energy into every channel of social life and thought.

It is of passing interest, in illustration of the effect upon Science and Literature of Local Character, to point out that, in France, the Academy first established in 1635 was devoted to preservation of the purity of the national language, as we might indeed infer from the literary grace and fluent pliancy of its speech. In England, on the other hand, the Royal Society expressed the practical characteristics of the Nation by specifically attending, in its earlier history, to questions of scientific application to the needs of life and commerce,—the construction of lightning-conductors for Cathedrals and war-vessels, for example, the ventilation of gaols, and the modes of obviating the corrosion of the copper sheathing of ships.* And this practically adaptive bent is especially manifest in the minuter ramifications of our varied Professional work in this Country. Although, to employ the language of the Schools, we can trace, in affiliation, the *Form* of the early tendency in Greece in its modern representatives, the *Matter* of the form has widely changed: the congregation of disciples for submissive reception of a Master's doctrines, or discussion of the tenets of a Philosophical Sect, is transformed into a varied Commonwealth, fused into unison by the same Spirit, but with open and responsive minds compacting facts into natural systems instead of superposing preconceptions upon facts.

But although the form of Society created in Greece failed to become naturalised in the Roman Empire, the characteristic spirit of that Empire, void though it was of instinctive attraction towards Scientific union, descended in diverse expression into the later Italian mind, and exalted that Kingdom as the illustrious originator of all modern Scientific Institutions. It has always appeared to me an interesting example of Evolution that the local

* Weld: History of the Royal Society: Vol. II.

type of established political order; of rigorous discipline in unified action; of systematised law; exhibited in the ancient Roman world in territorial acquisition and legal organisation, reappeared in Italy, at the prophetic Renaissance, in the transfigured form of intellectual conquest, and of discipline applied to the domain of physical and mental phenomena of Nature. This Revival originated in ardent and inappeasable longing for the vitality and creative power of Classical Literature and Art. The material ideal became subordinate to the immaterial: the sword and phalanx succumbed to the chisel and the pen; the legal instincts were incarnated in Scientific formulae and rules; and the mind discovered its legitimate birthright and heritage in higher themes and finer impulses than those which found their circumscription within unintellectual bounds.

The Intellectual Palingenesis of the cultivated and formative Races—the races endowed with a distinctive mission for the enlightenment of the World,—dawned in Italy, prior to the 15th Century, as a serene and humanistic sequel to prolonged social and mental darkness into which Roman Civilisation had vanished, and over which ecclesiastical despotism had disastrously reigned. Dowered with gifts of Classic Literature by Grecian refugees, and fired by the native strains of Dante, Petrarch, and Boccaccio, the Renaissance heralded, in lucent and broadening outline, the ample and propitious advent of an intellectual and memorable day.*

The circumstances precedent and preparatory to that condition which is consonant with mental freedom and spontaneous absorption in Science and Art, comprise also the absence or impotence of Ecclesiastical tyranny which, in the anterior stages of Science, had lavished its energies in barbaric efforts to stem the advance of intellectual and moral liberty which, in its gross and perverted interpretation, signified merely the freedom of compelled and abject submission. In Galton's finely suggestive work on *Hereditary Genius*,† a graphic and realistic picture is painted of the saddening and far-reaching consequences of this organised oppression upon the finer development of the Race by seclusion of the gentler types of character and repression of the types of originating and enterprising force. The Revival involved, again, the possession of sufficiency of insight into the marvels and complexities of Nature to cherish the hope of keener and more assured vision; the fusion of genial feeling and the formative

* Cajori: *History of Mathematics*: page 138.

† Chapter on the Influences that affect the Natural Ability of Nations.

[*Vide also* Lecky's *History of Rationalism in Europe*: Vol. I: cap. iii.]

force of mental collision between the toilers in a common Scientific pursuit, so refined from the selfish meagreness of individual effort and personal repute as to rank supreme the attainment of the general weal: the helpful equilibrium of National peace and rigid order, without which the calm concentration of intellectual energy languishes and dies; and the single love of Truth as symbolised in the Earth and Skies and Human Relationships awaiting but the deciphering power for its perfect interpretation.

Sequent to the Accademia del Cimento, multitudes of Associations originated from time to time in Italy, flushed with the renovated ardour of intellectual dominion over the phenomena of Nature and of unimpeded access to æsthetic life. The spirit of this impetuous Reformation—the coronation of Thought and Feeling upon the field where mental emancipation had been wrested from priestly bigotry,—swept slowly into England and then diffused its influence into France. It is pardonable to remember that the Royal Society formed the primary and most famous embodiment of this reinvigorated impulse if on no larger ground,—and could a more memorable claim be advanced?—than the issue of the *Principia*. For it has been justly suggested on evidence that, owing to Newton's deep aversion from publication with its possible controversies, the *Principia*, in all probability, would have remained unfinished and unknown had not this Society acted as the medium of that unsurpassed and unsurpassable flight of genius. The light of the Sun would be invisible were no conveying Ether to exist.

The descriptive form which Scientific Societies assumed has variously been expressed in the generic terms of Academy, Institute, and Society. These Titles no doubt once possessed distinctive shades of meaning based upon their etymologies, but, with the synonymising tendency of language, the signs of demarcation have definitely disappeared, and no useful purpose would be served beyond one of purely archæological interest in attempting to ascertain the original definitions. It is historically curious, however, to observe that the designation of "Academy" has been usually retained on the Continent; the name of "Institute" was employed in France; while British usage has consecrated to scientific service the title of "Society."

The term "Academy",—whether derived from Greek words signifying "medicine" and "people",* or, in another form of the phrase, "remoteness from the people",† or, as tradition

* *ἄκος*; *δῆμος*.

† *ἐκὰς*; *δῆμος*.

imputes, enshrining the name of the reputed owner* of the property,—was first applied to the historic school of Plato which assembled in the grove contiguous to Athens. And in loyal perpetuation of a title so illustriously allied with a creative mind, the various Associations, devoted to the culture of Arts and Philosophy, which ultimately were founded in Greece,—rare in number but rare also in a higher sense in intellectual stimulus and fruitful ancestry,—were designated by the name. The symbol was transmitted to the Learned and Scientific Institutions which luxuriantly flourished in Italy and other continental countries ; and I understand that, to the present day, the word “ Academy ” in the European languages, except in English, is restricted to a Central organ of sound information and correct taste in intellectual subjects.

So far as I can discover, one of the earliest uses of the term “ Institute ” occurred in France. When the aristocracy of talent was engulfed, with the aristocracy of wealth and rank, in the French Revolution, a decree, passed by the Republican Convention in the year III of the New Calendar, suppressed, in 1793, the existing Academy of Sciences, and ordained that an “ Institut National ” should be established in its place. It would appear that, like all profound and frequently unreasoning social upheavals,—unreasoning, I mean, in the extreme measures by which they are accompanied,—the French designed, by the cancelment of this ancient and honoured name, to sever their modern history from the natural and inevitable links of tradition, just as we observe in the English Reformation a corresponding and retrogressive policy in the introduction of our barbarous pronunciation of Latin as part of the abrogation of the Past which had cultivated the Italian mode.

In England, the term “ Society ” seems from the outset to have grown naturalised as distinguishing communions of congenial minds for the prosecution of scientific method and research.

V.—THE DIFFERENTIATION OF SCIENTIFIC SOCIETIES ; AND REINTEGRATION.

I have thus traced the process of integration or union of individuals into the form of Scientific Societies, which possesses its analogue in the Organic and Inorganic kingdoms in the production of a composite organism with extended and minuter

* *Academus* or *Ecademus*.

relations to exterior agencies and a resulting mode of completer existence. But similar to the corresponding order in the region of Nature, the action of incessant influences gradually effects a differentiation of the homogeneous Body, and a separation of parts ensues as a fresh development of life and activity. In a complex organism surrounded by increasingly complex conditions, a sequent specialisation of function to different departments of its Corporate Being inevitably inaugurates the commencing stages of an orderly process which fractures the attained unity into diversity, though each section is still mutually co-operant, and contributes, in due and essential proportion, to the integrity and equipment of the Whole. In the intellectual and scientific region, these influences comprise the unfolding variety of Nature; and the multiform requirements of the Social Body, developed by the progress of Civilisation and the interactions of Commercial and Industrial interests. Confronting these imperious and determinate demands with the comparatively stationary range of the human Intellect, the approach of the period is proclaimed when the totality of labour must be segregated into distinct divisions. The Scientific unit, dealing hitherto with the entire domain of Science, becomes necessarily partitioned into sections, restricted to individuated provinces of knowledge, while the Professions in turn assume more minutely distinctive functions, exclusively concerned with limited forms of practical application, and concurrently become endowed with a finer Scientific finish and capacity as the principles and methods of particular Sciences,—more efficiently promoted by their own divisions,—receive a detailed planancy and possess a more manageable and executive shape.

But I need not burden you by describing the modes in which this allotment of service has naturally occurred by the detachment of Societies from the original Body, and by the wider ramifications of Professional work. I will simply adduce an illustration gathered from the experience of our Royal Society. In 1807, the Geological Society, and, in 1820, the Astronomical Society, were founded by Fellows of the Royal Society under the compelling stress of the deeper disclosures of Nature and their imperative appeal for specialised Intellect; and, while faithful in allegiance to the noble Original whence they derived their start and impetus, they contributed most powerfully,—and in the only competent manner,—to the rapid and solid advancement of these departments of Search. As more appropriate to the nature of our Institute, I would particularly cite the establishment of the

Society of Arts in 1753 for the promotion of Arts, Manufactures, and Commerce, whose definite sphere was found harmonious with the supremacy and province of the Royal Society, since the latter had long discontinued the practical experiments which formed a characteristic feature of its earlier career.

I may note, in passing, Mr. Spencer's proposition that, after the several agencies had been constituted in protection of the most intimate necessities of social existence, the various Professions gradually emerged to exercise the function of what he terms the "augmentation of life";* and as a Professional man, the Actuary obviously may claim inclusion within the boundary of this phrase not merely on the general grounds of admission assigned by Mr. Spencer, but also, in view of the scope of the description, with a unique title to honourable position in the hierarchy so defined.

But another phase of Evolution succeeds this differentiation of function; and, in this further stage, we perceive a process of reintegration. For not only are the separated Bodies connected by the continuous and diffused spirit of the original unity; by the uniform character of the aim of each towards the realisation of a larger common end; but specific modes of consolidation, with preservation of distinctive features, sustain the harmony through the machinery of General Meetings of Societies, of which, I need only mention, in illustration, the British Association for the Advancement of Science, and similar Congresses in our department of work.

This tendency to a wider union is again promoted, in the Sciences and Professions, by the publication of Journals, which possess the two-fold merit of intellectual intercourse between the several Bodies, and the higher merit of avoiding that wasteful dissipation of mental energy, so saddening in the more primitive records of Culture, exhibited in the independent rediscovery of truths and methods which, already secured, had unhappily remained unknown. Resources once acquired are thus permanently amassed in the Treasury of Knowledge for universal and remunerative expenditure.

To adopt an astronomical simile, and transfer to the intellectual firmament the phenomena of the physical universe, the scattered particles of similar phasis coalesce into discrete masses; these again divide into minuter spheres; but still an even more pervasive unity links together the diversified system, and the concurring splendour is ampler and more vivid than the sum of the originally distinctive lights.

* Spencer: *Principles of Sociology*: Vol. III: Part vii: cap. i.

We observe, throughout, in the Scientific Kingdom, that persistent principle of Rhythm, which Tyndall and Spencer* long ago discerned as an invariable factor in the organic and inorganic worlds; consolidation followed by divergency; and divergency again compacted into a finer and completer oneness.

VI.—THE CONDITIONS OF EXISTENCE OF SCIENTIFIC SOCIETIES.

The intellectual *acquisitions* of one stage of Science are transmitted as *powers* and *capacities* to its successor; and this hereditary endowment forms an effective force in sustaining the integrity and vigour of the organised Body. But the bequest confers no absolute and indefeasible title to permanent stability unless it be attended by a ready capability of adaptation to the varying conditions by which the Body is affected and modified. The universal principle, which Sir William Grove† termed “Antagonism”, prevails alike in the material universe and in the world of Science and Profession as the touchstone of their fate; the incessant interaction, to employ technical language, between Environment and Organism; between the changing modes and intensities of the one, and the power or impotence in the other of responsive adjustment. In my subsequent suggestions, I shall incidentally deal with this subject, but a reference here is demanded in order to complete the order of Evolution which I have endeavoured to discover, in agreement with the processes observable in the Material sphere, within the compass of development of Scientific and Professional Associations.

VII.—THE BEARING OF THE PRECEDING ANALYSIS UPON THE INSTITUTE OF ACTUARIES AND ITS ASSOCIATED PROFESSION.

It is a misfortune that no connected history of the Origin and Development of Scientific Societies, on Natural principles, has, so far as I can ascertain, been hitherto attempted, for the subject is one of abiding and instructive interest in the region of intellectual and professional enterprise as affected by social influences; and a consecutive record would reveal, for the guidance of succeeding Institutions, with impressive and luminous effect, the conditions which sustain vitality and the failure of which is premonitive of atrophy and decrepitude. I am, therefore, compelled, in the absence of any chronicle, to deduce, from a miscellaneous and

* Spencer: First Principles: cap. x.

† Lecture at the Royal Institution: 20th April 1858.

fragmentary reading, the teachings and directions which the fortunes of such Societies suggest in relation to our Institute.

In approaching this delicate and difficult subject, I desire to emphasise the distinction which I have delineated between the functions and obligations of a purely Scientific Society and those incumbent upon a Professional Body. The former remains content, as I have mentioned, with the acquisition of facts and their generalised uniformities; its scheme embraces no essential relationship with practical work: it is allied to practice simply as a general theorem to particular cases elsewhere presented; or, figuratively, the Science may be described as extracting the massive blocks of knowledge from Nature's quarry with which the individual Arts and their organic embodiments, the Professions, may erect their appropriate structures adapted to the necessities of life. The *raison-d'être*, on the other hand, of a Professional Body like our own is ineradicably involved in the direct and continuous application of scientific and commercial principles to specific pursuits. Though this demarcation limits to an extent the pertinence of examples furnished by purely Scientific Societies, there yet exist certain general principles expressed in their history which remain relevant to the permanent features of all practical Institutions.

In proceeding to apply those teachings to the Institute of Actuaries and the Profession, I desire also decisively to point out that I venture to utter merely my personal judgments, without committal of the Institute or its Members; and my observations will necessarily assume the form of hopeful prevision without trenching upon the province of definite formal suggestion of the machinery of achievement.

I confess, too, that in touching upon this question, I am apparently violating one of the principles at which I hinted in my former Address: should you blame me, I promise to accept the correction humbly and heartily; pleading only in mitigation that my intention is dictated solely by deep affection of our Institute and pride in the Profession.

All valid change, as Coleridge* indicated, implies (i) its practicability, (ii) its adaptation to existing circumstances, and (iii) its necessity or desirability as a means of accomplishing more effectively the purpose for which the Institution was formed. And, in the survey of any suggested modification, he justly insisted that a distinct conception of the requisite ends should be

* The Friend: Section I. Essay viii.

framed, while a calm and kindly feeling should permeate the discussion.

It is obvious that, in order to maintain adequately its prescribed and destined *rôle*, a Professional Body must be promptly but surely capable of approximately exact response to the varying conditions of its position and the altered requirements which those conditions demand: hence it is imperative that its Constitution should prove of so elastic a character (its adjuncts and mechanism for achieving its professed aim) as to admit of periodical adjustment without abrupt dislocation or discontinuity with its historic past. The grave misfortune of many Societies has resided in the rigid nature of their original framework, which has frequently produced attendant complexity and friction upon any needful project of reform. Moreover we must not forget that a justly conservative spirit in the members of a Body is often pardonably and honourably warranted by the possession of fine traditions which seem indicative of the unwisdom of change. And again it is simply reasonable that any proposed modification should not be exclusively or largely regarded from the point of view of senior members whose career is far advanced, but essentially from that of the younger men whose position and prospects will be affected by the change. The lengthy and matured experience of the former should merely form a portion of the data on which the general judgment should be based.

But environment is incessantly altering; both material, mental, and Social ideals may remain constant, and yet the machinery for their attainment will vary with ampler times. Scientific and Professional Institutions, like other organisms, are but functional expressions of existing requirements and relations; necessity of adaptation, accordingly, not *merum arbitrium*, must, if permanency and efficient fitness are to be conserved, be accepted as a fact of Nature and human life; and the pre-requisite conditions simply consist of sagacity, of judgment, and of quiet, maturing wisdom in seizing apt occasions for adjusting, either instantaneously or (more generally) by graduated degrees, the application of Power to the work designed as viewed under wider and clearer light. The end of the journey looms distinct and constant, but the road to be traversed, though ever tending to the goal, will often deviate and diverge. It has accordingly been the fate of Societies, or rather their propitious fortune, to effect this modification from time to time as a title to remembrance and renown.

A.—PUBLIC RECOGNITION.

As a general preliminary remark, I may notice that valid considerations may be urged in favour of the Continental system of direct State encouragement of Scientific Societies, and equally pertinent reasons may be alleged in support of their complete independence. *Non nostrum inter vos tantas componere lites!* But such competing arguments, by the way, scarcely affect,—at least, in a pecuniary sense,—Professional Bodies which find their only unfettered exercise of power in the capacity of self-expression; though, obviously, reasonable inter-relations should exist,—public service on the one side correlated with honour and appeal on the other. And to whatever cause we may attribute the result, it cannot be denied that the Institute does not possess that prominent and authentic position in connexion with the State and Official Legal Institutions to which its admirable history and public utility undoubtedly present an imposing claim. In an organic system in Nature, the symmetric efficiency of the Whole is only achieved by the harmonious interaction of the specialised functions of the Parts; where definite assistance is required for the performance of the general work, it is derived from that individual section which is solely competent of specific supply; and the analogy may be extended to social Institutions and the central governing Body in the State. The latter, it is evident, should depend for its completeness of action, where complicated legislature is attempted, upon provision of counsel from authoritative sources; and it is only by abstracting its nutriment from these specialised organs that an adequate embodiment of its constructive and executive power can be realised. In numberless instances, legislation upon difficult and far-reaching social problems might have been expressed in a more serviceable form; with the absence of misinterpretation; the avoidance of results frequently contravening the intention and spirit of the Enactment; with the prevention of admission of failure which Interpreting and Amending Acts implicitly proclaim, had the Institute of Actuaries as a Body been consulted upon the scope and arrangements of the Bill.

I need only refer to the ineffective and defective provisions of the Married Women's Property Acts as an illustration; and to this I summarily add many wide subjects of a financial character, and all questions affecting Assurance business. When both natural analogies and repeated experience enforce the importance of

technical reference, to what cause must be assigned this fatal omission of appeal? I do not propose to enter into detailed suggestions, so far as the Institute is concerned, since some of my subsequent observations may partially involve my views; but I apprehend that, as regards references to individual Actuaries both by the State and generally, one reason, at all events, may rest in the absence of definition of the title, Actuary. And without some stable and recognised definition, the difficulty will to an extent continue to exist.

B.—THE TITLE OF ACTUARY.

I have devoted considerable attention, for the purposes of this Address, to elucidating the origin and history of our Professional name. The subject, in its earliest stages, is obscure, and, on that ground, the more deeply interesting; and in an Appendix I have furnished the results, in chronological sequence, at which I have arrived respecting its employment from the date of the Roman Consulship to the present time.

It is obvious that no definition of a rigorous and comprehensive character can be verbally expressed, and, on this basis, permanently fixed by Act of Parliament, but the term must be distinguished by way of specific inclusion and consequent exclusion: by the decree, in short, that an Actuary shall be a Fellow of the Institute of Actuaries or of the Faculty of Actuaries.

You will remember that, in the first Number of our *Journal*, the draft of a Bill for presentation to Parliament was submitted, in which an attempt at definition was made by legally limiting the title to certain persons specified in the schedule, and to persons who should thereafter satisfy the tests prescribed by the Institute.

I am hopeful that some feasible plan may be devised of obtaining a legislative acknowledgment of the name and the functions which it implies. The dignified position of the Institute, and the services which its publications and researches have conferred upon Society, amply warrant this pretension to distinct recognition.

C.—EXTENSION OF SCOPE.

Mr. Spencer has continually insisted, with a prodigality of illustration simply marvellous, upon the analogies observable between natural organisms and their development, and human institutions: and one of the most pervasive uniformities to be deciphered, upon which I have already enlarged, is the dependence

of supremacy of life and capacity upon congruity between the Body and its exterior circumstances: surrounding conditions vary: encompassing relations are modified: external forces change their incidence, direction, and intensity,—one increasing with the diminution of another,—while cyclic or periodic variations frequently occur. Triumphant progress is assured when a modification of conditions is swiftly and aptly confronted by fit adjustment in the affected structure; failure of coincident adaptation announces the relinquishment of authority and the advent of decrepitude and decay. An organism, apparently designed for lofty destinies, may thus resign its deputed Trust, or its supremacy may prove a genuine gift through possession of spontaneous wisdom and inherent ability to change with change. Authentic evidence is furnished by the history of our Institute that this primal and aboriginal power of permanent service is its natural dower, and upon us there rests the honoured responsibility, and especially with urgent weight upon our successors, of ensuring, with keen foresight, wise administration, calm and unprejudiced survey, that persistent adaptability should be amply maintained. I venture accordingly to think that the period is rapidly approaching when the revision of our Bye-Laws will require our earnest thought. It is, for example, deserving of serious discussion whether it may not be possible, for the closer integrity and representative completeness of our Corporate life, to extend our range, by including the Managers of Life Offices, who are not Actuaries, in some suitable form and recognised status which shall satisfy their just and legitimate expectations and prove in honourable accord with the high official positions and functions which they sustain in our common sphere; and thus, while still preserving the distinctive province of the Institute, with the technical qualification exclusively defined by its Fellowship, expand and confirm more securely its Representative character. Where a Professional Body, concerned with a limited business, exhibits a diversity of nature and service, the judicious adoption of a scheme of this description tends to distinct consolidation, and excludes the feebleness produced by the formation of different Associations engaged in the same commercial affairs. Such a process aids in the creation and conservation of an *esprit de corps*, with an undivided stimulus and aim; and, thus compacted, a Professional Body presents a practically homogeneous unity, so that its Corporate action, as the expression of its Corporate Will, concentrates the weight and power which alone an organic Profession can command.

The question of appropriate membership to persons pursuing cognate professions, and to persons partially but importantly connected with our practical labours, is also worthy of thoughtful consideration.

The necessity of comprehensiveness and aggregation has been perceived and realised by many eminent Professional Bodies. In the Institution of Civil Engineers (established in 1818 and incorporated by Royal Charter in 1828) I find included distinguished persons, who, though not engaged in technical practice, are competent from their position to render assistance in the prosecution of Public Works; persons again who are eminent in Science and experienced in undertakings connected with the Engineering profession. These are styled "Honorary Members." Members also are admitted, under the designation of "Associates", who, without being Civil Engineers are devoted to pursuits which constitute branches of Engineering, or who, from connexion with Science or Arts or otherwise, are qualified to concur in the advancement of professional knowledge. The Royal Institute of British Architects (incorporated in 1837) comprises "Honorary Associates" who, though not adopting the practice of the Profession, are capable and likely from position, or from prominence in Arts, Science, or Literature, or in matters relating to Architecture, to aid in promoting the objects of the Institute. The Institution of Mechanical Engineers (founded in 1847) admits Members, under the name of "Associate Members", who are attached to any departments of work connected with the practice or Science of Engineering.

I suggest, too, that admission to justly adequate position in our Society of persons holding Actuarial appointments in the Assurance world, and testifying to successful knowledge and capacity as experts, should not be too rigidly limited by the test of examination alone.

In Scientific Societies, it is true that a complex of separate Institutions constitutes a mark of intellectual vigour and enterprise, indicative of enlarging insight into Nature as each successive discovery widens the prospect and stimulates expectant energy; but, in the sphere of a restricted Professional Body, a multiplicity of independent Associations forms an index of weakness unless definite and recognised relationship can be established and maintained. It may not be feasible to amalgamate the various Institutions concerned with our labours, but a genuine scheme of affiliation and associated action and

sympathy may be created, so that, presenting the front of an unscattered Union, the force of aggregated strength and condensed will may be devoted to the attainment of any general end. An instance of the power of allied effort is furnished by the modifications in the Finance Act of 1894 secured by the united weight and representative influence of the Institute of Actuaries and the Life Offices' Association. And this efficiency of sporadic combination may be permanently registered in the form of recognised and formal connection. In this mode of Federation we could also embody the conception of conserving to each section the pursuit of its individual activity and the exercise of its differentiated duties, while maintaining loyal subordination to, and cordial co-operation with, the integrity of the Total which the units compose. This hopeful and helpful consolidation of the several Societies, ministering within our Profession, produced by a fitting and consistent expansion of the range of the Institute, seems to me one of those practical dreams which may be converted into reality by wise and reciprocal concessions. It does not lie within the province I have prescribed for myself to describe any detailed method: with confidence in the future, I am content at present to dream the dream.

It is worthy of record that the Astronomical Observatories of Paris and Greenwich were founded almost contemporaneously with the creation of the Royal Academy of Sciences and the Royal Society,—as though essentially associating Theoretical learning with Practical application. The Royal Society was originally constituted the Director and Visitor of our Observatory, and, since 1847, has shared that duty with the Royal Astronomical Society.

Sir Isaac Newton frequently expressed the hope that Societies for Scientific pursuits should be established throughout the Kingdom; and we all regard with pride the numerous Institutions devoted to our business which, animated by the genius of the Institute, have been organised in many important Cities. I have taken considerable trouble to acquaint myself with their history, and to study the Journals which they publish, and I have been impressed with the value of their contributions to our store of knowledge: especially I have perceived, with unmeasured delight, the testimony they afford of mental activity and eager work in the younger men. The Institute may justly claim a meed of praise in the origination of this intellectual and professional extension. A further hope, accordingly, which I entertain, rests

in the expectation that not merely in London and Scotland, but also in the Provinces and Ireland, these Associations may be affiliated to our central Body as mutually receiving and dispensing counsel and strength. It formed a Capital feature of Bacon's New Atlantis that accessions of knowledge should be procured from external provinces and treasured in "Solomon's House", through the agency of the "Luminis Mercatores", or Merchants of Light.

One of the most efficacious means of integration, to employ Mr. Spencer's phrase, consists in Congresses at stated periods: they cement the scattered energies of the Profession by concentration of toil and personal sympathy: by the stimulating and revealing power of individual intercourse; by the attrition of business asperities through the direct and subtle influence of social goodwill; while they also foster that fusing spirit which connects the several branches of a Profession into a disciplined and genial communion. It may, I trust, be feasible hereafter, by promoting Congresses of the Members of Central and Provincial Institutions, to extend and solidify this unifying attempt.

D.—TEACHING AND EXAMINATIONS.

The teaching aims and educational machinery of a Profession are obviously a product of experience, and should be guided and modified by the indications of that experience and the varying requirements of our Professional work. I have referred to the unhappy and natural tendency in Institutions towards framing Rules of Constitution of so inelastic and formal a character as to operate adversely upon that progress which they were intended to promote,—rules so formed indeed as though a tentative and initial stage could adequately represent and sum up the future possibilities of Professional existence and scope: as though, to speak figuratively but justly, the scattered and glimmering light of dawn presented the perfect and illuminated day. The framework of Constitution should clearly be so plastic as to possess an adaptive flexibility, while distinctly precise in spirit, for inclusion of that administrative and executive expansion which accumulating experience may dictate. Hence, in most instances, the Constitution of a Society will demand, for accomplishment of its contemplated purpose, a remodelling of shape in obedience to successive needs,—the definite form being preserved intact with pliant variety of expression. A similar remark applies to the

scheme of education and examination which a Professional Body adopts. We seek to produce, not mere technical experts, but men of judgment : not simply adepts in the conduct of processes, but wise masters of the processes they employ. And though undoubtedly the quality of judgment must be native, developable by actual experience, still the resources and skill of a Professional educator are competent of service in aiding natural endowment by means of the character of his teaching and his examination tests. The very etymology of the word proclaims the lesson. The genuine Educator is not simply the Instructor; he not merely supplies the knowledge which furnishes the basis of judicious work; but primarily and essentially his function is the educating or bringing forth, into prominence and strengthened energy, the faculties which must preside over the utilisation of facts and the application of method. The scheme of the Scientific New Atlantis included the class of selected Novices trained with observant zeal to sustain the traditions of the honoured dead.

In the department of Education, the history of the Institute compels our admiration of the wisdom and judgment with which these imperative obligations have been recognised and discharged; and I desire simply to add a few observations upon the subject of examinations,—bearing fully in mind the profound comment of a distinguished Statesman that examinations do not constitute the ornaments of our mental health, but simply supply the medicines of our mental infirmities.

Recognising the fact that our duty is to promote the cultivation of faculty, and that our essential aim is the closest union of Theory with Practice, our examination arrangements should be sedulously directed towards assisting faculties,—regal among which is the practical judgment,—to develop from a rudimentary to a completer form. The examinations therefore should be devised as a graduated and related scheme throughout their entire course for the express purpose of educating into growing efficiency, the appropriate mental capacities and their application to practical use. Intellectual or mathematical conundrums; mere exercises of mathematical demonstration or analytical ingenuity; detailed arithmetical correctness, are futile, and indeed destructive of the required success. And in intimate connexion with the subject, I will venture to assert that the teaching of the Institute should be intrinsically allied with the system of its examinations,—connected portions of one continuous and consistent scheme,—so that the same process,—in natural and adjusted advance,—may pre-

dominate throughout. The series of examinations again should be particularly designed, with deliberate care, to the cultivation of *judgment* and its exercise upon concrete cases in successive stages of effort, as soon as the elementary facts and instrumental methods have been securely grasped. Reasoned and practical knowledge of essential principles as embodied in our mathematical formulæ, born of mental labour, and not simply minute refinements, should constitute our working ideal. Vast mechanical dexterity may coexist with an absence of mastery of method and its judicious use. A simple question necessitating this thoughtful acquaintance with principles and their steady employment is infinitely superior in its educational and professional value to a complex theoretical problem involving even a supreme amount of mental power and mathematical adroitness. The effective exertion of independent self-originated thought on a student's part in solving an elementary problem of practice forms the genuine test of the adequacy of our system of education and examination. Moreover, it must always be remembered that our examinations are simply qualifying tests and not competitive struggles; and this conception should imperatively prescribe their organisation and the nature and compass of the questions proposed. This suggestion also hints at the undesirability of furnishing for study a specific set of books, which is only appropriate where the examinations assume a competitive, and therefore a largely uneducative, character, when surveyed under a true conception of the genuine aim.

E.—ADVICE TO STUDENTS.

This reference to examinations provides a natural mode of transit to some brief observations of encouragement and advice to our students. I would remind them that, although examinations must necessarily form a means, and an essential means, of admission to our ranks, besides possessing the virtue of an aid to mental and moral discipline and aptitude of intellectual concentration,—the ultimate sources of power,—they in themselves constitute but imperfect evidences of Professional fitness, and insecure and precarious modes of obtaining and measuring that soundness of knowledge and validity of judgment which express, in every sphere of work, the finished and efficient force of Professional training. I might almost completely summarise my remarks upon the suitable self-education of a student by bringing into relevant relief the ancient dispute between the Analytical and

Geometrical forms of mathematical instruction.* The Analyst is the speaker of symbolic language; he employs his shorthand signs and the established processes of their relationship and evolvment with exclusion, or rather non-presentation to the mind, of the sensible phenomena or actual conceptions which they represent; his therefore is a purely formal method; while the Geometrical student carries consciously throughout his investigations and calculations the mental notions corresponding to the sensible facts with which his inductions and deductions are concerned. The former course is attended with diminished mental strain; while the latter, difficult in its nature, is instinct with possibilities of the highest and most vigorous intellectual discipline. In illustration of this distinction, consider any of our fundamental symbolic expressions. A student may be master of his representative signs, and, by ascertained algebraical processes, and combinations with other expressions, may deduce various transformations; but if he restrict himself to this symbolic scheme alone, he becomes, from the true aspect of education, but a mechanical, however skilful, workman; the student, however, who realises the mental conceptions which the symbols depict, and rigorously carries these notions in his mind throughout his calculations and deductions, is possessed of the larger gift of increasing intellectual clearness and resource, and competent, therefore, to deal more efficiently with actual practical cases which his work hereafter will present. The intellect thus exercises its free and legitimate sway, and the symbols assume their appropriate position of abbreviated language acting as the vehicle of reasoned thought. Follow then closely the actual original writing inscribed by Truth in place only of the brief shorthand transcript into which it is translated.

Permit me again to impress upon the student, the supreme importance of even a minute amount of knowledge and its application which is secured by an earnest, patient, and thoughtful exercise of his own mind as surpassing, and indeed surpassing beyond calculable measure, the completest accretion of learning which is mainly of a remembered form. Each life should be an independent and original life in the construction of character and mind; each faculty trained by strenuous self-effort which mere recollective habit simply blunts; each result only accepted when grasped and confirmed by arduous personal mental toil, so that memory may be relegated, in the scheme of Professional education,

* Whewell: *Of a Liberal Education*: Section V.

Whewell: *Thoughts on the Study of Mathematics*.

to its natural office of a storehouse of facts which self-disciplined power has acquired and tested as a portion of the intellectual structure itself.

Be assured therefore that you have not comprehended your formulæ and your equations until you have proved able to translate them into the intellectual conceptions which they pictorially represent, and unless you perceive, not simply the mathematical connexion between the several portions of your symbolic train of reasoning but are capable also of deciphering and realising a rigid and necessary consistency and reasoned sequence between the mental notions which are involved. The natural relationship of the actual facts must be recognised ; not merely the formal congruence of the pictured signs.

Bear also in mind the truth that no one can be competently possessed of the import of a proposition until he has acquired the power of expressing it intelligently and securely in his own words. It is a mark of imperfect training and unprogressive education,—a sign indeed of mental retrogression or impotence,—when we continually find in our examinations and elsewhere that a Professional proposition is simply repeated by each student in the language in which it was enunciated by his tutor. In such unhappily frequent instances, the student merely proclaims himself to be,—not an organic bundle of disciplined and alert faculties, but,—simply a common repository of information in which his individuality possesses no part or lot by thoughtful toil. The future of a student is essentially poised upon this pivot : honour and success await the one whose progress signifies the exercise of faculty : failure is predicted by a mechanical dependence upon the energies of others.

I add a word upon the supreme value, as an index of future worth, of that self-reliance in the student which is cultivated by his sole individual search after sources and facts of knowledge instead of ignoble and supine contentment with prescribed courses of study and indicated books. The great Lessing has inspired us by the saying,—and all experience and reflection emphasise its force,—that if the Almighty offered him in one hand the Truth already acquired, and, in the other, the strenuous search after Truth, he should humbly but unhesitatingly prefer the toil of search rather than that degrading abnegation of faculty which he possessed in trust for ardent and self-denying cultivation.

The plastic period of youth alone furnishes the facile occasion

for gradually moulding those indurated Habits (laborious of later acquisition) which will enrich and invigorate all after-life.

Gentlemen-Students, I have delineated a lofty Ideal; but Ideals form the stimulus and reward of life; and so, with a cheerful word of hope to you, I finish my exhortations.

VIII.—CONCLUSION.

It is now my duty and privilege,—compounded, as all final duties are, of pleasant and mournful feeling,—the sadness of retrospect and the hopefulness of vision that scans the future,—to utter a few closing sentences of personal import. For although some months of happy ministry in office still stretch before me, my Address this evening is valedictory in its tone. The recollections of the year are deeply permeated with an abiding sense of the courtesy and goodwill which I have experienced from the Members,—I gratefully extend this expression to each individual Member. Your kindly interpretation of my motives has constantly atoned for my defects of personal service; and as I survey the past, and perceive the frequent occasions on which, with wider knowledge and more sustained sympathy, I could more adequately have fulfilled my duties, the regret which I naturally feel is largely lost, except as a stimulus to the future, in the memory of invariable help and generous encouragement. And intimately interweaved with this experience is the impressive sense I have gained of the distinguished mission of the Institute and its prompt response to its minutest obligations; no sign of decrepitude impairs our hopes; the omens are clearly predictive of enlarged and diffusive power; and, with wise, deliberate, and mature adaptation as its sphere expands, the pride in its history which animates its existing Members will suffer no diminution of intensity or amplitude in the minds of our successors upon whom its fortunes will honourably rest.

APPENDIX.

THE TITLE OF ACTUARY.

1. I have consulted the chief authority in antiquity for the meaning of the Term,—C. Suetonius Tranquillus,—and in his work entitled “*De Vitâ Cæsarum*”: Liber I: Divus Iulius: § 55, the following passage occurs: “Pro Quinto Metello (id est, “oratio) non immerito Augustus existimat magis ab actuaris

“exceptam male subsequentibus verba dicentis quam ab ipso editam.” Here it would appear, from the expression “male subsequentibus”, that the Actuarii were simply a shorthand writer; but from a wider examination I am inclined to think that this is possibly too restricted an interpretation, and to conjecture that the reference rather describes *one* only of the functions which he at times exercised.

In the absence of Newspapers in the ancient world, it was difficult to adequately acquaint the citizens with important knowledge, such, for example, as the decrees of the Senate. These decrees, it is true, were committed to writing under the direction of certain Senators, and deposited as State archives in the *Aerarium* or Treasury; but without public promulgation. Accordingly in the First Consulship of Julius Cæsar, B.C. 59, an edict was issued that Senatorial decisions and other official and civic information should be exhibited for the guidance of the people. These public chronicles of official proceedings and social events were generically termed “Acta.” The *Acta diurna*; *Acta publica*; *Acta urbana*, contained a record of the births, marriages, divorces, and deaths; accounts of money transactions between the Treasury and the Provinces; reports of the edicts of magistrates; and extracts from the *Acta Senatus*.

But these several *Acta* were not the same as the *Acta Senatus* from which, as I have stated, extracts were incorporated. Now it is clear that the Actuarii were a clerk or subordinate officer, who, in pursuance of the edict of Cæsar, attended the sittings of the Senate; and to him was committed the duty of compiling their *Acta*, under the superintendence of a superior official, from notes taken “*inter loquendum*.” The Actuarii accordingly made notes of the proceedings in the Senate, and subsequently compiled the *Acta* or records for publication. His designation is sometimes stated to be “*Actarius*”, which more closely identifies his functions with the preparation of the *Acta*. An Actuarii also compiled the *Acta* of a more general character which I have described, and it is stated that he was assisted by the *Notarii* or reporters who took down in shorthand the proceedings in the legal courts and elsewhere. For a system of shorthand or tachygraphy had been introduced into Rome during the last century of her freedom, and its origin is variously attributed to Cicero, to Mæcenas, and to Ennius. Another class of officials, whose name I have already mentioned, was especially termed the “*Notarius*”, who (as the etymology “*Nota*”, or mark, suggests) was essentially a

shorthand writer: these persons were generally slaves or libertini; and wealthy Romans frequently employed them in their personal service for the purpose, *inter alia*, of copying for private use the contents of the published Acta. At a later stage, the Notarii attended the Emperors as private secretaries, and the class then naturally became constituted of persons of distinction. It was inevitable that the titles of Actuarius and Notarius should continually be regarded as synonymous terms; and after much patient enquiry, I venture to conclude (i) that the various Acta were compiled for public use by the Actuarius; that in the preparation of this work he collected the social events from the official Registers; the decisions of the Magistrates from the shorthand reports of the Notarii who attended the Courts; but that the Acta Senatûs were compiled by him from notes of its proceedings which he himself, and he alone, had taken in shorthand; and (ii) that ultimately the terms Actuarius and Notarius became frequently interchangeable in colloquial usage, based on the fact that tachygraphy was connected with each profession, notwithstanding the distinctive circumstance that shorthand reporting formed the exclusive province of the one, but was associated with the higher function in the other of official compilation.

II.—Passing to more modern times, we find that the Registrar of the Lower House of Convocation is still styled the Actuary. I have failed to obtain much detailed information upon this stage of the history of the name. In 1667, Chamberlayne refers to the Actuary as taking down the decisions of Judges in the Court of Arches; in 1702, Bishop Gibson mentions that he was an officer of the Archbishop, and cites the following passage from the fees established by Archbishop Whitgift (1583–1603), for the Vicar-General's office:—"Feoda Actuaria Domûs Inferioris Convocationis solvenda"; while, in 1717, the term is defined by Blount as the title of the scribe who registered the Acts of Convocation.

It seems to me reasonable to suppose that the name was adopted as appropriate to the compilation of the Acts or Proceedings of an important Ecclesiastical Body in analogy with its relation to the Acta of the Roman Senate.

It is probable that the Actuary of the old Amicable Life Office was termed synonymously the Registrar on the basis of this ecclesiastical precedent.

III.—I have been able to trace the history of the title in Acts of Parliament through the courtesy and kindness of Mr. E. W. Brabrook:

- i. In the Friendly Societies' Act of 1819 (59 Geo. III, c. 128) it was enacted that a Justice should not confirm any Tables of Payments or Benefits or any Rules dependent upon their calculation unless they were approved by two persons, at the least, who were known to be professional Actuaries or persons skilled in calculation; and that a Society should not be dissolved unless a Certificate had been obtained from two or more professional Actuaries or persons skilled in calculation.
- ii. The first mention of the Actuary in connexion with Savings' Banks occurs in the Savings Banks' Act of 1824 (5 Geo. IV, c. 62) where the Certificate of transfer of deposits from one Savings' Bank to another must be furnished by two or three Trustees and Managers attested by the Secretary or Actuary of the Savings' Bank in question: the annual Statement also was to be countersigned by the Secretary or Actuary of the Bank.
- iii. This Act was repealed in 1828 by the Act 9 Geo. IV, c. 92, which however re-enacted the provisions relating to the Actuary's attestation of transfers, and his counter-signature to the Annual Accounts.
- iv. The Friendly Societies' Act of 1819 was repealed in 1829 by the Act of 10 Geo. IV., c. 56, and the provisions referring to professional Actuaries were not re-enacted. Moreover there is no reference to Actuaries in the provisions relating to the Tables of Contributions and Allowances.
- v. The Loan Societies' Act of 1840 (3 & 4 Vict., c. 110),—which is still in existence,—provided that a Barrister should not certify the Rules of a Society (where the Society adopted a scheme differing from that contained in the Schedule to the Act) unless a Certificate had been obtained from the Actuary to the National Debt Office.
- vi. In the Savings Banks' Act of 1844 (7 & 8 Vict., c. 83) provision was made for the punishment of an Actuary or other Officer who, receiving deposits, failed to hand them to the Managers.

- vii. The Friendly Societies' Act of 1846 (9 & 10 Vict., c. 27) required that the Registrar should not certify the Rules of a Society unless it adopted Tables certified by the Actuary to the Commissioners for the Reduction of the National Debt, or by some person who had been connected as Actuary for at least 5 years with a Life Insurance Company in London, Edinburgh, or Dublin.
- viii. This Act was repealed by the Friendly Societies' Act of 1850 (13 & 14 Vict., c. 115), but a similar provision to the preceding was inserted applicable to a class of Societies to be termed "Certified Friendly Societies"; and in the Schedule a form of Actuarial Certificate was prescribed.
- ix. By the Friendly Societies' Act of 1855 (18 & 19 Vict., c. 63) all previous Acts were repealed, and the requisition of an Actuarial Certificate was restricted to the case of a Society which only assured to its members a certain Annuity or Superannuation, immediate or deferred. The definition of Actuary was amended, and there was substituted for the qualification of having been at least 5 years connected as Actuary with an Assurance Company that of being the Actuary of some Life Office and of having exercised the profession of Actuary for at least 5 years.
- x. The Friendly Societies' Act of 1858 (21 & 22 Vict., c. 101) provided that Societies desirous of dissolving should be enabled to refer the appropriation or division of their funds to an Actuary as there defined.
- xi. This Act was repealed by the Friendly Societies' Act of 1860 (23 & 24 Vict., c. 58).
- xii. By the Savings Banks' Act of 1863 (26 & 27 Vict., c. 87) the previous Savings Banks' Acts were repealed, but clauses were re-enacted relating to the Actuary's punishment if omitting to pay over deposits; his counter-signature of the Annual Returns; and his signature to Certificates of the transfer of funds. This Act is still in force.

This series of Acts, besides revealing the insecure stability of our Legislative measures, is unhappily defective in assisting us to the conception entertained in the earlier stage of the definition of an Actuary. The mischievous conjunction "or" contained in the Act of 1819,—so fertile, in its loose employment, of misunderstanding and misinterpretation,—totally obscures the subject, for we are unable to ascertain with certainty whether the "or" indicated that the mention of "persons skilled in calculation" was intended as a mere synonym or alternative definition of the Actuary, or whether it simply placed together, as equally authoritative, *two* different qualified Referees.

Mortality Experience of Assured Lives and Annuitants in France.

By G. F. HARDY, F.I.A., *Actuary of the English and Scottish Law Life Assurance Association.*

[Read before the Institute, 29 March 1897.]

THE publication of the tables of mortality based upon the experience, collected by the Committee representing the four principal insurance companies of France, puts us in possession of an extensive and valuable body of statistics bearing upon the mortality of assured lives and annuitants in that country, and at the same time places the Actuarial profession under a considerable obligation to the Committee of French Actuaries to whom we are indebted for the collection and publication of the data and the resulting monetary tables. As in this country, the life assurance companies on the Continent had for a long time, necessarily, to depend upon statistics of mortality based upon observations of the population generally, which, whatever the skill with which they were prepared, were not only affected by the uncertainty attaching to all such statistics, but were not in any case specially suited to represent the probable mortality, either of assured lives or annuitants. The earliest of these tables, to which any value could be attached, are those of Deparcieux and Duvillard, corresponding in point of date, approximately, to the Northampton and Carlisle Tables in England, and it was not until 1860 that the three companies, the Compagnie d'Assurances Generales, Union, and Nationale, published their annuity experience, nor until 1874 that the first of these companies published a table representing the mortality experience

of their assurants. These latter tables, however, were based upon a relatively small number of facts, and in November 1876, a committee of six (subsequently reduced to four) French Companies determined to construct a mortality table representing their joint annuity experience, and that of a seventh company, "le Monde", this work being entrusted to the Actuaries of the six companies under the presidency of M. de Kertanguy, Actuary to the "Generale." Eleven years later, the same committee decided upon the construction of a second table, based upon the experience of assured lives. The data in the case of the annuitants extended in its final form, from 1819 down to the close of the year 1889, and in the case of the assurants, from 1819 to 1887. The general results of the experience for both annuitants and assured lives, were published at the time of the Great Exhibition of 1889, together with an adjustment of the mortality tables by Woolhouse's formula. This adjustment was subsequently replaced by an elaborate graduation by Mr. Makeham's formula, upon which all the subsequent monetary tables have been based.

EXTENT OF THE DATA.

The extent of the data, both as to annuitants and assured lives, is very considerable, as will be seen from the following figures:

Table	Lives	Years of Life	Deaths
Annuityants, Male ...	} 67,247 {	244,292	16,410
„ Female ...		391,617	20,506
Assurants, Male ...	} 229,143 {	1,408,398	18,926
„ Female ...		382,258	3,691

In respect to the assured lives, therefore, the experience is rather more extensive than that of the Institute, while the annuity experience is somewhat larger than that of the Government Annuity Experience in this country, published in 1884.

MODE OF COLLECTION.

Without discussing all the details of the methods employed in the collection and tabulation of the experience, it will be useful to indicate generally the lines followed, which were practically identical in respect of both tables. A separate tabulation was made in both cases for male and female lives, and in the case of

assured lives for the various classes of assurances most usually granted by the French Companies, although the figures for these special groups have not yet been separately published. In both cases the experience has been based upon lives, and not policies.

In computing the ages of the exposed to risk, it was assumed that all persons born between 1 July in any year and the following 30 June, were born on the intervening 1 January, and that, similarly, all lives entering under observation between 1 July and the following 30 June entered on the intervening 1 January. This latter assumption, however, appears to have been modified in the case of entrants, between the 1 July and 31 December, who were assumed to be at risk for a quarter of a year in the calendar year of entry. This course would seem to be inconsistent with the original assumption, and to have the effect of over-estimating the exposed to risk throughout the table, by approximately one-eighth of the entrants at each age, thus somewhat under-estimating the average rate of mortality at the insuring ages.

In each case the data was arranged in a form permitting of the calculation of "select" tables for each age at entry. The tables at present published, however, consist simply of the aggregate mortality tables for all ages at entry combined, being similar in this respect to the H^M and H^F Tables.

Although the exposed to risk and deaths are shown separately for males and females, the graduated tables upon which the monetary results are based refer only to both sexes in combination.

With respect to the assured lives, in which females represent about one-fifth of the whole experience, this step is perhaps less open to objection than in the case of the annuitants. Assuming that it is usual to assure female lives on precisely the same terms as male lives, and that the proportion of the former to the latter among new entrants is, on the whole, fairly constant, it may be argued with much plausibility that the proper table to employ for the calculation of premiums or reserves is one based upon the experience of both sexes. The principal objection, I apprehend, that can be urged against this proposition is that, owing to the differing incidence of the mortality at the various periods of life among female as compared with male lives, the effect of such a step would probably be to lower somewhat both the net annual premiums and the reserves, and that therefore it is an error on the safe side to exclude the female lives from the

experience and to base all calculations upon the male mortality table only.

Whatever may be said, however, as to the utility of an H^{MF} table in relation to assured lives, among whom it is common, although not universal, to treat the sexes upon the same terms, there can be no doubt, I think, as to the desirability of distinguishing between the sexes in the case of annuitants, among whom the factor of sex appears so powerfully to influence the mortality.

With the view of testing how far the final tables, which represent fairly the aggregate mortality of the two sexes, depart from the original facts when the sexes are treated separately, I have computed the expected deaths of both males and females at various ages amongst the annuitants and assured lives, respectively, the figures being given in Tables 5 and 6. From these it will be seen that while the deaths among the male annuitants exceeded the expected number by about 15 per-cent, those among the female annuitants fell short by nearly 10 per-cent, and that, therefore, annuities granted upon the lives of females, upon the basis of the mixed table, must result, in the long run, in considerable loss to the companies; leaving out of account, of course, any counter-vailing profit from interest.

METHOD OF GRADUATION.

As already stated, both tables were ultimately graduated upon the basis of Mr. Makeham's formula. The constants were determined in each case by a very elaborate process. In the first instance, approximate values of the constants were obtained by the so-called "aggregate" method described in the paper by Mr. King and myself in vol. xxii, *J.I.A.* Corrections to these constants were then obtained by the method of "least squares", an equation of condition being formed at each age between the observed and adjusted value of $\text{Log } p_x$ and the corrections to the three constants in the formula. Thus, starting from the initial equation $\log p_x = a + \beta c^x$, we have $\log p_x = a_0 + \beta_0 c_0^x + \delta a_0 + c_0^x \delta \beta_0 + \beta_0 x c_0^{x-1} \delta c_0$, where p_x is the observed value of the probability of surviving a year at age x , a_0 , β_0 , &c., are the first approximations to the value of the three Makeham constants and δa_0 , &c., the corrections to such first approximations, to be obtained by the method of least squares.

These equations were weighted in proportion to the square

root of the "number of observations" at the given age. It is not clear whether by the "number of observations" is meant the number of lives exposed to risk, or the number of observed deaths. The latter would, of course, be the more correct proceeding, as it has been variously shown that the mean error in the value of q_x and hence, generally, in the value of $\log p$ in any series of observations is nearly proportionate to $\frac{\sqrt{nq}}{n}$ where n is the number of exposures, and nq the adjusted or expected deaths, which, for this purpose, may be taken as equal to the observed deaths.

Improved values of the constants having been thus obtained, the process was again repeated, and the finally adopted values were brought out as under, using the customary symbols for the constants.

Constant	Annuity	Assured Lives	Constant	Annuity	Assured Lives
$c \dots$	1.1001136	1.0916817	$\log c \dots$.0414376	.0380960
$g \dots$	0.9993868	0.9984400	$\log g \dots$	-.0002664	-.0006780
$s \dots$	0.9944272	0.9949930	$\log s \dots$	-.0024270	-.0021800

It will be seen that the value of $\log c$ is somewhat below the average value found for this quantity, in the case of the Assured Lives table, and somewhat above that value in the case of the Annuity table.

It is to be remarked that Makeham's formula was not applied below age 25 in either table, and that, up to that age, the two tables were made identical, on the ground that on the one hand the observations of annuity at these ages were very few in number, and on the other hand that the lives assured at these ages relating principally to contingent and deferred assurances, might be considered to be of the same class as annuity.

Below age 25 the adjusted rates of mortality were obtained on the assumption that q_x could be represented for this portion of the table as an algebraical function of the sixth degree,

$$q_x = m_0 + m_1x + m_2x^2 + m_3x^3 + m_4x^4 + m_5x^5 + m_6x^6.$$

Seven equations were then obtained to determine the seven constants by equating the above expression to the values q_{26} , q_{25} , $(q_0 + q_1)$, $(q_2 + \dots + q_7)$, $(q_8 + \dots + q_{12})$, $(q_{13} + \dots + q_{18})$, and $(q_{19} + \dots + q_{24})$.

In determining the constants, the original facts for ages 23 to 70 only in the case of assurers, and from 40 to 89 only in the case of annuitants, were used. It is not quite clear why the data beyond age 70 should not have been utilized in determining the constants for the "Assured Lives" table; as a matter of fact, however, the graduated table represents the observed rates of mortality above age 70 fairly well, the adjusted number of deaths being 1,215, as against 1,200 observed. At ages over 82, it is true these numbers become 102 and 80 respectively, but such a deviation at these ages in a table relating to Assured Lives, cannot be considered as of moment. Of greater consequence, perhaps, is the fact that, at ages over 89, the data for which were neglected in determining the constants for the annuity table, the adjusted deaths were 1,101 to 899 observed, a feature, no doubt, due to the unusually large value of the constant c used in the formula.

It may be as well here to refer to the use of the method of "least squares" in this connection. The method is, of course, not so strictly applicable to the construction of a mortality table as in the determination of physical constants, as the positive and negative deviations from the mean results are not equal in amount, nor equally probable. The assumption, however, that they are so, is a very convenient one, and sufficiently near the truth to justify its employment in practice.

In the use of the method, it is clear that two alternative assumptions may be made which lead to different modes of procedure.

In the first place, we may assume that the mortality during adult life, among a group of persons similarly constituted to those under observation, but so numerous as to practically eliminate accidental irregularities, would strictly follow Makcham's law. Upon this assumption, the differences between the observed rates of mortality and those given by the formula, are merely accidental differences, due to the limited number of facts observed at individual ages, and the constants should be so determined as to make the sum of the squares of these deviations a minimum, the equations of condition at each age being multiplied by the square root of the number of deaths, so as to make the probability of a deviation of given amount in such a function as $\log p_x$ equally likely at all ages.

Secondly, it may, in the alternative, be assumed that the true law of mortality only approximately follows that given by

Mr. Makeham's formula. The deviations are then of two kinds; on the one hand the usual accidental irregularities, and on the other real divergencies between the law of the series when divested of these irregularities and the law represented by the formula. From this point of view, the true law of the series should be approximately ascertained by removing the accidental irregularities by some suitable method of graduation, and such constants should be adopted for the formula as will make the monetary values (for example, the annuities) based thereon correspond as nearly as possible to those deduced from the preliminary graduation, special weight being given to those ages at which such correspondence is of greatest importance in practice.

With respect to the use of Mr. Makeham's formula at all for purposes of graduation, it has to be borne in mind that the object of an adjusted mortality table cannot, strictly speaking, be to exhibit the results of any past experience. These are already given in the unadjusted table, and, of course, are not capable of alteration. The object in general of graduation is to produce a table which it is believed will be applicable to a similar body of lives in the future. As it is quite impossible that any two bodies of assured lives or annuitants can precisely resemble one another, when separated by at least a generation in point of time, and subject to all the changes of circumstance and condition involved, it is clearly out of the question to expect that the rates of mortality observed in the past will be reproduced with precision in the future, and it is, therefore, sufficient, for all practical purposes, if the adjusted table, upon which future results are to be estimated, accords, within a reasonable degree of approximation, with the original unadjusted results, more especially if no important systematic error can be pointed out in the formula. The applicability, therefore, of Mr. Makeham's formula can only be determined by trial in each individual case, and where it is found, as is the case, for example, in the H^M Table, that the monetary results based thereon are practically identical with those obtained by other methods of adjustment involving no assumption as to a law of mortality, there need be no hesitation in employing the formula in question, having regard to the immense advantage it affords, in many directions, in calculation of life contingencies.

In the tables we are now considering, the mortality of the assured lives (both sexes) is very well represented by the formula,

the differences between the actual and adjusted number of deaths, as shown in Table 5, not being greater than should be expected, and showing no tendency to a systematic departure of the graduated table from the original facts. In the Annuity Table, however, the agreement is not so satisfactory, as is, in fact, pointed out by its authors. It will be seen that from age 25 to 54 the actual deaths are largely in excess, while from 55-69 they are more than 6 per-cent below the adjusted numbers, being again in excess during the following 15 years of age, and about 8 per-cent below (on nearly 4,000 deaths) after age 85. The reasons for this want of agreement, which is, to a considerable extent, noticeable in other attempts to adjust the rates of mortality of annuitants by Makeham's formula, are, in the first place, that the mortality is disturbed for so large a portion of the table by new entrants, and, in the second place, that the male annuitants, in particular, under observation at the younger ages, are, on the whole, very inferior, in point of vitality, to assured lives of the same age, as may be at once seen by comparing the rates of mortality of male annuitants and assurers in Table 7.

Without stopping to consider particularly the cause of this phenomenon (the effect of which upon the character of the mortality table may, however, be fairly represented by employing Mr. Makeham's further development of Gompertz's formula) the question arises as to how far such a feature should be retained in a graduated mortality table intended to be used as a basis for the grant of annuities. The method employed by the French Actuaries in determining the constants for the annuitants' mortality table, in which the rate of mortality at age 25 was made identical with that for the same age in the table of assured lives, and the very limited experience between ages 25 and 40 was discarded, has had the effect of removing this feature of a heavy annuitant mortality at the younger ages, and keeping the rates throughout below those for assured lives of similar age. It appears to me that this is a perfectly sound proceeding, and that it is not easy to justify the use of rates of mortality, in a table upon which a Company grants annuities, higher than the rates it assumes in granting assurances.

The divergence in this respect between the Government Annuitants (male) and the H^M "Select" Tables, which are the Standard Tables for annuities and assurances in this country is, of

course, well known. It will be seen from the following values of annuities at 3 per-cent, according to the two tables, that up to age 30 it is equivalent to a difference of about five years in the age:

Age	Government Annuitants (Male) Select	HM "Select" (Sprague)	Excess of HM over Government Annuitants
20	20·560	21·342	·782
25	19·600	20·667	1·067
30	18·588	19·671	1·083
35	17·516	18·491	·975
40	16·376	17·132	·756
45	15·152	15·616	·464
50	13·813	13·998	·185

The lives upon whom annuities are granted at ages below, say 40–45, clearly form a mixed class, and include, from various causes which need not here be discussed, a large proportion of under-average lives, who, if proposed for assurance, would be accepted only at an extra premium. As, however, many annuitants at these ages will be quite equal, in point of vitality, to the average of assured lives, and as, moreover, a Company may, at any time, be called upon to grant both an annuity and an assurance upon the same life, it would, I consider, be desirable in practice boldly to substitute for the observed rates of mortality in the annuitant experience the rates representing the mortality of assured lives wherever the latter rates are the lower.

The value of Makeham's formula as an instrument for the adjustment of mortality tables must depend largely upon the facility with which it can be adapted to "Select" tables. That this is theoretically possible I have shown elsewhere.* This is, perhaps, scarcely the place to deal with the practical application of the theory, but the following result of a first rough attempt to represent Dr. Sprague's Select tables may be of interest:

Taking the value of

$$\mu_{[x]+t} = \{A + Hx + Bc^x\} - \phi(t)(A + Bc^x),$$

where the terms in the large bracket represent approximately the

* See *J.I.A.*, vol. xxxi, p. 359.

mortality of the $\Pi^{(5)}$ Table, and the value of $\phi(t)$ is taken arbitrarily as follows:

$t=0$	$\phi(t)=\frac{35}{70}$
1	$\frac{20}{70}$
2	$\frac{10}{70}$
3	$\frac{4}{70}$
4	$\frac{1}{70}$
5 and upwards	.	0

the resulting mortality table yields the following net annual premiums at 4 per-cent:

Age	Sprague's Select	Makeham's Graduation	Difference	
			+	-
20	1.391	1.370021
25	1.509	1.516	.007	...
30	1.714	1.719	.005	...
35	1.992	1.993	.001	...
40	2.361	2.362	.001	...
45	2.851	2.854	.003	...
50	3.488	3.510	.022	...
55	4.358	4.388	.030	...
60	5.541	5.561	.020	...
65	7.138	7.138000

The nature of these differences would indicate that the graduated results might easily be brought still closer to the "Select" values, but it will be conceded that the agreement is close enough for practical purposes.

RESULTS OF THE EXPERIENCE.

The main interest in the new table will, I apprehend, centre in a comparison of the results obtained in respect of both classes of lives with those obtained in this country. In the absence of select tables, which, however, it is to be hoped that the Committee of French Actuaries will see their way to publishing, it is only possible to make a comparison with aggregate tables, such

as the H^M and H^F , and with similar tables for the Government Annuitants. For this purpose, adjusted tables not having been prepared for the separate sexes, I have computed the mortality tables which follow by means of the unadjusted rates of mortality.

The number of annuitants observed below age 40 is too small to make the results of any value, but beyond that age, and generally throughout life so far as the assured lives are concerned, there is a sufficiency of data to bring out tables regular enough for purposes of comparison.

The following figures, extracted from the more complete tables following this paper, give a comparison of the expectation of life for each sex, in the case of the assured lives, with the H^M and H^F tables, and, in the case of annuities, with the Government Annuity Experiences for all ages at entry.

Expectation of Life of Assured Lives and Annuitants in France (unadjusted) as compared with the Institute of Actuaries and Government Annuity Experience.

Age	ASSURED LIVES				ANNUITANTS				Age
	Male		Female		Male		Female		
	French Experience	H ^M	French Experience	H ^F	French Experience	English Government Annuity	French Experience	English Government Annuity	
20	41·60	42·06	43·54	40·82	20
25	38·01	38·41	40·30	37·41	25
30	34·13	34·68	36·65	34·50	30
35	30·22	31·02	33·06	31·45	35
40	26·50	27·40	29·37	28·25	26·86	...	30·36	..	40
45	22·88	23·79	25·63	24·99	23·99	...	27·13	...	45
50	19·36	20·31	21·95	21·62	20·61	20·51	23·49	23·27	50
55	16·09	16·96	18·50	18·19	17·50	17·57	19·89	19·66	55
60	13·02	13·83	14·90	14·85	14·34	14·61	16·25	16·26	60
65	10·45	11·01	11·96	11·77	11·46	11·57	12·90	12·98	65
70	8·04	8·50	9·21	9·08	8·80	8·95	9·90	10·00	70
75	6·05	6·38	7·25	6·93	6·67	6·84	7·40	7·45	75
80	4·69	4·72	6·26	5·45	4·89	5·05	5·40	5·46	80

Dealing first of all with the assured male lives, it will be seen that the average duration of life is, throughout, below that of the H^M Table, the difference between the two tables running fairly regularly. After reaching a minimum about age 25 of ·37 of a year, the difference rises to a maximum at age 50 of nearly one year, and thence diminishes almost uniformly to the end of life.

It is clear that these differences in the value of the "Expectation of life" must represent, at least at the older ages, substantial differences in the value of the net premiums, although in the absence of "select" tables, it is not possible to say accurately what these differences should be. A fairly trustworthy estimate may, however, be made if we assume the relation of the "select" to the "aggregate" tables to be similar (that, for example, the ratio of select to non-select premiums is the same in the French and in the H^M Experience). The following table exhibits the differences between the Institute 3 per-cent net premiums, and those indicated by the experience of French male assurants, and by the experience of the 23 German companies. At a time when so considerable a Continental business is done by English companies, I venture to think that the figures will not be without interest.

Three per-cent net Annual Premium per-cent, according to (1) H^M Table; (2) French Assured Lives (males); (3) Experience of 23 German Offices.

Age <i>x</i>	H ^M 100P _{<i>x</i>}	FRENCH ASSURED LIVES—MALE			23 GERMAN OFFICES		
		100P _{<i>x</i>}	Extra per-cent		100P _{<i>x</i>}	Extra per-cent	
			On Sum Assured	On H ^M Premium		On Sum Assured	On H ^M Premium
20	1·427	1·450	·023	1½ per-cent	1·588	·161	11 per-cent
25	1·625	1·618	·023	1½ per-cent	1·776	·151	9 per-cent
30	1·880	1·923	·043	2 per-cent	2·040	·160	8½ per-cent
35	2·193	2·272	·079	3½ per-cent	2·379	·186	8½ per-cent
40	2·589	2·706	·117	4½ per-cent	2·817	·228	9 per-cent
45	3·114	3·274	·160	5 per-cent	3·384	·270	9 per-cent
50	3·801	4·032	·231	6 per-cent	4·145	·344	9 per-cent
55	4·725	5·030	·305	6½ per-cent	5·156	·431	9 per-cent
60	5·987	6·413	·426	7 per-cent	6·502	·515	9 per-cent

The premiums at the younger ages for the French Male Assurants would, of course, be somewhat increased if the correction suggested above (the deduction of one-eighth of the entrants at each age) were made in the numbers exposed to risk—and the effect of this would probably be to bring the extra at these ages to nearly 5 per-cent of the H^M premium.

On the other hand, the female assured lives show a distinctly lighter mortality, especially at the younger ages, than that shown by the H^F table, and, so far from demanding any extra premium for whole term assurances as compared with the male lives, they show

at all ages a considerably higher expectation of life. It will be seen, however, on reference to Table 5, where the "expected" deaths for each sex are computed by the same standard, that from age 10 to 35 the female assurants exhibit a somewhat higher rate of mortality than the males, although the difference is not very material, amounting only to about '001 in the value of q_x , equivalent, say, to about 2s. 6d. per-cent in the premiums for short term risks at those ages, unless this comparison is disturbed by a different effect of selection in the two tables.

The annuity experience, both in respect of males and females, agrees very closely with that of the English Government experience. For ages below 50 the English figures are not available, but the differences in the value of the function \dot{e}_x for that age onward, between the French and English Annuitants, whether male or female, is so small that the tables might almost be said to be identical. If we assume that the effect of selection upon the mortality in the case of annuitants is nearly the same in both countries, it would appear that the same annuity-rates may be employed in practice both here and in France.

After what has been said above, as to the relation of the French Experience to our own Standard Tables, it will be unnecessary to do more than refer to the comparison made in Tables 8 and 9 of the values of annuities, according to the various mortality tables there stated.

The French "Assured Lives" Table will be seen to occupy a position nearly midway between the English and German standards; and it is evident, from the run of the annuity-values, that while the premiums are higher than under the H^M Table, the valuation reserves will be also somewhat larger, the same being true of the German Table. The French Annuitant Table occupies a position intermediate between the Government Male and Female Tables, except at the ages below 40, to which reference has already been made above.

It is only necessary to add, in conclusion, that the data collected by the French Actuaries is of such interest and value, that it is to be hoped they will see their way to publish the materials in a form suitable for the construction of tables for each age at entry, and thus lay the actuarial profession under still further obligation.

TABLE 1.

FRENCH ASSURED LIVES, MALES.—*Unadjusted Mortality Table.*

Age	q_x	l_x	d_x	\bar{e}_x	Age	q_x	l_x	d_x	\bar{e}_x
(1)	(2)	(3)	(4)	(5)	(1)	(2)	(3)	(4)	(5)
0	·0329	10,000	329	51·94	50	·0167	6,305	105	19·36
1	·0335	9,671	324	...	51	·0180	6,200	112	...
2	·0214	9,347	200	...	52	·0195	6,088	119	...
3	·0141	9,147	129	...	53	·0212	5,969	127	...
4	·0132	9,018	119	...	54	·0217	5,842	127	...
5	·0080	8,890	71	53·12	55	·0232	5,715	133	16·09
6	·0067	8,828	59	...	56	·0241	5,582	134	...
7	·0054	8,769	47	...	57	·0248	5,448	135	...
8	·0049	8,722	43	...	58	·0287	5,313	152	...
9	·0049	8,679	43	...	59	·0312	5,161	161	...
10	·0032	8,636	28	49·67	60	·0361	5,000	181	13·02
11	·0028	8,608	24	...	61	·0387	4,819	187	...
12	·0032	8,584	27	...	62	·0390	4,632	181	...
13	·0028	8,557	24	...	63	·0432	4,451	192	...
14	·0012	8,533	36	...	64	·0465	4,259	198	...
15	·0049	8,497	42	45·44	65	·0460	4,061	187	10·45
16	·0052	8,455	44	...	66	·0507	3,874	196	...
17	·0053	8,411	45	...	67	·0513	3,678	189	...
18	·0065	8,366	54	...	68	·0599	3,489	209	...
19	·0062	8,312	52	...	69	·0695	3,280	228	...
20	·0093	8,260	77	41·60	70	·0690	3,052	210	8·04
21	·0065	8,183	53	...	71	·0785	2,842	223	...
22	·0069	8,130	56	...	72	·0898	2,619	235	...
23	·0050	8,074	40	...	73	·0855	2,384	204	...
24	·0065	8,034	52	...	74	·0891	2,180	194	...
25	·0057	7,982	46	38·04	75	·1250	1,988	248	6·05
26	·0056	7,936	44	...	76	·0863	1,738	150	...
27	·0062	7,892	49	...	77	·1330	1,588	211	...
28	·0060	7,843	47	...	78	·1520	1,377	209	...
29	·0066	7,796	51	...	79	·1520	1,168	178	...
30	·0056	7,745	43	34·13	80	·1380	990	137	4·69
31	·0072	7,702	55	...	81	·1330	853	113	...
32	·0069	7,647	53	...	82	·2330	740	172	...
33	·0069	7,594	52	...	83	·1850	568	105	...
34	·0075	7,542	57	...	84	·1390	463	64	...
35	·0083	7,485	62	30·22	85	·2440	399	97	3·33
36	·0088	7,423	65	...	86	·1850	302	56	...
37	·0093	7,358	68	...	87	·2110	246	52	...
38	·0092	7,290	67	...	88	·2860	194	55	...
39	·0095	7,223	69	...	89	·3330	139	46	...
40	·0103	7,154	74	26·50	90	·0000	93	...	2·17
41	·0110	7,080	78	...	91	·3330	93	31	...
42	·0108	7,002	76	...	92	1·0000	62	62	...
43	·0116	6,926	80	...	93
44	·0119	6,846	82	...	94
45	·0121	6,764	82	22·88	95
46	·0139	6,682	93	...					
47	·0148	6,589	98	...					
48	·0145	6,491	94	...					
49	·0144	6,397	92	...					

TABLE 2.

FRENCH ASSURED LIVES, FEMALES.—*Unadjusted Mortality Table.*

Age	q_x	l_x	d_x	\bar{e}_x	Age	q_x	l_x	d_x	\bar{e}_x
(1)	(2)	(3)	(4)	(5)	(1)	(2)	(3)	(4)	(5)
0	·0323	10,000	323	53·54	50	·0166	6,364	106	21·95
1	·0285	9,677	276	...	51	·0139	6,258	87	...
2	·0201	9,401	189	...	52	·0127	6,171	78	...
3	·0153	9,212	141	...	53	·0161	6,093	98	...
4	·0120	9,071	109	...	54	·0169	5,995	101	...
5	·0100	8,962	90	54·51	55	·0161	5,894	95	18·50
6	·0078	8,872	69	...	56	·0146	5,799	85	...
7	·0053	8,803	47	...	57	·0161	5,714	92	...
8	·0033	8,756	29	...	58	·0163	5,622	92	...
9	·0028	8,727	24	...	59	·0204	5,530	113	...
10	·0051	8,703	44	51·08	60	·0248	5,417	134	14·90
11	·0045	8,659	39	...	61	·0256	5,283	135	...
12	·0040	8,620	34	...	62	·0283	5,148	146	...
13	·0042	8,586	36	...	63	·0308	5,002	154	...
14	·0039	8,550	33	...	64	·0441	4,848	214	...
15	·0069	8,517	59	47·14	65	·0351	4,634	163	11·06
16	·0068	8,458	58	...	66	·0405	4,471	181	...
17	·0063	8,400	53	...	67	·0312	4,290	134	...
18	·0065	8,347	51	...	68	·0581	4,156	242	...
19	·0040	8,293	33	...	69	·0452	3,914	177	...
20	·0088	8,269	73	43·54	70	·0691	3,737	258	9·21
21	·0081	8,187	66	...	71	·0930	3,479	324	...
22	·0113	8,121	92	...	72	·0640	3,155	202	...
23	·0068	8,029	55	...	73	·0706	2,953	208	...
24	·0065	7,974	52	...	74	·0500	2,745	137	...
25	·0056	7,922	44	40·30	75	·1360	2,608	355	7·25
26	·0085	7,878	67	...	76	·0850	2,253	191	...
27	·0070	7,811	55	...	77	·1110	2,062	229	...
28	·0077	7,756	60	...	78	·0761	1,833	139	...
29	·0063	7,696	48	...	79	·1480	1,694	250	...
30	·0084	7,648	64	36·65	80	·1080	1,444	156	6·26
31	·0079	7,584	60	...	81	·0741	1,288	96	...
32	·0085	7,524	64	...	82	·1440	1,192	172	...
33	·0076	7,460	57	...	83	·1580	1,020	161	...
34	·0078	7,403	58	...	84	·0313	859	27	...
35	·0090	7,345	66	33·06	85	·1724	832	143	4·26
36	·0087	7,279	63	...	86	·1000	689	69	...
37	·0077	7,216	56	...	87	·1714	620	106	...
38	·0075	7,160	54	...	88	·2140	514	110	...
39	·0088	7,106	63	...	89	·2110	404	85	...
40	·0081	7,043	57	29·37	90	·3330	319	106	3·24
41	·0096	6,986	67	...	91	·5000	213	107	...
42	·0106	6,919	73	...	92	·0000	106
43	·0077	6,846	53	...	93	·0000	106
44	·0095	6,793	65	...	94	·5000	106	53	...
45	·0108	6,728	73	25·63	95	...	53
46	·0089	6,655	59	...					
47	·0126	6,596	83	...					
48	·0118	6,513	77	...					
49	·0112	6,436	72	...					

TABLE 3.
FRENCH ANNUITANTS, MALES.—*Unadjusted Mortality Table.*

Age	q_x	l_x	d_x	\bar{e}_x	Age
(1)	(2)	(3)	(4)	(5)	(6)
40	·0221	10,000	221	26·86	40
41	·0159	9,779	156	...	41
42	·0054	9,623	52	...	42
43	·0256	9,571	245	...	43
44	·0138	9,326	129	...	44
45	·0161	9,197	148	23·99	45
46	·0156	9,049	141	...	46
47	·0097	8,908	86	...	47
48	·0126	8,822	111	...	48
49	·0181	8,711	158	...	49
50	·0162	8,553	139	20·61	50
51	·0157	8,414	132	...	51
52	·0198	8,282	164	...	52
53	·0240	8,118	195	...	53
54	·0232	7,923	181	...	54
55	·0211	7,739	163	17·50	55
56	·0188	7,576	142	...	56
57	·0241	7,434	179	...	57
58	·0253	7,255	184	...	58
59	·0261	7,071	185	...	59
60	·0296	6,886	204	14·34	60
61	·0282	6,682	188	...	61
62	·0331	6,494	215	...	62
63	·0323	6,279	203	...	63
64	·3394	6,076	239	...	64
65	·0392	5,837	229	11·46	65
66	·0422	5,608	237	...	66
67	·0446	5,371	240	...	67
68	·0501	5,131	257	...	68
69	·0521	4,874	254	...	69
70	·0596	4,620	275	8·80	70
71	·0704	4,345	306	...	71
72	·0700	4,039	283	...	72
73	·0768	3,756	288	...	73
74	·0849	3,468	294	...	74
75	·0906	3,174	288	6·67	75
76	·0985	2,886	284	...	76
77	·1031	2,602	268	...	77
78	·1150	2,334	268	...	78
79	·1286	2,066	266	...	79
80	·1419	1,800	255	4·89	80
81	·1500	1,545	232	...	81
82	·1675	1,313	220	...	82
83	·1777	1,093	193	...	83
84	·1865	900	168	...	84
85	·2140	732	157	3·66	85
86	·2170	575	125	...	86
87	·2150	450	99	...	87
88	·2440	351	86	...	88
89	·2570	265	68	...	89
90	·2780	197	55	2·91	90
91	·2590	142	37	...	91
92	·2800	105	29	...	92
93	·2500	76	19	...	93
94	·3260	57	19	...	94
95	·3490	38	13	2·0	95
96	·2000	25	13	...	96
97	·3640	13	4	...	97
98	·2300	8	2	...	98
99	·3750	6	2	...	99
100	·4000	4	2	...	100
101	1·0000	2	2	...	101

TABLE 4.
FRENCH ANNUITANTS, FEMALES.—*Unadjusted Mortality Table.*

Age	q_x	l_x	d_x	\bar{e}_x	Age
(1)	(2)	(3)	(4)	(5)	(6)
40	·0081	10,000	81	30·36	40
41	·0149	9,919	148	...	41
42	·0131	9,771	128	...	42
43	·0113	9,643	109	...	43
44	·0141	9,531	134	...	44
45	·0097	9,400	91	27·13	45
46	·0078	9,309	73	...	46
47	·0102	9,236	94	...	47
48	·0108	9,142	99	...	48
49	·0145	9,043	131	...	49
50	·0091	8,912	84	23·49	50
51	·0140	8,828	124	...	51
52	·0119	8,704	104	...	52
53	·0153	8,600	132	...	53
54	·0145	8,468	123	...	54
55	·0120	8,345	100	19·89	55
56	·0113	8,245	118	...	56
57	·0137	8,127	111	...	57
58	·0173	8,016	139	...	58
59	·0186	7,877	146	...	59
60	·0195	7,731	151	16·25	60
61	·0200	7,580	152	...	61
62	·0219	7,428	163	...	62
63	·0257	7,265	187	...	63
64	·0255	7,078	181	...	64
65	·0263	6,897	181	12·90	65
66	·0326	6,716	219	...	66
67	·0363	6,497	236	...	67
68	·0392	6,261	245	...	68
69	·0403	6,016	242	...	69
70	·0481	5,774	278	9·90	70
71	·0520	5,496	286	...	71
72	·0572	5,210	298	...	72
73	·0624	4,912	306	...	73
74	·0666	4,616	307	...	74
75	·0739	4,309	318	7·40	75
76	·0835	3,991	333	...	76
77	·0885	3,658	324	...	77
78	·0982	3,334	327	...	78
79	·1130	3,007	340	...	79
80	·1259	2,667	336	5·40	80
81	·1373	2,331	319	...	81
82	·1490	2,012	300	...	82
83	·1547	1,712	265	...	83
84	·1640	1,447	237	...	84
85	·1720	1,210	208	4·11	85
86	·1860	1,002	186	...	86
87	·2100	816	171	...	87
88	·2120	645	137	...	88
89	·2490	508	126	...	89
90	·2460	382	94	3·14	90
91	·2610	288	75	...	91
92	·2860	213	61	...	92
93	·2600	152	40	...	93
94	·3360	112	38	...	94
95	·2060	71	15	2·81	95
96	·2210	59	13	...	96
97	·2500	46	12	...	97
98	·6130	34	21	...	98
99	·4450	13	6	...	99
100	·5000	7	3	2·21	100
101	·5000	4	2	...	101
102	·0000	2	102
103	·0000	2	103
104	·0000	2	104
105	·0000	2	105
106	·0001	106

TABLE 5.

FRENCH ASSURED LIVES EXPERIENCE.

Comparison of Actual with Expected Deaths according to the Adjusted Mortality Table.

Ages	MALES			FEMALES			BOTH SEXES		
	Expected Deaths	Actual Deaths	Actual percentage of Expected	Expected Deaths	Actual Deaths	Actual percentage of Expected	Expected Deaths	Actual Deaths	Actual percentage of Expected
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
0-4	246.3	256	104	212.8	238	98	489.1	494	101
5-9	157.2	162	103	164.0	159	97	321.2	321	100
10-14	147.7	119	80	156.2	166	106	303.9	285	94
15-19	198.4	191	96	191.3	203	106	389.7	394	100
20-24	131.4	131	100	127.4	154	121	258.8	285	110
25-29	578.8	535	92	241.8	259	107	820.6	794	96
30-34	1,472.6	1,397	95	364.5	395	108	1,837.1	1,792	98
35-39	2,270.9	2,358	104	144.4	426	96	2,715.3	2,784	103
40-44	2,681.5	2,787	104	451.0	385	85	3,132.5	3,172	101
45-49	2,696.1	2,725	101	409.5	329	80	3,105.9	3,054	98
50-54	2,447.3	2,543	104	349.8	289	83	2,797.1	2,832	101
55-59	2,062.7	2,086	101	289.3	186	64	2,352.0	2,272	97
60-64	1,515.6	1,652	109	223.6	180	81	1,739.2	1,832	105
65-69	983.1	986	100	157.2	120	76	1,140.3	1,106	97
70-74	560.2	572	102	103.3	93	90	663.5	665	100
75-79	277.3	297	107	67.2	64	95	344.5	361	105
80-84	104.4	99	95	41.3	25	60	145.7	124	85
85-89	28.5	27	96	22.6	15	70	51.1	42	83
90-95	2.7	3		6.1	5		8.8	8	
All ages	18,563.0	18,926	102	4,053.3	3,691	91	22,616.3	22,617	100

TABLE 6.
FRENCH ANNUITANTS' EXPERIENCE.
*Comparison of Actual with Expected Deaths according to the
Adjusted Mortality Table.*

Ages	MALES			FEMALES			BOTH SEXES		
	Expected Deaths	Actual Deaths	Actual percentage of Expected	Expected Deaths	Actual Deaths	Actual percentage of Expected	Expected Deaths	Actual Deaths	Actual percentage of Expected
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
0-4	1.3	4	...	1.1	2.4	4	...
5-9	.7	1	103	.7	...	100	1.4	1	101
10-14	.85	2		1.3	2	
15-19	1.0	2		1.0	1		2.0	3	
20-24	1.4	1	260	1.8	1	175	3.2	2	205
25-29	1.9	3		3.1	6		5.0	9	
30-34	3.5	11		6.6	11		10.1	22	
35-39	6.7	7	155	15.2	10	118	21.9	17	131
40-44	17.2	31		38.8	54		56.0	85	
45-49	47.3	62		100.8	99		148.1	161	
50-54	156.7	220	140	300.9	276	92	457.6	496	108
55-59	437.3	514	118	788.6	614	78	1,225.9	1,128	92
60-69	1,130.3	1,305	116	1,884.7	1,508	80	3,015.0	2,813	93
65-69	2,183.6	2,388	109	3,445.3	2,889	84	5,628.9	5,277	94
70-74	3,205.4	3,674	115	4,743.3	4,303	91	7,948.7	7,977	100
75-79	3,399.9	3,773	111	4,989.0	4,704	94	8,388.9	8,477	101
80-84	2,530.7	2,829	112	3,746.2	3,726	99	6,276.9	6,555	104
85-89	1,195.8	1,245	104	1,911.1	1,743	91	3,106.9	2,988	96
90-94	339.1	294	87	580.9	488	84	920.0	782	85
95-99	60.8	41	70	108.2	67	62	169.0	108	65
100-106	4.7	5		7.0	4		11.7	9	
All ages	14,726.1	16,410	115	22,674.7	20,506	90	37,400.8	36,916	99

TABLE 7.
COMPARISON OF THE MORTALITY OF ANNUITANTS AND
ASSURED LIVES.

Ages	SUM OF THE UNADJUSTED VALUES OF q_x FOR AGES GIVEN IN COL. 1							
	Male Lives				Female Lives			
	Government Annuitants	HM	French Annuitants	French Assurants	Government Annuitants	HF	French Annuitants	French Assurants
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
25-29	.133	.035	.050	.030	.077	.058	.060	.035
30-34	.061	.041	.113	.034	.047	.059	.051	.040
35-39	.078	.047	.044	.045	.047	.060	.025	.042
40-44	.087	.054	.083	.056	.055	.065	.062	.046
45-49	.088	.068	.072	.070	.055	.069	.053	.055
50-54	.109	.089	.099	.097	.064	.079	.065	.076
55-59	.125	.121	.115	.132	.090	.100	.076	.084

TABLE 8.
COMPARISON OF ANNUITY-VALUES AT 3 PER-CENT INTEREST.
Assured Lives.

Age	American 30 Offices	HM	French Experience 1892	German 23 Offices	Age
(1)	(2)	(3)	(4)	(5)	(6)
15	23·088	23·153	22·954	...	15
20	22·290	22·013	22·046	21·222	20
25	21·360	21·038	21·053	20·329	25
30	20·283	19·867	19·836	19·198	30
35	19·049	18·587	18·470	17·896	35
40	17·652	17·176	16·957	16·452	40
45	16·096	15·594	15·311	14·883	45
50	14·396	13·896	13·556	13·172	50
55	12·582	12·094	11·731	11·394	55
60	10·702	10·236	9·889	9·622	60
65	8·820	8·418	8·091	7·897	65
70	7·009	6·657	6·402	6·256	70
75	5·343	5·061	4·880	4·796	75
80	3·881	3·742	3·570	3·520	80
85	2·646	2·739	2·493	2·549	85
90	1·560	1·740	1·651	1·516	90
95	0·800	0·415	1·023	0·599	95

Annuity-takers.

Age	Government Annuity-takers— Males, Select	Government Annuity-takers— Females, Select	French Experience 1892	Carlisle	Age
(1)	(2)	(3)	(4)	(5)	(6)
15	21·471	23·093	23·497	22·582	15
20	20·561	22·292	22·695	21·695	20
25	19·601	21·415	21·831	20·666	25
30	18·588	20·451	20·751	19·557	30
35	17·515	19·380	19·514	18·434	35
40	16·376	18·180	18·114	17·142	40
45	15·152	16·820	16·553	15·863	45
50	13·813	15·270	14·844	14·302	50
55	12·309	13·607	13·017	12·409	55
60	10·601	11·791	11·116	10·491	60
65	8·902	9·909	9·204	8·917	65
70	7·299	8·000	7·355	7·123	70
75	5·809	6·366	5·645	5·512	75
80	4·553	4·937	4·140	4·365	80
85	2·885	3·229	85
90	1·894	2·499	90
95	1·157	2·757	95

DISCUSSION.

The PRESIDENT (Mr. T. E. Young) said the original and exhaustive character of Mr. Hardy's writings was so fully recognized, that but few preliminary observations from him were requisite; for in whatever field of knowledge Mr. Hardy was engaged, he always gleaned so vigorously and vigilantly, that the work of his successors was largely confined to the very pleasing duty of simply attributing praise. He (the President) need not stay to point out the special opportuneness of the subject which Mr. Hardy had introduced in connection with the important, indeed almost national, investigation in which they themselves were engaged. He had now to call upon the referees to discuss the paper, in pursuance of the practice adopted, and to ask other members to favour the meeting with their views.

Mr. A. B. ADLARD said that at the beginning of the paper the author directed attention to the obligation under which the actuarial profession is placed to the Committee of French Actuaries for collecting and publishing so valuable an addition to the statistics of mortality among assured lives and annuitants; and he was sure that the expression of such an obligation was one in which they would all most heartily concur. At the same time, he was sure that all who were present would also concur in expressing their thanks and obligation to Mr. Hardy for the trouble he had taken in preparing his paper, in which the general results of the investigations of their French *confrères* were presented in so concise and interesting a form. As the paper partook greatly of the nature of a review of the volume which was published in Paris in 1895, he did not think there was very much to be said by way of criticism—that is, criticism of Mr. Hardy's paper—but at the same time there were many points of interest which arose in connection with the subject. Under the heading "Mode of Collection", Mr. Hardy was not quite accurate in his enumeration of the different classes of assurances as to which the particulars were collected for separate tabulation if thought desirable. They were as follows: (1) Assurances for the whole of life; and for these *white* cards were used. (2) Endowment assurances, called by the French actuaries "*Assurances mixtes*", because they belong to both of the two classes into which their transactions are primarily divided—i.e., "*Assurances en cas de décès*", and "*Assurances en cas de vie*", and for these endowment assurances *blue* cards were used. Then (3) there were what are called "*Assurances à terme fixe*", under which the sum assured is payable at a fixed date, whether the assured be dead or alive, while the payment of the premiums depends upon his continued existence; and for these *rose-coloured* cards were used. Then (4) there were ordinary temporary assurances, for which they used *grey* cards; and lastly, there were what the French call "*Les contre-assurances*", for which the colour of the cards was *vert d'eau*—a very pale green. These "*contre-assurances*" were not contingent survivorships, but were temporary increasing assurances, and he believed were usually effected in connection with an endowment, or a deferred annuity, so as to secure a return of the premiums paid in the event of death occurring before the benefit was entered upon. The

distinction of sex, as regards assured lives, was indicated by a double bar being placed across the cards relating to female lives. As regards annuitants, *violet* cards were used for male lives, and *dark green* for female lives. He was indebted to the courtesy of Mons. Léon Marie (the actuary of Le Phénix) for the specimen cards, which, he thought, might be of some interest to those present. On examining them, one could not but be struck with the variety of colours made use of, necessitating, one would almost think, a preliminary examination of the clerks to be employed, similar to that which was adopted in the case of signalmen on railways, to make quite sure that they were not afflicted with colour blindness. As regards the calculation of the number *exposed to risk*, he had very carefully followed the description of the course adopted in the volume itself (and beyond that they could not go, because the figures were not given); and he quite agreed with Mr. Hardy that the effect of the method appeared to be to over-estimate the numbers exposed to risk, by approximately one-eighth of the number of entrants, at each age. As regards the mode in which the tables were finally presented, it was much to be regretted, he thought, that, although the ungraduated results were given separately for each sex, the graduated tables, upon which all the monetary results were based, were only given for males and females combined. They knew it had always been the practice of French companies to charge the same rates of premium for assurances, and to grant the same rates of annuities, for males and females; and when these rates were based upon a combined experience such as those now under consideration, an assumption was practically involved that the relative numbers of the sexes would always coincide with those in the observations upon which the rates were based—an assumption which, of course, might or might not in the main be realized. In the case of annuities, if any considerable increase in the proportion of female lives should occur, the result must of course be very detrimental to the company granting the annuities. As to the method of graduation adopted, they all knew that Makeham's formula invariably produced a curve of absolute smoothness, whatever might be the irregularities, either characteristic or accidental, of the initial series, and on that account it was not suitable for universal adoption. With regard to the experience of the French *assured* lives, both the sexes combined, the table of comparison of *actual* with *expected* deaths given by Mr. Hardy, shows that by use of the adjusted values of q_x the actual deaths were practically reproduced throughout; and there could therefore be no doubt whatever as to the suitability of Mr. Makeham's formula for the table. As regards the *annuitants'* experience, where the characteristic irregularities of the original series were of a marked nature, as might readily be seen by reference to the table giving the sum of the unadjusted values of q_x in quinquennial groups, the method was clearly not so suitable, as admitted by the authors, and as shown in Mr. Hardy's table immediately above the one last referred to. Still, when they considered that a table of this nature was only likely to be used for annuity calculations and not for benefits of temporary duration, he thought that they would all concede that the agreement of the adjusted table with the original facts was quite close enough to admit of the adoption of the method,

particularly when the great advantages which it affords in the calculation of benefits involving two or more lives were taken into account. In connection with this point it was, he thought, a little surprising that, after having adopted Makeham's formula, and given the values of annuities upon 1, 2, 3, 4, &c., up to 9 lives of equal ages, the French actuaries should have gone to the trouble of calculating complete tables of the values of annuities on two lives, at no less than *five* rates of interest—a monument, certainly, to their industry and perseverance. It was, also, more than they would have expected to find, that complete tables, both of *temporary* and *deferred* annuities, at all the five rates of interest and on both tables of mortality, are given. When referring to the mode in which the French actuaries have dealt with the annuitants' mortality table at the younger ages, Mr. Hardy remarked that it was "not easy to justify the use of higher rates of mortality in a table upon which a company grants annuities, than the rates it assumes in granting assurances." That was a somewhat stronger statement than he (Mr. Adlard) would be prepared to follow, *so far as regarded lives under 40 or 45*, where, as it appeared to him, the medical examination of lives proposed for assurance would be far more effective in creating a select class than any so-called self-selection of the annuitants. Many of the annuities that were purchased at such ages would, he thought, be purchased by executors or trustees to facilitate the winding-up of estates, without any regard to the state of health of the nominees; and others would probably be purchased by persons upon whom the cares of life had pressed somewhat heavily in their earlier years, and from which they were desirous of freeing themselves; and such would be hardly likely to form a select class. At the older ages at which annuities were usually purchased, the case was, of course, quite different; but at the *younger ages, to which those remarks exclusively applied*, he himself did not see any objection to adopting Mr. Sprague's H³¹ Select Table as the basis for premiums for assurances, and at the same time the Government Annuitants' Male Select Table as the basis for annuities on male lives. He could not leave that division of the paper without remarking on the very great success of Mr. Hardy's first rough attempt (as he called it) to represent Mr. Sprague's Select Table by means of Makeham's formula. With regard to the results of the experience, they had had their attention drawn to the fact that among French *male* assured lives the mortality was heavier than that shown by the H³¹ Table; and this, he thought, might to some extent be accounted for by the circumstance that the French assurance companies have no rated-up lives. With them the practice had been either to accept a proposal in accordance with their ordinary tariff, or to reject it. So it might happen, one would think, on some occasions, that lives which were not exactly what would be called first-class, and which would, according to the practice in England, be accepted with an addition of perhaps 3, 5, or even 7 years to the age, might, in the case of the French companies, be accepted at the ordinary rates, as there might not appear to be sufficient reason to justify their rejection altogether. In considering the subject, it at first occurred to him that the comparatively high rate of mortality among the

male assured lives might, perhaps, have been partly attributable to the wars in which France had been engaged; but that idea was abandoned on finding that at the annual meeting of La Générale (one of the four companies) in July 1871, immediately after the Franco-German War, the manager said in his speech: "I do not mention the claims occasioned by the war, because they are not numerous enough to exercise a sensible influence on our accounts"; and it also appeared, from another speech made at the same meeting, that death in war, or from wounds received in war, was not covered by their guarantees; and that, although they had given a liberal interpretation to their policies in favour of those required to serve in the National Guard, even when they had volunteered, the adoption of that course entailed upon them no loss of importance. The wars, therefore, could not have had any material effect on the mortality experience; and he could not help thinking that the circumstance to which he had alluded, of the French having no rated-up lives, lay at the root of the comparatively high rate of mortality shown among the French male assured lives. With regard to the female assured lives, among whom the mortality was comparatively light, he had been trying to find out some solution, but he could not say that he had met with any definite success. At first he thought that the birth-rate being so much lower in France than in England—22·8 per thousand as compared with 31·2 on an average of 5 years—might have had something to do with it; but, on making a further comparison with the birth rates in other European countries, and finding that the ratios in some of those countries were also lower than in England, he felt obliged to give up that idea. Then, there was also the fact that the mortality amongst infants in France was considerably higher than in England; and he had thought that perhaps this might have some favourable influence upon the mortality in after life, on the principle of what had been called the survival of the fittest; but on finding that the mortality among infants under 5 years of age was almost as high in Germany as in France, and at the same time that the mortality among German female lives, as shown by the experience of the twenty-three German offices, was much heavier than in the case of the French female assured lives, he could not do otherwise than conclude that the rate of infantile mortality could not have much to do with the matter. The only possible solution which he could suggest was, that there might be some difference in the measure of severity with which the proposals for assurances on female lives were scrutinized by the respective assurance companies of France and England. As far as his own knowledge went, the medical examination of female lives in England had been *less* severe than in the case of male lives; and it was, of course, within the limits of possibility that there might be a difference in this respect in the practice of French companies. Should that be so, such a difference in practice might be quite sufficient to account for the comparatively light rate of mortality among female assured lives shown in the French tables. He joined very heartily with Mr. Hardy in expressing a hope that their French *confrères*, notwithstanding the fact that they had already received *un grand prix* at the International Exhibition, would not consider

that their labours were completely ended, but that, acting on the principle "*nil actum reputans dum quid superesset agendum*", they would, before long, publish the necessary materials for the construction of select tables, and thus (as Mr. Hardy had put it) place the actuarial profession under a still further obligation.

Mr. GEOFFREY MARKS echoed very heartily the President's statement as to the welcome always accorded to any paper by Mr. Hardy. He believed he was speaking for many present in saying that he viewed any contribution from that gentleman with particular pleasure, coming as it did not only from so distinguished a member of the profession, but from so kindly and able a teacher. They were, as the President said, accustomed to original work from Mr. Hardy, and anything coming from him was sure to bear very strongly the stamp of his own views. Mr. Adlard's comments had been so full that very little was left for other speakers. There were, however, one or two points as to the practical part of the paper to which he should like to refer. No doubt the information which their French colleagues had been at the pains to collect was very valuable, from the fact that it was practically the first thing of its kind in France. Of course, independently of their theoretical value, the investigations had a strong bearing on questions in which they were themselves interested as practical men. The large extent to which, of late years, business on the Continent had developed, must have very often forced on the attention of gentlemen responsible for the management of offices the enquiry whether or not the rates they were charging were adequate to meet the requirements of Continental risks. He believed it was the practice for most English offices to charge the same rates for Continental lives as for home lives. There was, no doubt, some additional risk; and the conclusions drawn by Mr. Hardy from the data collected by the French actuaries confirmed that view. Mr. Adlard had said that the war risk had very little influence on the result. He (Mr. Marks) was not quite sure whether that was so, or whether it could be permanently excluded from the view of English offices, although the Continental offices, on whose experience the existing tables were founded, did not, he thought, undertake war risks as part of the ordinary contract. He believed that that was so in Germany, and perhaps the President might be able to say something on that point. It seemed very clear that the English offices should, on Mr. Hardy's results, charge something in the nature of an extra premium to cover the difference in the vitality; and he thought they should also charge something to cover the war risk, because he did not think it was the practice of English offices to exclude that risk from their contracts, however the Continental offices might treat it. An addition to the premium, as Mr. Hardy had suggested, of 5 per-cent for French lives, and 10 per-cent for German lives, seemed desirable. What the rate should be on the borderland he did not know. The question as to the exposed-to-risk involved rather a curious point. In reading Mr. Hardy's paper, he was at first disposed to differ from him; but, on talking the matter over with the author, he was bound to say that he thought he was perfectly correct. Certainly the numbers exposed to risk were over-estimated. He could not quite see how it arose, but he thought it must be susceptible of some explanation, and

that it must be a slip, from the simple fact that no explanation was offered by the French actuaries in their preface to the tables. They stated it as if it were quite an ordinary thing, which would be obvious to the reader, but it was not so. Indeed, there seemed to be no reason whatever for it, and he was at a loss to understand it. One thing that had struck him was, why so much importance should appear to be attached to the annuity experience as compared with that derived from the life assurance risks. In all the French investigations, the investigation of the annuity experience was preceded by that of the assurance experience; a course which did not seem warranted by the comparative volume of business. Mr. Hardy's note as to the applicability of Makeham's formula to the graduation of the tables was very interesting. On setting out the unadjusted rates of mortality graphically, it was evident that the ages, say from 25 to 75, would be very suitably treated by Makeham's formula. It would have been better, and probably would have produced more satisfactory results, if, instead of a first adjustment by Woolhouse, the French actuaries had adopted some graphic adjustment, at least of the extremities of the tables, as a preliminary to the use of Makeham's formula. It seemed very clear that Makeham's formula did not apply to annuitant experience, either at home or abroad. He thought—it might be a heresy—that there was a strong probability that the very remarkable vitality which was experienced, both in the United Kingdom and abroad, among annuitants was to some extent due to fraud. It was, perhaps, not a right thing to say, but they knew in their experience that the means employed to identify the claimants of annuities were somewhat lax, especially in the offices which had a large business. It would be impossible to keep a proper watch on every claimant for an annuity. They did their best, but the best might be better. They knew that in the early part of the century, very considerable frauds were practised on the Government, and he thought it would pay offices if they looked a little more closely after the older annuitants when they made their claims.

Mr. T. G. ACKLAND said that Mr. Hardy had taken up a subject which was somewhat out of the ordinary course—and for that they were the more indebted to him—in examining the interesting volume which had been published by the four French companies. Those who were at all familiar with the practice of the French companies in the past, would be aware that the tables generally employed, until this recent publication was available—those of Deparcieux and Duvillard—were extremely unsatisfactory; and that the valuation methods followed approximated closely to those which were known in England as the hypothetical or re-assurance method, with certain modifications. The extent of the data was stated by Mr. Hardy, and he referred to it as “rather more extensive than that of the Institute.” Perhaps it might be well to modify that, by explaining that Mr. Hardy referred to the H³¹ Institute experience published in 1869. He thought they were already in a position to say that, in connection with the present investigation, their data would very largely exceed the figures of the earlier experience. Mr. Adlard had referred to the special classes of assurance granted by the French assurance companies. The *assurances à terme fixe* were of a special character. He supposed

Mr. Hardy did not include them in his list, because they were practically unknown to English companies, except to those transacting a Continental business. The *contre-assurance*, he had no doubt, as Mr. Adlard had stated, referred to what were sometimes spoken of among us as counter-assurances, against the return of premium in the case mainly of pure endowments, and deferred annuities and assurances. It was somewhat interesting to note that the female lives, of which so much had been said, were in the French experience about as 1 to 5 of the whole experience, while in the Institute Tables they were about as 1 to 9—that was, there were relatively about twice as many females in the French experience as in the Institute experience. Turning to the table of expectations of assured lives and annuitants in France, as compared with the Institute of Actuaries and Government annuity experience, they would note that the female vitality of assured lives in France was throughout superior to the Institute H^F table, and he was quite unable to suggest any explanation of that remarkable fact. They were prepared to see that the male assured experience showed a higher mortality in France; but that the females should throughout at every age show a superior vitality, was certainly a singular fact. In the case of male lives, it might be that the effects of war, to which Mr. Adlard referred, were not fully seen at the date to which he referred, July 1871; and that although actually death was not to any great extent caused by war, yet a material deterioration might have subsequently affected the mortality. It was also, he believed, a fact that although the French companies did not as a rule rate-up their lives, they did occasionally transfer proposals from the whole-term class to the endowment assurances class, when they were below the standard of health; and that might also have a material effect upon the table. The table given by the author of “3 per-cent net annual premium according to the H^M Table, the French assured lives, and the experience of 23 German offices”, would, he thought, have been of more value if it had been expressed in the form of select premiums. The premiums in the table being throughout based on aggregate tables, the comparisons could hardly be fully relied upon, although they were no doubt the best obtainable under the circumstances. It would also be very interesting if a corresponding table could be given for female lives, but he did not know whether the data would permit of that.

The paper divided itself into two separate parts; the one relating to the French mortality tables, and the other—which he thought of at least equal interest—to the special application of the method of graduation by the formula of Makeham. He thought they had not had before them for a long time so interesting a communication as that which Mr. Hardy made to them in deducing the select premiums on the basis of the H^M Table by the formula of Makeham, and he wished that the author, instead of devoting a portion only of his paper to this interesting subject, had more fully gone into this question for their benefit. So far as he could follow the formula which Mr. Hardy gave for the application of Makeham’s formula, he had adopted the second modification of Makeham; so that the expression within brackets, representing the mortality of the $H^{M(5)}$ Table, was deduced from that “further modification” which Makeham

published only a short time before his death. Apparently, the second portion of Mr. Hardy's expression was based upon the earlier of Makeham's formulas, stated in the familiar form which represented the force of mortality as made up of a constant and an expression increasing geometrically; the effect of selection being ingeniously arrived at by the adoption of arbitrary fractions, the numerators of which represented the series of figurate numbers of the fourth order. He should like to ask Mr. Hardy whether it was not a fact, assuming that he had adopted the second modification of Makeham, that the application to joint lives was somewhat more cumbersome than if the simpler first modification had been adopted throughout. He believed that Makeham had shown, in his paper of 1890 above referred to, that they were able in this case still to avail themselves of the law of uniform seniority, but under somewhat modified circumstances, as to the rate of interest involved. Turning now to table 5 appended to the paper, it seemed to be a remarkable fact that there should be the excess of the actual deaths over the expected in the case of female lives between ages 10 and 35; and this did not appear to be altogether consistent with what one would expect from the general comparisons of the rates of mortality; but no doubt the facts were as stated. The table 7, "Comparison of the mortality prevailing among annuitants and assured lives", showed on the unadjusted facts, when summed in groups of five, a very irregular progression; and it looked as if the original data were unusually irregular, or else that at some ages there were hardly sufficient data to produce a regular or accurate curve. Finally, in one small point of criticism, he would venture to ask whether the word "assurant" was a word fully admitted into the English language, and if Mr. Hardy was responsible for it, or whether it had received a prior sponsor?*

Mr. A. H. BAILEY thought the word "assurant" was a convenient word, and a very good suggestion, as obviating the difficulty now found in discriminating between the *assured* and the *assurer*, terms which are now frequently confounded.

Mr. H. W. MAXLY said they were very much indebted to Mr. Hardy for the paper which he had read. Those members who were, like himself, unfamiliar with the French language, could now read, mark, learn and inwardly digest the important results which the French actuaries had produced in their great work, without the trouble of translating it. He hardly liked to raise the question of the addition of the one quarter of the entrants in the last half of the year to the numbers at risk during the first year of observation, after others, better qualified than himself, had said they were quite certain it was wrong; but he thought there might be a possible explanation found in the want of more complete information as to how the data was manipulated. For instance, it was important to know what was done with the deaths that occurred in the latter half of the year amongst those who entered during that period; and also how the withdrawals were treated in the first year of exposure. If a large number of those who entered in the latter half of the year did not renew their policies, and the withdrawals were treated as being under

* The word "assurant" was often used by the late Dr. Farr.—ED. J.I.A.

observation for half a year only, and if the deaths which occurred among the new entrants in the latter half of the year were brought into account, then they would produce a rate of mortality which was too high. It might be, that to correct this, a rough adjustment, by adding to the exposed to risk in the first year a quarter of the numbers brought forward, was made. He only threw out that as a possible explanation of what otherwise seemed to be an unintelligible proceeding. With regard to graduation, he thought Mr. Hardy had thrown out a hint which might be extremely useful in future graduations. A good graduation was at no time a very easy thing to accomplish. It required a great deal of labour, of time, and of thought, if it was to be done well. He felt sure they might be able to get an excellent graduation, if Makeham's formula were applied to a table already well graduated by the graphic method. He did not think there was any mode of graduation worth considering, but the graphic, and Makeham's method. He quite agreed with the author, that the ease with which, by Makeham's formula, the functions which they constantly required in practice could be obtained, far outweighed any slight deviation from the original facts exhibited by the application of Makeham's method; and that there need be no hesitation in employing that formula, provided the values of the annuities by the adjusted table were practically identical with those obtained by other methods of adjustment involving no assumption as to the law of mortality. He had himself recently succeeded in making a graphic graduation which would very closely comply with the theory of errors; and if any of the young actuaries would like to try their hand at making a graduation by Makeham's method from those figures, he should be very pleased indeed to give them the opportunity. While on the question of graduation, he would like to point out what their friend, Mr. Bailey, had so often insisted upon—that there was a striking increase in the rate of mortality between the ages of 20 and 25. In the unadjusted mortality table—males—it would be seen that before the age of 20 and after the age of 22 the rates of mortality ran fairly well, but curiously enough, at the age of 20 there was a very considerable increase; and amongst the female lives it was even more marked where there was a very large increase from age 21 to age 24. A graphic curve drawn to represent the mortality prevailing amongst both those classes would show what might be called a "hump" at those ages, and this was true of almost every table constructed from actual observation.

Mr. F. T. M. BYERS asked whether the assured lives in these experiences were mainly French lives? It occurred to him that it was possible that the French companies had got a large number of lives from Switzerland, particularly from French Switzerland, because the Swiss were a provident and an assuring community. The mortality amongst the Swiss was said to be high, and he believed it was attributed to phthisis and pulmonary diseases in the valleys of Switzerland, those parts being very densely populated. If the experience of the French companies included a large number of Swiss lives, it was possible the rate of mortality among the French lives assured might be more favourable than that which appeared in the table Mr. Hardy had brought under their notice.

The PRESIDENT, in conveying to Mr. Hardy their cordial acknowledgments of his very valuable and thoughtful paper, said he could not avoid expressing the gratification they naturally felt that the investigations of their French colleagues had been prompted by the example set by the Institute in past years; and the gratification was enhanced by the fact that the two methods of graduation which they had adopted were those which had been devised by two of their most distinguished mathematicians, formerly connected with the Institute, Mr. Woolhouse and Mr. Makeham, the former in connection with the preliminary observations, and the latter with those results in their revised and completed form. Having had an opportunity of perusing the introduction to the French volume, he was very forcibly reminded of the crusade which was started by Sir David Brewster a considerable number of years ago, with the object of securing what was termed "the endowment of research." It was very interesting to observe that when those preliminary results were submitted at the great Exhibition in Paris in 1859, the jury not merely awarded a Grand Prix to the work, but presented, in addition, a Gold Medal apiece to the four actuaries concerned in its preparation. He thought it was obvious from this fact, that the opinion expressed in the immortal work of "Tristram Shandy", "that they managed these things better in France", still held good. He should like to add that Mr. Finlaison, whose judgment on that subject was extremely impressive, had written to him as follows: "I think the general confirmation of our standard English Tables is remarkable, particularly as to the relative rates of mortality of the two sexes. What induced the French actuaries to adopt one annuity rate for males and females after the superiority of female life was proved by their own observations, and confirmed by what has been long known here, it is difficult to understand. I should have thought that the preponderance of the number of female annuitants might have convinced them of the risk of using an amalgamated table."

He (the President) thought it might not be unfitting on his part to take this opportunity of calling to their remembrance the fact that, within the last few days, the most distinguished member of the Institute as a pure mathematician had passed away, in the person of Professor Sylvester. Mr. Sylvester's connection with the profession lay in the fact that he was the first actuary of the Equity and Law, from 1844 to 1855; and they had the pleasure that evening of seeing their friend Mr. Bailey, who was his immediate successor, and Mr. BurrIDGE, who became his successor at a remoter period. Professor Sylvester's researches lay quite apart from the routine of their work, and proved of so recondite a character that, as in the case of all original minds, many years must necessarily elapse before the value and meaning of his results could be appreciated and applied; but with regard to his native force and power of intellect, they might very fairly say, if they adopted Professor Tyndall's method of graphic comparison, that Professor Sylvester, in the region of pure mathematics, ranked at no very considerable distance from the master mind of Newton himself.

He congratulated Mr. Hardy, in begging him to respond, that, owing to the exhaustiveness and insight displayed in the paper, he

occupied the happy position of having mainly to reply to approbation and praise.

MR. HARDY, in replying, thanked the speakers and the members generally for the appreciative way in which they had received his somewhat short and perhaps rather sketchy paper. Mr. Adlard, he said, threw a considerable amount of light on the subject, and he was much obliged to him for pointing out that he had not accurately described the classes of assurance into which the experience had been divided. With respect to the question of the relative mortality of annuitants and assured lives, Mr. Adlard thought that there was sufficient reason in the observed higher rates of annuitants to retain those rates in constructing monetary tables. But they had to meet the fact that frequently now-a-days they were asked to grant a double contract on the same life. He had known several cases of somewhat large magnitude, in which offices had been asked to grant an annuity and an assurance at the same time, on the same life, and it was quite clear, in a case of that kind, it would not only be an anomaly to use an annuity table which gave a higher rate of mortality than the assurance table, but it would also involve the office in certain loss. The extra rate of mortality observed amongst the French lives had been commented on by various speakers, and various suggestions had been made as to how it could be accounted for. He did not attempt in his paper any explanation, because none of those explanations appeared to him to quite meet the case. War mortality, for example, scarcely seemed to him an adequate explanation, as the extra mortality was observable not specially at the younger ages, but in a more marked degree as they reached the older periods of life. Then as to the suggestion that the extra mortality is due to the fact that rated-up lives were not taken by the French companies, that at first sight seemed to be a satisfactory, or at least a partial, explanation; but exactly the same thing took place in America—the offices there did not take rated-up lives, and the American Offices' Experience was very much better than their own. Mr. Marks had referred to the effect upon the recorded annuitant mortality of possible frauds. Of course, it was difficult to say to what extent fraud might affect the recorded deaths in the case of annuitants, but he should not think himself that it would be to any serious extent. He should be inclined to say that offices were more liable to be content with insufficient proof of age in the case of annuitants, especially the older annuitants, than to be lax in respect of proof of existence. The older the life the more difficult it was to produce absolutely satisfactory proof of age, and the consequence was that they were often asked to accept as proof of age evidence which was not absolutely satisfactory and conclusive; and if, in those cases, they got certain over-statements of age, then, of course, their experience would suffer. He was glad to find that Mr. Bailey approved of the word "assurant." It was not a word of his own invention, although he was not prepared to say at the moment where he found it. Mr. Ackland had referred to the comparison of the premiums, and had rightly said that they could not properly compare those premiums unless they had select tables to compare them by. That was strictly true. He thought they might,

however, make a fairly approximate comparison, and when they bore in mind that the average duration of the assured lives on which the French Table was based was about 8 years, against their $9\frac{1}{4}$ in the H^M Table, they would see that the effect of selection there would probably be greater proportionately than in their own tables, and therefore the French experience would have the benefit of that larger proportionate number of selected lives. If they had select tables, instead of the aggregate tables they had to be content with at present, they would no doubt find that there was a still greater difference between the French experience and their own. Referring to the question of the graduation by Makeham's formula of select tables, the example given was, as Mr. Ackland pointed out, according to Makeham's second modification of Gompertz, because he wished to bring the tables as near to the original H^{M(5)} as possible, although he did not attach much importance to the twists in that table at young ages. Of course it would be quite possible, by carefully selecting constants, to get a fairly correct agreement between the adjusted and unadjusted table without using his second modification. Mr. Byers had made some interesting remarks as to the mortality of Swiss lives, which, he said, were subject to a much higher rate than the average, and that fact might possibly to some extent account for the higher mortality amongst the French offices. But he did not think it would go far in that direction, because they could scarcely suppose that the Swiss lives would form a very large percentage of the whole lives assured by these four French companies. He was very glad to find that both his referees were in accord with him as to the procedure of the French actuaries in calculating the "exposed to risk." Mr. Manly did not appear to be quite convinced on the point. He thought, however, that a reference to the specimen schedule and the description of the process given in the work would bear out the opinion he had expressed in the paper. He was glad to say, owing to the suggestion kindly made by Mr. Young, there were several copies of the French experience now in the library, so that the members of the Institute would have an opportunity of going to the fountain head, and of forming their own judgment, not only on the point under discussion, but on all other points connected with it.

On the Mortality of Assured Lives in West Africa.

THE question of the rate of mortality experienced by residents on the West Coast of Africa, has recently attracted considerable attention among actuaries. And indeed it is one of increasing importance. Partly owing to the growth of the Liverpool trade with the West Coast, partly owing to the spread of European ideas (including that of life assurance) among the natives, the number of proposals to life offices from residents in Africa is each

year becoming greater. It is most desirable, therefore, that some attempt should be made as soon as possible to measure the extra risk of residence on the West Coast, and it is with the object of assisting such an attempt that the present note is offered to the readers of the *Journal*.

It is clear that for the purpose in view, the contributions on the subject in the July 1897 number are not altogether suitable. For Mr. A. E. Sprague's and Mr. Hart's notes do not deal with *assured* lives, while Mr. Pagden's letter, though not open to this objection, deals with an experience so small, and of so special a character, that its results are scarcely reliable. Yet these results are so startling that it appears desirable to test whether they are or are not confirmed by the experience of other offices which have accepted West African risks, and more especially those whose experience has extended over a considerable period. Two such offices, the "Star" and the "London and Lancashire", were approached on the subject, and very kindly furnished the requisite data for the proposed investigation. We take this opportunity to express to the managers of these offices, our appreciation of their public spirit in the matter. The work was entrusted to Mr. A. Levine and Mr. C. T. Weeden, and it is hoped that the following account will not prove uninteresting.

563 cards in all were contributed, but after eliminating duplicates, and a few entrants subsequent to 31 December 1896, there remained 523 lives under observation. The bulk of the lives were traders or professional men, in a good position, resident in Agra on the West Coast, or Freetown, Sierra Leone. The experience extends over a period of 40 years. The total years of life observed were 4,712, so that the average duration of the exposure was almost exactly nine years. There were altogether 119 deaths and 260 withdrawals, while 144 lives were existing at the close of the observations. Among the withdrawals were a few lives who had permanently returned to Europe; the date of withdrawal was taken as the date of such permanent return. The subsequent experience of these lives, to which reference will be made later on, was in the first instance disregarded. A preliminary examination of the cards seemed to show that the experience was almost equally made up of European and of native lives, and it was found, in fact, that of the 523 lives, 263 were Europeans, and 260 West African natives. As the latter were not affected by the phenomenon known as acclimatization, it

appeared undesirable to have a mixed experience, and it was decided, therefore, to treat the two classes separately.

Taking first the European lives, the 263 entrants were distributed as follows :

Ages	20-25	26-30	31-35	36-40	41-45	46-66
Entrants	28	71	70	43	25	26

The total years of life observed were 1,957, and there were 54 deaths, 133 withdrawals, and 76 existing. The average annual rate of mortality was therefore only 2.76 per-cent as compared with 4.7 found by Mr. Sprague, and 5.2 by Mr. Hart. The nature of the experience in each of the age groups is exhibited in the following tables :

AGES AT ENTRY 20-25		
Year of Duration	Exposed to Risk	Deaths
1	26	1
2	20	0
3	16	0
4	15	1
5	12	0
6-29	122	3
Total	<u>211</u>	<u>5</u>

AGES AT ENTRY 26-30		
Year of Duration	Exposed to Risk	Deaths
1	68	0
2	52	2
3	43	1
4	38	0
5	36	0
6-33	325	8
Total	<u>562</u>	<u>11</u>

AGES AT ENTRY 31-35		
Year of Duration	Exposed to Risk	Deaths
1	66	2
2	49	2
3	42	0
4	36	2
5	33	1
6-33	274	8
Total	<u>500</u>	<u>15</u>

AGES AT ENTRY 36-40		
Year of Duration	Exposed to Risk	Deaths
1	43	2
2	31	0
3	25	0
4	20	0
5	18	0
6-37	213	5
Total	<u>350</u>	<u>7</u>

AGES AT ENTRY 41-45		
Year of Duration	Exposed to Risk	Deaths
1	25	1
2	20	0
3	16	0
4	15	0
5	13	3
6-28	93	4
Total	<u>182</u>	<u>8</u>

AGES AT ENTRY 46-66		
Year of Duration	Exposed to Risk	Deaths
1	22	1
2	18	0
3	15	0
4	15	2
5	10	0
6-22	72	5
Total	<u>152</u>	<u>8</u>

Or, taking all the ages together, and again ages 26-40 together, we have

ALL AGES AT ENTRY			AGES AT ENTRY 26-40		
Year of Duration	Exposed to Risk	Deaths	Year of Duration	Exposed to Risk	Deaths
1	250	7	1	177	4
2	190	1	2	132	4
3	157	1	3	110	1
4	139	5	4	94	2
5	122	4	5	87	1
6-37	1,099	33	6-37	812	21
Total	<u>1,957</u>	<u>54</u>	Total	<u>1,412</u>	<u>33</u>
Average annual rate of mortality, 2.76 per-cent.			Average annual rate of mortality, 2.34 per-cent.		

The data in the separate groups are too few to yield any reliable rates of mortality, but this objection has not the same force when applied to the table for ages 26-40 combined. These are the so-called "assuring ages," and they include the bulk of the entrants. And although caution must be exercised in drawing any inference from an experience of only 184 entrants, yet it may be urged that these figures show at least that Mr. Sprague's and Mr. Hart's rates are probably too high, and that Mr. Pagden's are quite exceptional. Having regard, too, to the fact that the present experience covers a period of nearly 40 years, and that in recent years many improvements in sanitation, &c., have been introduced on the West Coast, it seems likely that future results will be even more favourable.

No attempt has been made to trace the effects of acclimatization, and in fact none was possible, as the materials available did not show the condition as regards acclimatization of the lives at the date of assurance. A considerable number of the lives had, we understand, then already become acclimatized. By grouping the years of duration in various ways, different apparent laws of mortality can be obtained, but none of these can be regarded as satisfactory in view of the limited character of the experience. The table for ages 26-40 given above may be taken, however, as at least an indication that the mortality is probably heaviest in the first two years of exposure; so that acclimatization would begin to operate in the third year.

A table is added giving the experience according to ages attained; the figures are not important in themselves, but they may be useful in facilitating a comparison with the H^M Table.

The data for the first five years, and for the subsequent experience are given separately.

Age attained x	First 5 Years		After 5 Years		Age attained x'	First 5 Years		After 5 Years	
	E_x	d_x	E_x	d_x		E_x	d_x	E_x	d_x
20	1	50	7	1	36	1
21	2	51	8	1	34	4
22	6	52	8	...	28	1
23	15	53	9	...	26	2
24	16	2	54	9	...	21	...
25	17	55	8	...	19	...
26	28	...	1	...	56	5	1	20	...
27	34	...	5	1	57	3	...	20	...
28	40	...	8	...	58	1	1	19	...
29	42	...	7	...	59	1	...	17	1
30	46	2	9	1	60	0	...	16	1
31	44	1	16	...	61	0	...	15	...
32	46	...	21	...	62	0	...	14	1
33	45	...	24	...	63	0	...	11	...
34	49	1	25	...	64	0	...	10	...
35	44	...	35	...	65	0	...	9	1
36	40	3	40	4	66	1	...	7	...
37	38	2	35	...	67	1	...	6	1
38	43	2	37	1	68	1	...	4	...
39	37	...	43	2	69	1	...	4	1
40	27	1	44	1	70	1	...	3	...
41	15	...	49	...	71	0	...	4	2
42	19	...	46	2	72	2	...
43	21	...	47	...	73	2	...
44	15	1	48	...	74	2	1
45	16	...	47	2	75	1	...
46	13	...	43	...	76	1	...
47	15	...	41	...	77
48	11	1	40	...					
49	9	1	37	2	Totals	858	21	1,099	33

Taking next the native lives, there were 260 entrants distributed thus:

Ages	16-25	26-30	31-35	36-40	41-45	46-61
Entrants	21	53	58	61	36	31

The total years of life observed were 2,755; and there were 65 deaths, 127 withdrawals, and 68 existing. The average annual rate of mortality was thus 2.36 per-cent, as compared with 2.76 per-cent for the Europeans. It will be noticed that the average duration of exposure is greater, which is accounted for partly, of course, by the fact that we have here no cases of "permanent return to Europe", and consequent neglect of a portion of the experience. For purposes of comparison we give

the following tables, similar in form to those given above for the European lives:

AGES AT ENTRY 16-25			AGES AT ENTRY 26-30		
Year of Duration	Exposed to Risk	Deaths	Year of Duration	Exposed to Risk	Deaths
1	21	0	1	52	1
2	15	0	2	41	0
3	14	0	3	37	1
4	12	1	4	32	0
5	10	0	5	28	1
6-36	132	5	6-38	418	9
Total	<u>204</u>	<u>6</u>	Total	<u>608</u>	<u>12</u>

AGES AT ENTRY 31-35			AGES AT ENTRY 36-40		
Year of Duration	Exposed to Risk	Deaths	Year of Duration	Exposed to Risk	Deaths
1	57	1	1	58	0
2	49	0	2	50	1
3	45	1	3	46	0
4	41	0	4	42	1
5	35	1	5	36	1
6-39	417	8	6-40	449	10
Total	<u>644</u>	<u>11</u>	Total	<u>681</u>	<u>13</u>

AGES AT ENTRY 41-45			AGES AT ENTRY 46-51		
Year of Duration	Exposed to Risk	Deaths	Year of Duration	Exposed to Risk	Deaths
1	35	0	1	31	2
2	33	0	2	29	2
3	28	0	3	25	3
4	28	1	4	20	0
5	23	0	5	18	0
6-27	168	9	6-42	180	6
Total	<u>315</u>	<u>10</u>	Total	<u>303</u>	<u>13</u>

ALL AGES AT ENTRY			AGES AT ENTRY 26-40		
Year of Duration	Exposed to Risk	Deaths	Year of Duration	Exposed to Risk	Deaths
1	254	4	1	167	2
2	217	3	2	140	1
3	195	5	3	128	2
4	175	3	4	115	1
5	150	3	5	99	3
6-42	1,764	47	6-40	1,284	27
Total	<u>2,755</u>	<u>65</u>	Total	<u>1,933</u>	<u>36</u>
Average annual rate of mortality, 2.36 per-cent.			Average annual rate of mortality, 1.86 per-cent.		

But in dealing with native lives, the element of acclimatization does not enter at all. These tables, therefore, which proceed by the duration, are useful only to trace the effects of selection. In

a small experience like the present, however, it will probably be more useful to find the rates of mortality for *ages attained*, regardless of duration, and to compare the results with the H^M Table. The following table shows the experience on this basis:

Age attained = e	E_e	d_e	Age attained = x	E_x	d_x	Age attained = j	E_j	d_j
16	1	...	41	103	3	66	22	2
17	1	...	42	105	3	67	20	3
18	1	...	43	102	1	68	16	1
19	0	...	44	101	...	69	15	...
20	0	...	45	100	...	70	11	2
21	1	...	46	90	2	71	9	2
22	3	...	47	81	1	72	5	...
23	8	...	48	87	2	73	4	...
24	11	...	49	85	...	74	4	...
25	17	...	50	85	3	75	4	...
26	20	...	51	80	5	76	4	1
27	29	1	52	74	4	77	2	...
28	39	1	53	68	2	78	2	...
29	41	...	54	62	1	79	1	...
30	48	...	55	62	2	80	1	...
31	58	1	56	53	2	81	1	...
32	61	2	57	46	1	82	1	...
33	67	...	58	39	3	83	1	...
34	69	2	59	35	3	84	1	...
35	73	1	60	31	...	85	1	...
36	80	1	61	33	2	86	1	...
37	88	1	62	29	...	87	1	...
38	88	...	63	27	...			
39	96	1	64	24	2			
40	101	1	65	22	...	Total	2,755	65

Or, grouping the ages, we have

Ages	Exposed to Risk	Actual Deaths	Expected Deaths according to H^M Table	Excess
16-30	220	2	1.6	0.4
31-40	781	10	7.3	2.7
41-45	511	7	5.7	1.3
46-50	431	8	6.2	1.8
51-55	346	14	6.4	7.6
56-60	204	11	5.2	5.8
61-87	262	13	11.0	-1.0
	2,755	65	46.4	18.6

It is worthy of notice that not till ages 51 and upwards does the rate of mortality appear very much in excess of the H^M rate. Moreover, the progression appears so regular that it may be

regarded as a tolerably accurate exhibition of the facts. And although, as already mentioned, the experience is too small to justify any *absolute* conclusions, yet it seems to suggest that for the younger assuring ages the annual extra mortality in the case of West African Native Lives is not more than 1 per-cent.

It may be useful in conclusion to give briefly the facts relating to the European Lives who passed out of the experience by their permanent return to Europe. The number of these was 18, and their average age at the date of such return was 43·5. Their subsequent experience shows 5 deaths, 5 withdrawals, and 8 existing. The years of life were 129, so that the average rate of mortality was 3·87 per-cent, considerably greater, as might be expected, than the average rate for the whole experience.

A. L.

Are Events resulting from the exercise of the Human Will suitable for the Deduction of Serviceable Probabilities?

The following interesting and instructive correspondence on an important subject has been placed at our disposal by the writers, and we need not make any apology in submitting it to the readers of the Journal.—ED. *J.I.A.*

I.

Letter, dated 10 March 1897, from M. HENRI ADAN, of Brussels, President of the Association of Belgian Actuaries, and Corresponding Member of the Institute of Actuaries, to MR. T. E. YOUNG, President of the Institute of Actuaries.

I SHOULD be most obliged if you would be kind enough to give me your opinion on the following point.

You know the Austrian Scheme for compensation for accidents of Labour and the allowances it provides, particularly in case of death of the Workman.

Such a Scheme necessitates Family Tables showing the probability of marriage, the probability of bachelorhood, the probability of issue, the probability of the existence of children under 15 years of age, the probability that the deceased is the only support of ascendants, the probability of poverty of the latter.

If, as I think, Family Tables are supposed to show laws of probabilities, I must confess that, although admiring the ingenious calculations made by Actuaries on the subject, I cannot bring myself to recognize their practical value, because

the facts considered belong to man's *free Will*. (While the statistical element of other facts, such as pauperism, the supporting of pauper ascendants, are not well known).

If facts proceeding from man's free Will can be made the subject of calculations which will show their course in the past, I think it would be somewhat rash to draw deductions therefrom with reference to the future, and it would be dangerous to apply such deductions to financial schemes.

If there is no indiscretion, I should be very glad if you would give me summarily your views on the subject.

P.S.—I know the papers on the subject published in the *Journal of the Institute of Actuaries*, such as on Family Annuities, by Mr. G. King, and others by Mr. Sprague, but I do not find in these whether, as a principle, facts deriving from man's free Will may be submitted to probability calculations.

II.

Memorandum, dated 15 March 1897, from MR. YOUNG to M. ADAM.

If I do not misapprehend the purport of your letter of the 10th inst., I understand the question to be whether the Calculus of Probabilities can be validly applied to facts dependent on the exercise of the Human Will, and particularly, whether the deduced numerical results can form a safe foundation for financial schemes dealing with the future.

The question is undoubtedly a difficult one, and necessarily involves the metaphysics of the nature of the Human Will.

In the facts of Mortality we are concerned with an event which is *certain*, and only require to determine its rate and mode of incidence from age to age, which again is almost absolutely, though not entirely, independent of the Will. One probability, therefore, alone is involved. In the facts of marriage, on the other hand, we are concerned with a double probability—the probability of marriage itself, and the probability of its occurrence at different ages—both being dependent on the exercise of the individual Will as influenced by social conditions, personal elements, and material circumstances.

The problem, as I have said, involves

- (i) the psychological nature of the Will,
- (ii) the visible results of the exercise of Will as evidencing its nature and conditions.

I venture, therefore, with much diffidence, to hazard a few remarks.

I do not pretend in any degree to have fathomed the subject, though I have long meditated upon it; but I confess I can form no valid conception of the Will except under the form of a Motive Theory. In other words, I can in no intelligible way regard the Will as an Arbitrary Force, acting in irregular or capricious modes, but rather as a Force which is influenced by Motives or Springs of Action, and which follows the Motive or Influence which happens at the moment to be the strongest or most attractive; and, further, that that Motive is shown to be the strongest in whose direction the Will operates. It seems to me that even apparently exceptional cases confirm this view. A man is surrounded, we may observe, by conflicting appeals to different courses of action among which the Will must choose; he fails to respond to any of them, and, hence, appears to possess an arbitrary power; that is to say, the Motive Theory seems to fail. But, from much observation and reflection, I think that the Theory still furnishes the explanation. For, at the moment of choice, a desire, *e.g.*, to act differently from other people, and thus to show his superiority, may affect his mind; or a feeling of obstinacy may prevail; or some higher, but not evident, emotion or influence may enter, and thus the man may select a course quite distinct from the several courses which we notice as presented to him. But the feelings I have named—superiority and obstinacy—are also *Motives*; they are appeals to the action of the Will in certain particular modes: and, hence, a Motive Theory of Will, or a Will that is governed by Motives, appears to be a valid truth.

Hence, as the Will is not arbitrary, but is controlled by Motives so that a sequence universally exists between Motive and Act, we may regard its manifestations as the subject of intelligent study and prevision. If the Will were arbitrary, no science of Will, or calculations expressing its results, would be possible; if the Will be always influenced by definite Motives, a science of probabilities, that is, of future forecasting, is feasible. In other words, Arbitrariness excludes *uniformity* of action, and, therefore, of measurement or probability; but the conception of a Will which is impelled by the strongest Motives involves *uniformity* of action *in the long run*, and is, therefore, properly the subject of calculations for future guidance. The proposition seems correct that, given hereafter the occurrence of the same Motives and

circumstances (internal and external) which impelled a man to a particular action in the past, that same action will result again. (It is, of course, involved, that the individual *Character*—the consolidated result of heredity, and past personal exercises of Will—is a prime factor in the case, since that character will be more congenial to one class of Motives than to another, and, hence, the Will will generally follow the congruent class.)

I turn now to the second element which is necessary if a basis of calculation and prevision for the future is to be formed, viz.: the similarity of the nature of the Will as exhibited in different persons and races. If different Wills were differently constituted, so that the conception of one Will would require to be materially modified before it could be applied to another, no probabilities would be possible, for no uniformly average result could be obtained. I need not, however, dwell upon this point, for it will be conceded that human Nature is framed on the same essential lines. Hence, I conclude from (i) the nature of the Will as controlled by definite Motives and, therefore, capable of showing *uniformities* of action, and (ii) the generic similarity of all Wills, that a science of probabilities is feasible in relation to events which depend upon the operation of the Will.

Another consideration lies in this: some Motives appeal to one Will more vigorously and decisively than to another, and so generally: and in a combination of results of Will, that is, in average results deduced from the exercise of many Wills, we gain a fair and representative exhibition of the influence of *all* possible Motives.

Human acts then, or the results of Will, being examined, *e.g.*, in the case of Marriage—we possess, I conceive, a reasonable and (if the range of observation be adequate) a sound basis for mathematical probabilities, or uniformities for future application. And the *practical* value (which is the essential requirement) of such researches and calculations for future use will depend upon (i) the adequate extent of our observations, and (ii) the appropriateness and similarity of the class of facts we have examined, and from which we have obtained our calculations, with the class of facts to which we desire to apply those calculations.

Speaking now of many practical deductions and applications of probabilities of this character,—especially where the schemes considered are of wide extent and involve many difficult elements,—I am distinctly of opinion that to a large degree they are mere guess-work, for they generally fail in one, or more, or all of the following characteristics:

- (i) the results from which the probabilities are deduced may be too obscure, or imperfect, or irrelevant, or insufficient.
- (ii) the calculations may be applied to cases which are not sufficiently similar in nature and circumstances to the cases from which the calculations have been obtained.
- (iii) the application of the probabilities may not take account of the fact that the proposed scheme (based on the calculated probabilities) may so far affect the cases of the future as to render them to an extent inappropriate for such application.
- (iv) the calculations may be largely and specially determined by past conditions, personal and social; and it is often not adequately kept in view that a modification of those conditions in the future will also affect the probabilities employed, and affect, therefore, their suitability of application.

NOTE.—In the preceding remarks I have dealt mainly with *a priori* considerations, which indicate the reasonableness of anticipating that the results of Will may be regarded, under the circumstances I have mentioned, as proper data for calculation and expectation; the subject will be completed by taking into account the uniformities disclosed by the results of observation on the exercise of a Combination of Wills.

III.

Letter, dated 19 March 1897, from M. ADAM to MR. YOUNG.

I think you have understood perfectly well my question which indeed is: Are facts resulting from the exercise of Man's Free Will fit to be made the subject of statistical researches which would allow to deduce a law showing their probability of occurrence?

As you rightly observe, the question involves most delicate psychological elements, giving it a philosophical character; the varying nature of which seems to me scarcely fitting with the homogeneous, precise, elements wanted to form a safe foundation for probability calculation.

Now, in the case I am particularly aiming at, *i.e.*, in the case of the Austrian Law, and with reference to marriage as an action dependent on the exercise of the Human Will, the motives influencing the labourer's Will—social conditions—personal elements (character)—material circumstances—seem to me to

possess such power that the practical value of the calculations becomes delusive.

Indeed, it must be considered that one of the main generating elements of compensation is the wages paid, and that these are subject to fluctuations of every kind, resulting from the most various and unsteady causes.

Therefore, I am rather induced to consider as very doubtful the value of the Family Tables which have been admitted in Austria, and I feel inclined to lay, to a great extent, at the door of the confidence put in this instrument, the bad working of the scheme.

I understand that, from observations, a Mortality Table of the labouring classes according to their occupation may be deduced, but I cannot bring myself to admit that one may calculate and ascertain :

- (1) the probability that the labourer will marry or not.
- (2) that he will or not leave a widow, if the ages of man and wife are not previously known.
- (3) that he will or not leave children under fifteen years of age.
- (4) that he will or not leave ascendants in want.
- (5) that he will or not leave a widow who will marry again or not.

The value of appreciations given on that subject seems to me to belong to what you call mere guess-work, and would appear very disquieting to me as a foundation of a social scheme.

You will perceive that I am particularizing the question to which you give a more general, higher, and more philosophical scope.

External conditions will indeed mostly influence the Will, but still I do not think that one may deny it a certain independence, either as re-action against those conditions, or as pure spontaneousness free from influence.

Liberty is voluntary action exercising itself with a thorough knowledge of the matter and without an irresistible motive.

Mind's Nature is activity ; and the proof of its liberty is the original activity, the source of which lies in the mind.

This is the spontaneousness by virtue of which the mind acts of its own accord—there seems to be no reason for putting it completely aside.

IV.

Letter, dated 22 March, from MR. YOUNG to M. ADAM.

If we observe a sufficient number of events produced by the operation of the Human Will—sufficient in number and extent, I mean, to exclude the supposition of mere chance, or local or temporary conditions—these results, I conceive, would be acceptable as the basis for deducing probabilities for future guidance, assuming always, as I have mentioned, that the future circumstances are *pari materie* with the cases in question.

As an example of temporary conditions I might take the following case: the price of wheat is usually brought into relation with the rate of marriage, but, without fully accepting this specific view, it is a matter of experience that a time of low prices is usually a period of an increase in the marriage rate. Hence, I mean that if the events observed were restricted to years of low prices, the probabilities obtained would be quite inapplicable when we regard economic conditions generally.

My conclusion, therefore, is that marriage probabilities are safe data to proceed upon provided the proper conditions (scientific and practical) of collection and application are observed, and I think that such tables for various classes of the population may be prepared, which could be practically depended on for schemes involving the occurrence, or absence, of marriage. But here, again, I point out, referring to my previous letter, that it may even happen that the scheme itself, according to its nature and consequences, may affect generally the future tendency to marry, so that the cases are rendered unsuitable for the application of the probabilities by reason of the very scheme which these probabilities support.

Having said this, I am quite prepared to concur with you that probabilities—that is to say, probabilities in the sense in which we understand the doctrine—relating to the leaving of children under 15 years of age, to the leaving of parents in a condition of want, and similar events, are essentially, as far as I have been able to observe, of the nature of mere guess-work without any practical, or scientific, foundation whatever. This latter defect was that which impressed me the most seriously in considering the German Law of Assurance against Invalidity and Old Age.

When I thus speak, however, I do not, I should add, exclude the possibility of data upon these subjects becoming so extensive,

homogeneous, and consistent as to furnish a suitable foundation for deductions.

It seems to me to be the same thing—when observations dependent upon the exercise of the Will are collected, and show order and regularity—whether we say that the events are such as to afford guidance for the future, or that the Will, producing them, is uniform in its action.

REVIEW.

*Mortality Tables and Frequency-Curves.**

OF the various papers comprised in these two volumes, ranging as they do over a great variety of subjects, it is our intention to deal only with that giving its title to the collection, though there are several that might be expected to appeal specially to the actuary.

The paper in question was originally delivered as a lecture before the Leeds Philosophical and Literary Society (January 1895), and is a popular exposition of some of the results arrived at by the author in his memoir upon "Skew Variation in Homogeneous Material" (*Phil. Trans.*, Vol. 168, p. 313), the second of an important series of papers on the Mathematical Theory of Evolution.

The present paper is described by the author as "an attempt to 'resuscitate the mediæval conception of the relation between Death and Chance (as illustrated in the many 'Dances of Death' for which we are indebted to various artists), and to express it in a more 'modern scientific form.'" In other words, it is an attempt to represent the d_x column of the mortality table, the number of deaths occurring in successive years of age among the survivors of a given number of births, by a series of super-imposed frequency-curves.

Each such frequency-curve may be considered to represent a "chance distribution", defining the latter as "one in accordance with law, and one the nature of which [as a whole, but not that of 'individual cases']† can for all practical purposes be closely predicted."

This definition of a "chance distribution" is perhaps wider than would ordinarily be accepted as convenient, as among the various frequency-curves which Professor Pearson introduces, those represented by the familiar functions

$$\kappa \cdot \frac{n}{n-x} p^x q^{n-x} \text{ and } y_0 e^{-cx^2}$$

(the latter coinciding with the former when n is very large and for comparatively small values of n when $p=q=\frac{1}{2}$) alone represent "chance distributions" in the sense of coinciding with the distribution of the number of occurrences of a given event in a given number of trials, when the probability of the occurrence at each trial is known and fixed.

* *The Chances of Death, and other Studies in Evolution.* By KARL PEARSON, M.A., F.R.S. 2 vols. (Ed. Arnold.)

† Some such qualifying words seem necessary, as otherwise the term "chance distribution" would be extended to series of events such as, say, the number of lunar eclipses in successive years, in which each individual result can be predicted.

For a complete account of the various types of frequency-curves, and the method of fitting a curve to a given set of statistical results*, reference must be made to Professor Pearson's "Memoir on Skew Variation" above mentioned, or to Mr. Yule's recent papers in the *Statistical Society's Journal*, the whole of which ought to be of extreme interest to statisticians. It will be enough here to mention the types of curves dealt with by the author, which can be best presented in a tabular form.

Types of Frequency-Curves.

Character of Curve		Type	EXAMPLES, $y =$
Limited Range	In both directions	Skew . . .	I $y_0 \left(1 + \frac{x}{a_1}\right)^{va_1} \left(1 - \frac{x}{a_2}\right)^{va_2}$; $y_0 \left(\frac{x}{a_1} - 1\right)^{-va_1} \left(1 - \frac{x}{a_2}\right)^{va_2}$
		Symmetrical	II $y_0 \left(1 - \frac{x^2}{a^2}\right)^m$
	In one direction	Skew . . .	III $y_0 \left(1 + \frac{x}{a_1}\right)^{va_1} e^{-vx}$
			Skew . . .
Unlimited Range	In both directions	Symmetrical	V $y_0 \left(1 + \frac{x^2}{a^2}\right)^{-m}$; $y_0 e^{-cx^2}$

The first of these curves, $y = y_0 \left(1 + \frac{x}{a_1}\right)^{va_1} \left(1 - \frac{x}{a_2}\right)^{va_2}$, is obviously a generalized form for Types I, II, III, and V.

The value of x for which either function is a maximum, is termed by the author the "Mode", corresponding to the most probable event.

The average value of x as given by the relation

$$\frac{\int_{a_2}^{a_1} \phi(x) \cdot x \cdot dx}{\int_{a_2}^{a_1} \phi(x) dx}$$

is the "Mean", corresponding to the ordinate containing the centre of gravity of the curve.

The square root of the mean of the squares of the deviations from this last result is the "Standard Deviation."

Finally the want of agreement between the "Mode", *i.e.*, the most probable or most frequent event, and the "Mean", introduces the

* This method, where a single curve only is in question, consists in obtaining expressions in terms of the constants for the 2nd, 3rd, 4th, &c., "moments" of the curve, that is for the sum of the ordinates multiplied by the squares, cubes, &c., of their distance from the "mean", and comparing these expressions with the corresponding numerical values easily obtained from the unadjusted figures. This process is as convenient as it is ingenious, and the only drawback we see, and this is not perhaps important, is that undue weight would seem to be given to the extremes of the curve where the facts are usually very scanty.

conception of Skewness, the degree of which may be measured by the ratio $\text{Skewness} = \frac{\text{Mean} - \text{Mode}}{\text{Standard Deviation}}$.

The life table selected as a standard was that known as the English Life Table No. 4 (males) constructed by Dr. Ogle from the Registrar General's Returns for the ten years 1871-1880, and although the want of accuracy, with respect to the ages, in all statistics of the general population is notorious, and the heterogeneous nature of the data is to some extent a drawback, it is difficult to point out any better table for the purpose required, unless it be the Peerage Table of Messrs. Bailey and Day, where the facts are accurately known and the material homogeneous. Plotting out in a curve the numbers dying at each age out of 1,000 males born, the author finds that, unlike as the whole curve is to a frequency distribution, the old age portion strikingly resembles such a distribution, being very accurately represented by a Skew frequency-curve of Type III; and that when this portion of the mortality is removed, a second curve can be found representing the outstanding mortality for the next portion of the table, &c., until the period of infancy is reached.

The final expression arrived at by Professor Pearson, representing the whole of the death curve, is as follows:—Representing by \bar{d}_x the numbers dying *per annum* at the moment of attaining age x , out of 1,000 male births, so that $\bar{d}_x = l_x \mu_x = l_{x-\frac{1}{2}} - l_{x+\frac{1}{2}}$ nearly.

$$\begin{aligned} \bar{d}_x = & 15.2 \left(1 - \frac{x-71.5}{35} \right)^{7.7525} e^{.2215(x-71.5)} & \text{Old age} & \text{curve.} \\ + & 5.4 e^{-[.05524(x-41.5)]^2} & \text{Middle life} & \text{,,} \\ + & 2.6 e^{-[.09092(x-22.5)]^2} & \text{Youth} & \text{,,} \\ + & 8.5(x-2)^{.3271} e^{-.3271(x-3)} & \text{Childhood} & \text{,,} \\ + & 415.6(x+.75)^{-.5} e^{-.75(x+.75)} & \text{Infancy} & \text{,,} \end{aligned}$$

The constant factor in the curve representing the mortality of childhood is given in Professor Pearson's paper as 9.0. It should, however, in order to reproduce the total of 46.4 deaths assigned to this curve, be 8.5 as above.

The degree of approximation with which these curves represent the mortality table as a whole will be seen by the specimen ages and values, given in the table at the close of this note, which, for convenience in the avoidance of decimals, shows the numbers dying at successive ages out of 10,000 born.

It will be found that the agreement throughout is very close, when it is borne in mind that the author's primary object was not to graduate the English Life Table, but merely to give illustrations of the practical employment of frequency-curves in various classes of statistics. When the mode of construction and materials of the English Life Table are taken into account, it will be admitted that the deviations are probably within what may be termed the errors of observation.

Treating the curves in their order from Old Age downwards, it will be seen either from the above formula (or from the numbers in column 4 in the Table) that the Old Age curve is a Skew-curve,

having its maximum representing 152 deaths per annum out of 10,000 births at age $71\frac{1}{2}$, having a higher limit at age $106\frac{1}{2}$, and no lower limit, but running practically from about age 10 to 100.

The curves for Middle Life and Youth are both of the "normal" type $y = y_0 e^{-cx^2}$, the former with a maximum at age $41\frac{1}{2}$ and a range practically of about forty years on either side, the latter with a maximum at age $22\frac{1}{2}$ and range of about 22 years in either direction.

The curve representing the mortality of Childhood, commences suddenly at age 2, rising at first perpendicularly from the axis of x and rapidly reaching a maximum at age 3, from which point it continuously diminishes, having no upper age limit, but ceasing practically about age 22.

The curve of Infant mortality presents a very curious feature, as it will be seen to start from a point nine months before birth, at which point the rate of mortality is infinitely great. On this head, Professor Pearson says: "Try as I would, no theoretical frequency-curve could be made to fit it (the mortality of infancy left out-standing after subtracting the first four curves of old age, middle life, youth, and childhood) until I had shifted its start some 'nine months before birth.'"

We have thus a certain number of pre-natal deaths, amounting in fact to 605 for every 1,000 born alive. Such a result is theoretically interesting, but we cannot safely assume that the curve representing infant mortality may be carried back, without modification, into the pre-natal period.

On the question as to how far the incidence of special classes of diseases will be found to accord with the character of Professor Pearson's curves, a reference to the registered causes of death* shows that (omitting infantile deaths) the maximum number of deaths from diseases of the Nervous, Circulatory, Respiratory, and Urinary systems occur about age 70, the maximum deaths from diseases of the Digestive System and from Cancer between 60 and 65, and from old age mainly after 75, so that this group agrees well with the position of the maximum in the Old Age curve at age $71\frac{1}{2}$. No special class of disease appears to be represented by the curve of Middle Life mortality with its maximum at age $41\frac{1}{2}$; the deaths from violence, however, show a not very strongly marked maximum about age 40. The curve representing the period of youth, with its maximum at $22\frac{1}{2}$, is not well represented, the greatest number of deaths from Phthisis occurring about age 30, and a relatively unimportant maximum of deaths from Enteric Fever at age 20.

The deaths from Measles, and the smaller number from Scarlet Fever and Diphtheria, which are most numerous in the second, third, and fourth years of life respectively, may be considered to represent the deaths of childhood, this curve having a maximum at age 3. It is difficult, however, to see why this curve should begin abruptly at

* See Supplement to the 55th Annual Rept. of the Registrar General, Part I, page 4. A population table is, of course, not a Life Table, and in the former, assuming the population to be an increasing one, the points of maximum mortality corresponding to the various causes of death would all be shifted towards the younger ages; only slightly where the maximum is strongly marked, but more appreciably so where the maximum is feebly marked.

age 2, a proceeding that may, perhaps, be due to the author's too great respect for the "facts" supposed to underlie the figures of the mortality table.

For the excessive mortality of infancy, several large classes of disease are, of course, accountable.

Upon this aspect of the subject Professor Pearson remarks: "On the whole, it does not appear to be that the causes grouped together in these components—with the possible exception of childhood—refer so much to the special severity of certain diseases, as to the special prevalence during the periods considered of various susceptibilities, relative capacity to resist death—whatever be the form of its attack—pre-disposition peculiar, for example, to the periods covered by old age or middle age mortality."

Professor Pearson's curves naturally suggest comparison with other attempts to represent the mortality curve, such as that of Gompertz, where the number of survivors at age x is given by the formula $l_x = kg^{c^x}$ (which requires at least two changes in the constant c , say, at ages 15 and 55), or that of Makeham's modifications of Gompertz, where $l_x = ks^x g^{c^x}$, or, as in his later development, $ks^x w^{x^2} g^{c^x}$, which, as is known, represent very fairly most standard tables from about age 20 or 25 to the end of life, but which require at least one, usually two, supplementary curves to satisfactorily represent the mortality of the earlier years of life. Professor Pearson's resolution of the mortality table into a series of super-imposed curves has, perhaps, the advantage over Makeham's hypothesis, from a philosophical standpoint, of recognising certain broadly-marked periods of life, which, undoubtedly, have certain special physiological characteristics, and each of which may possibly have its own special law of mortality. On the other hand, the sufficiently close adherence, in general, of Makeham's formula to the observed mortality in adult life—the only period with which the actuary has in most cases to deal—combined with the great saving of labour resulting from the principle of "uniform seniority" embodied in the formula, render the latter, for many purposes, almost indispensable.

A curious point of distinction is, that in Professor Pearson's formula a necessary limit is put to the duration of human life, while according to Makeham's hypothesis a man might live to any age. This distinction, however, is of no practical significance. The probability of living to age 110, for example, computed according to the constants employed in the construction of the Life Table in the Text-Book (Part II), is about one in sixty-thousand millions; so that, if the formula be taken to represent the law of mortality for the race during the past 2,000 years, it is improbable that this age has been attained by any male life since the Christian era.

It need scarcely be said that, although this note has dealt mainly with the Author's application of "frequency-curves" to the complicated case of the mortality table, there are many other possible applications that will at once occur to the actuary—such, for example, as to the statistics of marriage or of issue, the numbers withdrawing in successive years of assurance, the number of superannuations in various years of age or of service, &c., &c. In the majority of such cases we believe it will be found that Professor Pearson has provided us with the means and methods of disentangling the general law

underlying the observed facts from the accidental irregularities inseparable in practice from all statistical data, and, therefore, of predicting with greater certainty than the rough facts enable us to do, the probable results of the operation of such law in the future.

Numbers Dying at the under-mentioned Ages out of 10,000 Births.

Ages last Birth-day	DEATHS ACCORDING TO PROFESSOR KARL PEARSON'S FREQUENCY-CURVES						English Life Table No. 4, Males	(7)-(8) + -		Ages last Birth-day
	Infancy	Child-hood	Youth	Middle Life	Old Age	Total				
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	
0	1,562	...	1	1,563	1,586	... 23	0	
1	535	...	1	536	512	24 ...	1	
2	208	74	1	1	...	284	265	19 ...	2	
3	86	82	1	1	...	170	171	... 1	3	
4	37	70	2	1	...	110	125	... 15	4	
5	16	57	2	1	...	76	73	3 ...	5	
6	7	44	3	2	...	56	57	... 1	6	
7	3	34	4	2	...	43	48	... 5	7	
8	2	26	5	2	...	35	40	... 5	8	
9	1	20	7	2	...	30	33	... 3	9	
10	...	15	8	3	1	27	29	... 2	10	
11	...	11	10	3	1	25	25	11	
12	...	8	11	4	1	24	24	12	
13	...	6	13	5	1	25	24	1 ...	13	
14	...	5	15	6	1	27	24	3 ...	14	
15-19	...	10	105	47	7	169	164	5 ...	15-19	
20-24	...	2	128	90	13	233	229	4 ...	20-24	
25-29	105	148	25	278	271	7 ...	25-29	
30-34	58	210	45	313	311	2 ...	30-34	
35-39	21	256	79	356	358	... 2	35-39	
40-44	5	267	134	406	407	... 1	40-44	
45-49	1	241	215	457	454	3 ...	45-49	
50-54	186	328	514	523	... 9	50-54	
55-59	124	466	590	597	... 7	55-59	
60-64	71	610	681	678	3 ...	60-64	
65-69	35	724	759	751	8 ...	65-69	
70-74	15	754	769	771	... 2	70-74	
75-79	6	661	667	676	... 9	75-79	
80-84	2	461	463	466	... 3	80-84	
85-89	231	231	228	3 ...	85-89	
90-94	71	71	68	3 ...	90-94	
95-99	11	11	11	95-99	
100 and upw'ds	}	1	1	1	100 and upw'ds	
Totals	2,457	464	507	1,731	4,841	10,000	10,000	88 88	Totals	

G. F. H.

ACTUARIAL NOTES.

I.—On the Generalization of Simpson's Rule for Joint-Life Annuities. By J. MAYHEW ALLEN, F.I.A.

THE object of this Note is to investigate the result of substituting for a joint-life annuity on a lives of different ages, an annuity on β lives of equal ages, namely:

$$a_{xyz} \dots \alpha \equiv a_{wuv} \dots \beta$$

or, what is the same thing,

$${}_n p_{xyz} \dots \alpha = {}_n p_{wuv} \dots \beta \quad \dots \quad (1)$$

for all values of n .

We have

$${}_n p_x \cdot {}_n p_y \cdot {}_n p_z \dots \alpha = ({}_n p_w)^\beta$$

$$\therefore {}_n p_w = \{ {}_n p_x \cdot {}_n p_y \cdot {}_n p_z \dots \alpha \}^{\frac{1}{\beta}} \quad \dots \quad (2)$$

and, putting $n=1$ in equation (2)

$$p_w = \{ p_x \cdot p_y \cdot p_z \dots \alpha \}^{\frac{1}{\beta}} \quad \dots \quad (3)$$

Also, since

$${}_n p = p \cdot {}^1 p \cdot {}^2 p \cdot {}^3 p \cdot \dots \cdot {}^{n-1} p,$$

${}_n p$ is seen to be a function of p , and can be represented by $f(p)$.

We thus have from equations (1), (2) and (3),

$$f[\{ p_x \cdot p_y \cdot p_z \dots \alpha \}^{\frac{1}{\beta}}] = \{ f(p_x) \cdot f(p_y) \cdot f(p_z) \dots \alpha \}^{\frac{1}{\beta}} \quad (4)$$

From (4) it follows that, in general, when α is not equal to β ,

$$f(p) = (p)^{\phi \cdot n} \quad \dots \quad (5)$$

but, in the particular case where $\alpha = \beta$,

$$f(p) = \psi(n) \cdot (p)^{\phi \cdot n} \quad \dots \quad (5a)$$

Dealing first with the case where α is not $= \beta$, we have

$${}_n p_x = (p_x)^{\phi \cdot n} \quad \dots \quad (6)$$

or, dividing both sides by p_x ,

$${}_{n-1} p_{x+1} = (p_x)^{\phi \cdot n-1} \quad \dots \quad (7)$$

Now

$${}_{n-1} p_{x+1} = (p_{x+1})^{\phi \cdot n-1}$$

also, by putting $n=2$ in equation (7), we get

$$p_{x+1} = (p_x)^{\phi \cdot 2-1}$$

$$\therefore (p_x)^{\phi(n)-1} = (p_x)^{\{ \phi(2)-1 \} \phi(n-1)}$$

whence

$$\phi(n) = 1 + c\phi(n-1)$$

where $\{ \phi(2)-1 \}$, being independent of n , is represented by the constant factor c .

Hence, since ${}_0 p_x = 1$, we have

$$\phi(0) = 0, \quad \phi(1) = 1, \quad \phi(2) = 1 + c, \quad \phi(3) = 1 + c + c^2, \quad \&c.,$$

and, generally, it follows at once by mathematical induction that

$$\phi(n) = 1 + c + c^2 + \dots + c^{n-1} = \frac{c^n - 1}{c - 1}$$

Thus, putting $x=0$, and $n=x$ in equation (6).

$$l_x = l_0 \left(\frac{l_1}{l_0} \right)^{\frac{c^x - 1}{c - 1}}$$

which transforms into $l_x = kg^{e^x}$

by writing g for $\left(\frac{l_1}{l_0}\right)^{\frac{1}{e-1}}$, and k for $\frac{l_0}{g}$.

This is Gompertz's law of mortality.

Dealing now with the case $\alpha = \beta$, we have

$${}_n p_x = \psi(n) \cdot (p_x)^{\phi n},$$

or, dividing both sides by p_x ,

$${}_{n-1} p_{x+1} = \psi(n) \cdot (p_x)^{\phi n-1} \quad . \quad . \quad . \quad . \quad (8)$$

$$\text{Now,} \quad {}_{n-1} p_{x+1} = \psi(n-1) \cdot (p_{x+1})^{\phi n-1};$$

also, by putting $n=2$ in equation (8),

$$p_{x+1} = \psi(2) \cdot (p_x)^{\phi 2-1}$$

$$\therefore \psi(n) \cdot (p_x)^{\phi n-1} = \psi(n-1) [\psi(2) \cdot (p_x)^{\phi 2-1}]^{\phi n-1} \quad . \quad (9)$$

Similarly,

$$\psi(n) \cdot (p_y)^{\phi n-1} = \psi(n-1) [\psi(2) \cdot (p_y)^{\phi 2-1}]^{\phi n-1} \quad . \quad (10)$$

where y is any other life.

Dividing the corresponding sides of equations (9) and (10) the one by the other, we get

$$\left(\frac{p_x}{p_y}\right)^{\phi n-1} = \left(\frac{p_x}{p_y}\right)^{\{\phi 2-1\} \phi n-1},$$

$$\text{whence} \quad \phi(n) = 1 + \{\phi(2) - 1\} \phi(n-1),$$

and, precisely as in dealing with the previous case, we find that

$$\phi(n) \text{ is of the form } \frac{c^n - 1}{c - 1}.$$

Returning, now, to equation (9), we see that the terms involving p_x cancel each other, and there remains

$$\psi(n) = \psi(n-1) \cdot \{\psi(2)\}^{\phi n-1}$$

$$\text{Now,} \quad 1 = {}_0 p_x = \psi(0) \cdot (p_x)^{\phi 0} = \psi(0) \cdot 1,$$

$$\therefore \psi(0) = 1$$

$$\psi(1) = 1$$

$$\psi(2) = \psi(2)$$

$$\psi(3) = \{\psi(2)\}^{1+\phi(2)}$$

$$\psi(4) = \{\psi(2)\}^{1+\phi(2)+\phi(3)}, \quad \&c.$$

and in general, by mathematical induction, it follows that

$$\begin{aligned}
\psi(n) &= \{\psi(2)\}^{1+\phi(2)+\phi(3)+\dots+\phi(n-1)} \\
&= \{\psi(2)\}^{\frac{c-1}{c-1}+\frac{c^2-1}{c-1}+\frac{c^3-1}{c-1}+\dots+\frac{c^{n-1}-1}{c-1}} \\
&= [\{\psi(2)\}^{\frac{1}{1-c}}]^{n-\frac{c^n-1}{c-1}} \\
&= s^{n-\frac{c^n-1}{c-1}}, \text{ if we represent } \{\psi(2)\}^{\frac{1}{1-c}} \text{ by } s; \text{ we}
\end{aligned}$$

thus have ${}_n p_x = s^{n-\frac{c^n-1}{c-1}} p_x^{\frac{c^n-1}{c-1}}$,

or, putting $x=0$, and $n=x$ in this equation,

$$l_x = l_0 \cdot s^x \cdot \left(\frac{l_1}{s l_0}\right)^{\frac{c^x-1}{c-1}};$$

and this, by writing g for $\left(\frac{l_1}{s l_0}\right)^{\frac{1}{c-1}}$, and k for $\frac{l_0}{g}$, transforms into

$$l_x = k s^x g^{c^x},$$

which is Makeham's law of mortality.

It is thus seen that, if for a joint-life annuity on any number of lives of different ages we can substitute a joint-life annuity on a number of lives of equal ages, then, if the number of substituted lives be not equal to the number of original lives Gompertz's law of mortality holds, but when the number of substituted lives is equal to the number of original lives, Makeham's law of mortality holds.

[The above may be more briefly proved as follows by an extension of the method suggested by Mr. Henderson:

Suppose that it is possible to replace the annuity on any number h of joint lives of different ages by an annuity on any other number k lives of equal ages, it is required to find what is the law of mortality.

The hypothesis requires that ${}_n p_{wxyz\dots k} = {}_n p_{xyz\dots h}$ for all values of n , where $xyz\dots$ are independent, but w is a function of them all.

This may be written $k \log {}_n p_w = \log {}_n p_x + \log {}_n p_y + \dots$. Differentiating with respect to n and changing the sign of both sides of the equation, we have

$$k \mu_{w+n} = \mu_{x+n} + \mu_{y+n} + \dots \quad (i)$$

for all values of n . Differentiating again with respect to n , and remembering that

$$\frac{d\mu_{w+n}}{dn} = \frac{d\mu_{w+n}}{dw}, \quad \frac{d\mu_{x+n}}{dn} = \frac{d\mu_{x+n}}{dx}, \text{ and so on,}$$

$$\text{we have} \quad k \frac{d\mu_{w+n}}{dw} = \frac{d\mu_{x+n}}{dx} + \frac{d\mu_{y+n}}{dy} + \dots \quad (ii)$$

Putting now $n=0$ in equations (i) and (ii), and noticing that in the latter the partial differential coefficients will then become complete, we get

$$k\mu_w = \mu_x + \mu_y + \mu_z + \dots \quad (\text{iii})$$

$$k \frac{d\mu_w}{dw} = \frac{d\mu_x}{dx} + \frac{d\mu_y}{dy} + \frac{d\mu_z}{dz} \quad (\text{iv})$$

Now, remembering that $xyz \dots$ are absolutely independent, and that w is a function of them all, we have, differentiating (iii) and (iv) with respect to x ,

$$\begin{aligned} k \frac{d\mu_w}{dw} \frac{dw}{dx} &= \frac{d\mu_x}{dx} \\ k \frac{d^2\mu_w}{dw^2} \frac{dw}{dx} &= \frac{d^2\mu_x}{dx^2} \\ \therefore \frac{\frac{d^2\mu_x}{dx^2}}{\frac{d\mu_x}{dx}} &= \frac{\frac{d^2\mu_w}{dw^2}}{\frac{d\mu_w}{dw}} \end{aligned}$$

and it is evident that a similar equation can be got for each of the variables $y, z \dots$ on the left hand, with w on the right.

Hence

$$\begin{aligned} \frac{\frac{d^2\mu_x}{dx^2}}{\frac{d\mu_x}{dx}} &= \frac{\frac{d^2\mu_y}{dy^2}}{\frac{d\mu_y}{dy}} = \frac{\frac{d^2\mu_z}{dz^2}}{\frac{d\mu_z}{dz}} = \&c. = \text{a constant,} \end{aligned}$$

since $x, y, z \dots$ are independent.

From this it follows that $\mu_x = A + Bc^x$, and hence, substituting in equation (iii), we have

$$k(\Lambda + Bc^w) = h\Lambda + B(c^x + c^y + \dots)$$

\therefore if h is not equal to k , $\Lambda = 0$.

Hence the following theorems:

- (a) If for h lives of different ages we may substitute one life, or any other number of lives (not h) of equal ages, Gompertz's law must hold.
- (β) If for h lives of different ages we may substitute h lives of equal ages, Makeham's law must hold.

A. L.]

II.—On the Equated Time of Payment. By A. LEVINE, F.I.A.

AT the end of the first chapter of his "Theory of Finance", Mr. George King refers to the equated time of payment, or average due date, of various sums of money due at different times by one

person to another. An approximate value for the equated time of payment is there found, which is given by the rule "multiply each amount by the time to elapse until it will fall due, and divide the sum of the products by the sum of the amounts." It is well known that this approximation favours the debtor, in other words, that it gives too large a value for the equated time of payment. But no proof of this fact appears to have been published, and it is hoped, therefore, that the following may be found of some interest.

Adopting Mr. King's notation, the correct value of the equated time of payment is x , which is given by the equation

$$(\bar{S}_1 + \bar{S}_2 + \bar{S}_3 + \dots) v^x = \bar{S}_1 v^{n_1} + \bar{S}_2 v^{n_2} + \bar{S}_3 v^{n_3} + \dots,$$

while the approximate value is

$$\frac{\bar{S}_1 n_1 + \bar{S}_2 n_2 + \bar{S}_3 n_3 + \&c.}{\bar{S}_1 + \bar{S}_2 + \bar{S}_3 + \&c.},$$

which may be denoted by y . It is required to prove that $x < y$. Let us consider $\bar{S}_1 + \bar{S}_2 + \bar{S}_3 + \dots$ quantities, of which

$$\begin{array}{lll} \bar{S}_1 & \text{are each equal to} & v^{n_1}, \\ \bar{S}_2 & \text{,,} & \text{,,} \quad v^{n_2}, \\ \bar{S}_3 & \text{,,} & \text{,,} \quad v^{n_3}, \text{ and so on.} \end{array}$$

The arithmetic mean of these quantities is evidently

$$\frac{\bar{S}_1 v^{n_1} + \bar{S}_2 v^{n_2} + \bar{S}_3 v^{n_3} + \dots}{\bar{S}_1 + \bar{S}_2 + \dots}, \text{ or } v^x;$$

the geometric mean of the quantities is

$$\{v^{\bar{S}_1 n_1} \times v^{\bar{S}_2 n_2} \times v^{\bar{S}_3 n_3} \times \dots\}^{\frac{1}{\bar{S}_1 + \bar{S}_2 + \bar{S}_3 + \dots}}$$

that is

$$v^{\frac{\bar{S}_1 n_1 + \bar{S}_2 n_2 + \bar{S}_3 n_3 + \dots}{\bar{S}_1 + \bar{S}_2 + \bar{S}_3 + \dots}}, \text{ or } v^y.$$

But the arithmetic mean of any number of positive quantities, which are not all equal, is always greater than their geometric mean.

Hence $v^x > v^y$; from which it follows that, as v is less than unity,

$$x < y$$

or

$$x < \frac{\bar{S}_1 n_1 + \bar{S}_2 n_2 + \bar{S}_3 n_3 + \dots}{\bar{S}_1 + \bar{S}_2 + \bar{S}_3 + \dots}.$$

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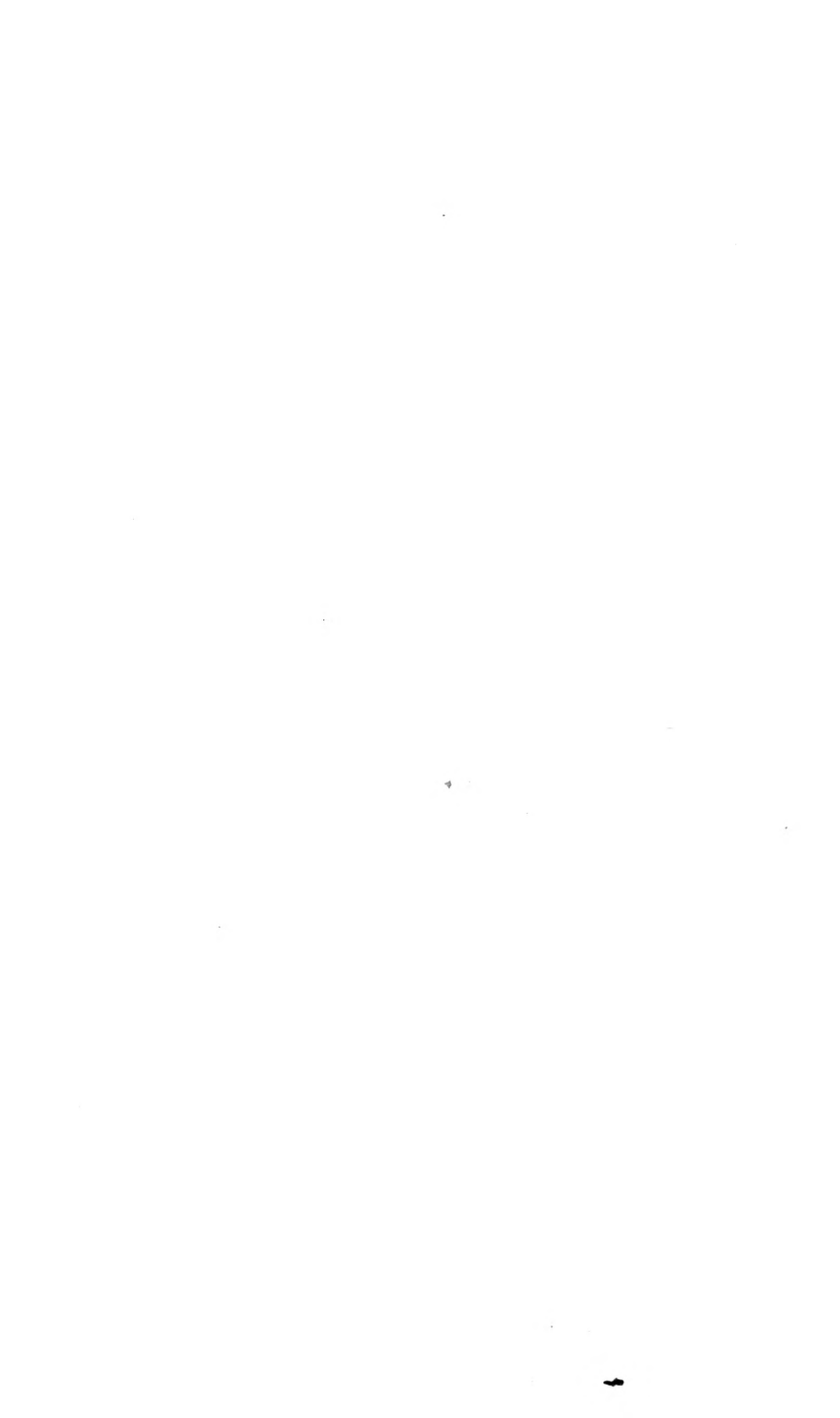
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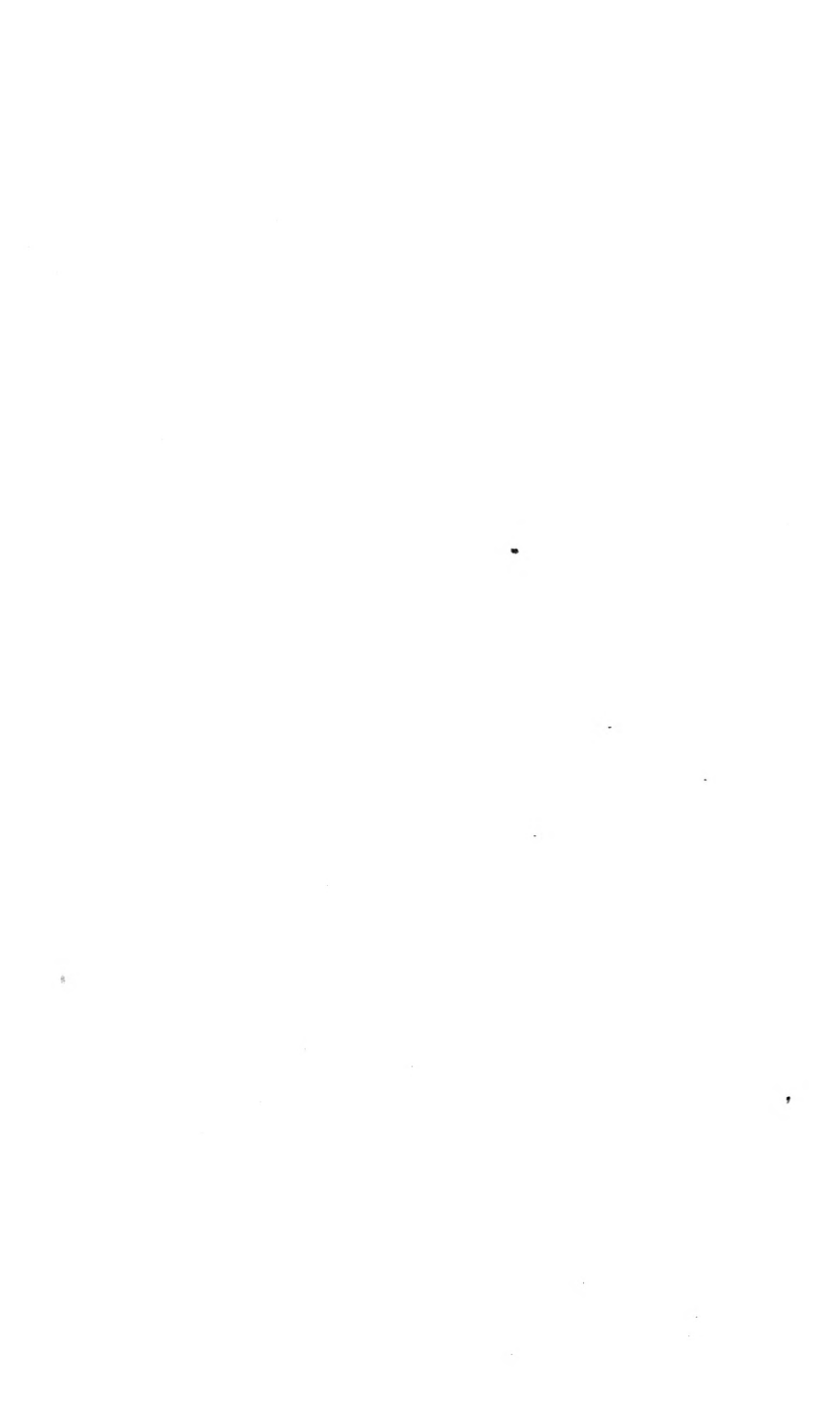
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